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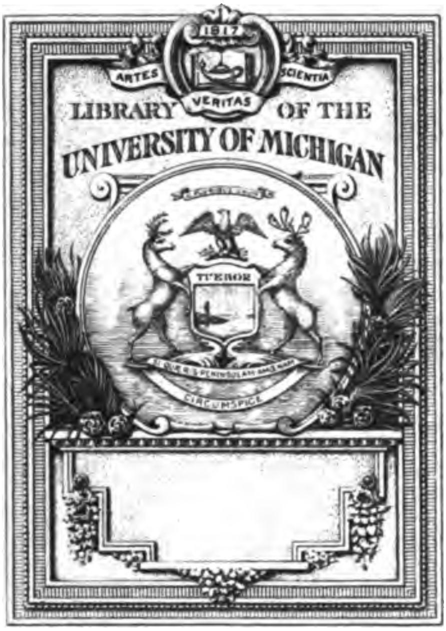
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THE PRACTICE

OF

ENGINEERING FIELD WORK.

THE PRACTICE
OF
ENGINEERING FIELD WORK,

2019

APPLIED TO
Land, Hydrographic, and Hydraulic
SURVEYING AND LEVELLING,
FOR
RAILWAYS, CANALS, HARBOURS, TOWNS' WATER SUPPLY,
RANGING CURVES AND CENTRE LINES,
GAUGING STREAMS, ETC.

INCLUDING THE DESCRIPTION AND USE OF
SURVEYING AND LEVELLING INSTRUMENTS,
AND THE PRACTICAL APPLICATION OF
TRIGONOMETRICAL TABLES.

ILLUSTRATED BY NUMEROUS PLANS AND DIAGRAMS.

By W. DAVIS HASKOLL,
CIVIL ENGINEER, AUTHOR OF "RAILWAY CONSTRUCTION,"
ETC.

LONDON:
ATCHLEY AND CO., 106, GREAT RUSSELL STREET,
BEDFORD SQUARE, (W.C.)
1858.

LONDON:
SAVILL AND EDWARDS, PRINTERS, CHANDOS STREET,
COVENT GARDEN.

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THE PRACTICE

OF

ENGINEERING FIELD WORK.

CHAPTER I.

Land Surveying.—Description of Gunter's Chain.—Chaining and Practical Observations on the Method of Chaining correctly, and with Despatch.—Chaining on Sloping Ground.—Stepping and Inclinometer.—Offsets.—Offset Staff.—Method of Offsetting.—Stations.—Field-book.—First Description of the details of a Survey.—Practical general Remarks.—Straight-Edge.

WHETHER it be proposed to construct such works as railways, or canals, docks, harbours, waterworks, roads, navigation or town improvements, large buildings, or any public or military works, a survey of the lands through or on which such works are to be constructed, is one of the first requirements, and this survey has to be laid down on paper according to certain adopted usage amongst engineers, and in conformity generally with some legal regulations of the country in which the survey is made, and which in England we term "Standing Orders;" it is also often required for large or small districts of land, either for public or private purposes. A good survey is a faithful representation on paper of any particular lands, either as to the whole or part; it is correct in length and breadth, and shows accurately the position of every fixture on the land, natural or artificial, as rivers, streams, roads, fences, buildings, &c. &c.

There is more or less difference observed in making surveys according to the purpose for which the plan is required; for the construction of a railway, a canal, a road, or the improvements of a river, a long narrow slip is required; for a dock, for waterworks, for town improvements, and many other purposes, where as much in width as in length of land may be required, another method has to be followed, though the same principles are carried out; for a maritime survey, where a great portion of the work is what we term "inaccessible," another system has to be adopted;

but they all resolve themselves into one thing, and that is, a correct plan of the lands and areas surveyed; although the methods pursued are different, the instruments are the same, and consist of certain lineal measures for ascertaining length and breadth, and certain instruments for measuring angles, whether the lines containing the angles are actually measured in length, or calculated, or merely laid down on paper according to the direction given by the lines containing the angles. For many military purposes, instead of using a chain, lines are sometimes paced by the military surveyor on foot, or sometimes on horseback; but for civil engineering, all linear measurements are taken with the chain, which is one of the most important of all our instruments, and with the use of which we must be perfectly familiar in all its appliances before we can do anything in engineering field-work, as far as surveying and levelling are concerned, or setting out works. Before proceeding with a description of this instrument and its use, we must observe that we take it for granted that the beginner is acquainted with the ordinary use of a pair of compasses, a drawing pen, a **T** square, and a set square; if unaccustomed to drawing, the best thing he can do is to apply himself first to tracing, and then to copying some drawings; dividing a foot square of paper into square *inches*, and these again into forty or fifty equal parts, horizontally and vertically, is very good practice; so is describing a hundred circles in one of two inches diameter. When he can readily do this, he will handle his compass, pen, and rule with ease and comfort.

The instrument generally used for taking land measurements is known as Gunter's Chain: it is 66 feet in length, and divided into 100 *links*, united each to each by three rings, usually of elliptic form. For very important and delicate purposes, where the sides of triangles are all calculated, a steel chain, jointed like a watch chain, may be used for measuring the base line, and which is always to be stretched to an equal degree; it is often laid in troughs, carefully adjusted horizontally, and allowances are made for extension and contraction from temperature; field engineering, however, but very rarely indeed requires the use of this chain, and we shall at once return to the use of "Gunter's Chain." This being 66 feet long, and divided into 100 links, each link will be 7·92 inches in length, or $\frac{66 \times 12}{100} = 7{\cdot}92$. Every ten links are marked by pieces of brass, shaped something like the fingers of the hand, the first ten being noted by one finger, the second ten by two fingers, the third by three, and the fourth ten, or forty links, by four fingers; fifty, or the centre of the chain, is noted by a round mark. From the other end of the chain, the same notation commences up to the middle of the chain again, so that one finger may note 10 or 90, accordingly as it is read from one end or the other; and in the same manner 20 may be 80, &c. &c. Beginners have therefore to be careful on this point, for very serious errors are often made, even by those who have had a good deal of practice.

An acre is equal to 4 roods, or 10 *square chains*, or 160 perches, or 4840 square yards, or 43,560 square feet, or 100,000 square links, and a length of 10 chains, by a width of 1 chain, is equal to an acre; or 1000 links, multiplied by 100, equal to 100,000, equal to one acre. If, therefore, the contents of a field, or of several fields, or of an entire survey, be found in square links, then dividing by 100,000 will give the acres; but this division is effected by pointing off the five last figures towards the right hand, when the remaining figures will give the acres; multiplying the figures cast off by 4, and again pointing off the five last on the right, will give the roods; and repeating this operation with 40 instead of four, will give perches and decimals, thus :—

$$
\begin{array}{lrl}
\text{Links. Links.} & \text{A.} & \\
850 \times 550 = & 4.67500 & \\
 & 4 & \\
\hline
 & 2.70000 & \\
 & 40 & \\
\hline
 & & \text{A. R. P.} \\
 & 28.00000 & \text{or } 4\text{—}2\text{—}28.00000.
\end{array}
$$

At each end of the chain is a brass handle, by which it is held when in use. With the chain must be provided ten arrows, made of stout wire, about fifteen inches long, pointed at one end for the purpose of driving into the ground, and with a ring at the other end for hanging on the finger.

The chain is used for measuring distances straight from one point to another, and which may be only one or two chains in length, or five or six miles, or more; in measuring a line of five miles, or 400 chains (eighty chains being equal to one statute mile), if the chain be only one inch too long or too short, we shall have an error of 400 inches, or half a chain, and so in proportion as the line measures more or less; hence the necessity for the length of the chain being correct. To make certain of this, a chain's length of 66 feet should always be set out on a level piece of ground, or along the coping of a wall, so as to afford opportunity of testing the chain by this standard as often as necessary. Even with an old chain, which may be supposed to have been well stretched by repeated use, this testing should always be performed once a week; with a new chain oftener, and care should be taken to straighten the links that have got bent. When it is remembered that a chain may be stretched tight some three or four hundred times a day, or more; dragged over rough ground, loaded with wet clods, through hedge and ditch, it will be easily perceived that a considerable elongation may take place; this should be corrected by shortening equally on each side of the 50-mark until the chain is reduced to its proper length, either by shortening two of the links, or taking off a couple of the rings.

The operation of using the chain is performed by two persons, called the "*leader*" and the "*follower*," and in the manner now to be described. The leader starts in the direction required with the ten arrows in his left hand, and one end of the chain in his right, the follower remaining at the starting point, there holding the other chain end.. Having dragged out the length of the chain, the leader holds one of the pins or arrows with the handle of the chain in his right hand, and according to the direction of the follower, sticks it into the ground on the line to be measured. This done, and on a signal being given, the leader leaving the first arrow sticking in the ground, and giving the chain a cast to the right to prevent its dragging against this arrow, proceeds until he has reached another chain's length; whilst the follower walks up to the pin first driven in, at which he holds his end of the chain whilst the second arrow is stuck in the ground by the leader according to the follower's directions. This operation is repeated until the leader has expended his ten arrows, upon driving the last of which into the ground, he calls out "*ten.*" Ten chains have thus been measured, and the Surveyor makes a note of this in the field-book. The leader now waits until the follower walks up to him; the latter makes a mark in the ground where the tenth arrow has been fixed, *both* leader and follower count the arrows to see that the number is complete, and the leader again taking the whole of them in his left hand, proceeds as before; thus a line of any length may be measured on a survey, care being taken never to omit noting in the field-book every tenth chain as soon as it is reached, for which reason it is imperatively necessary that the leader call out "*ten*" every time the ten arrows are expended; a fine, if necessary, will soon teach him.

The accuracy and dispatch with which a survey is carried on depend on no one thing more than on the chaining; for which reason it will be necessary to enter into a few details on this subject.

In chaining from one point to another it is necessary that the chainmen should have at least one distinct object before them. On a line of any considerable length, as a base or sub-base, it will be necessary, in the first instance, that such a line be ranged out; that is, that poles be stuck up at intervals along the line, so that the chainmen may guide their work by such poles, or ranging-rods, which will have to be placed generally on the heights, and in the hollows, through which the line is to pass, so that, in crossing over the undulating ground, there shall be always one, if not two, of the ranging-rods visible in the foreground. If this is not attended to it will often happen that, on a sudden, the chainmen will find themselves without any objects to guide them. Generally, however, a good follower will look out for this, and select some object in a hedge, so as to keep himself in line until the poles become again visible. The straight stem of a tree, a poplar, a chimney, or any other well-defined object, should always be selected as a fore object, when convenient, as this selection, judiciously made, often saves a great deal of time, by preventing the necessity of sending men forward with poles. This is very often the case on lines of forty or fifty chains, and even more. For short lines, some well-defined object in a bush, a gate-post, a stake in a hedge, may be selected; but it is absolutely necessary that the object be well defined, so as to prevent mistakes, which might prove very serious. Occasions often occur when it is useful to leave two or three marks behind, by which the leader may be able to guide himself. A stick cut out of a hedge, with a cleft made in it, so as to hold a bit of white paper, often answers the purpose very well; all these little matters save time, as they avoid the necessity of fetching poles, and then sending forward or backwards to stick them up. These paper marks are called whites, and are often useful in sighting a line through a wood.

In order to insure good chainage—certainly one of the most important things in a survey—many good surveyors prefer "*following*" the chain themselves; undoubtedly a good plan in many respects, but not without corresponding inconveniences, having to chain at the same time that he has to hold his field-book, and his "*offset-staff*," and, at the same time, having to keep his eyes about him, as to where his offsets have to be taken to, leaving stations, and making his notes. A little pains taken with chainmen, and a judicious enforcement of a little discipline, will almost always, in a few days, produce good chainmen. I will now endeavour to point out all that has to be observed to ensure good chainage.

On starting in the required direction, the leader should, instead of going at random, keep his eyes fixed on the forward object; this will ensure his being, at the chain's length, not very far off the line. If he is not at once persistently drilled into this, instead of being only a foot or a few inches off the right line, one way or the other, he will be generally many feet, perhaps yards, out of the true direction. When so far off, he will have to move many times backwards and forwards, as directed by the follower, before he will be on the line, by which time is lost at *every* chain; and when this is repeated two or three hundred times in the course of the day, sometimes a great deal more, it will at once be seen that it absorbs a very considerable portion of the working day. But if, on the other hand, he is at once taught to keep his eyes on the forward object, as he drags the chain along, he will, at the chain's length, almost always find himself within a foot or so of the mark, when a very few directions will bring him correctly on to the line.

When the leader has come to the chain end, he should turn half face about, so as to stand with his right side towards the follower, holding the end of the chain and one arrow in his right hand, and this arrow he should hold at not more than five or six inches from the point end. This will ensure the chain laying flat enough for all practical purposes, if the follower at the same time holds his hand down to the ground, and close to his arrow. The ~~follower~~ Leader should then hold his chain-end and arrow out before him, so that his body be not in the follower's line of sight. He should not at first stick his arrow into the ground, but hold it an inch or so off the ground, until he finds, by the given directions, that he is getting very near the right spot. On the follower's calling out "*mark*," the leader drives the pin into the ground when the chain is laid.

The next thing to be very careful of is, that the leader holds his arrow perfectly upright; until well drilled to his work, he is too apt to lean his arrow slantingly forward, so that correct chainage is not obtained. This generally arises from the follower leaving the leader to *straighten the chain by tightening* or pulling at it; this should not be; the follower should be taught to straighten the chain himself by *gently* casting it one side or the other as may be required, and he should not be allowed to call out to the leader, "*straighten the chain*," or "*tight*," which means the same thing, the leader having already quite strain enough on his arm by having to hold out the chain and pin sideways.

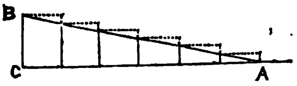
In the next place, with regard to the manner of the follower giving directions to the leader so that the latter may stick his arrow on the right line; it is too much the custom to allow the follower to give signals to the leader, either by waving his hand right or left, or working his head in those directions; nothing can be worse, as it compels the leader, who is stooping and holding out his hand loaded with the arrow and tight chain, to twist his head round to *see* the follower's signals, and consequently to have his eyes off his arrow, so that it may slant backward or forward without his being aware of it; besides it is unnecessarily fatiguing. The follower should be taught to signal to the leader by calling out "*to you,*" or "*from you,*" as the position of the arrow may require; the leader has then a better command over the chain; he can keep his eye on his arrow, and see that he holds it upright.

We must now say a few words as to being *over* cautious or *too* fastidious in getting the arrow *exactly* on the line; in doing this, however, we must particularly request our readers not to imagine that we in any way advocate slovenly surveying, which in engineering matters particularly is absolutely worthless; at the same time, however, a great deal of time may also be uselessly expended in "picking out pins' points." Perhaps one of the best illustrations we can afford as to our meaning, is a chain at the time of being used in chaining across a field freshly ploughed up, or through a field of wheat, clover, turnips, &c.; now, not all the pulling and casting we can give, will lay the chain *perfectly* straight; again, observe the innumerable little undulations in the surface of the ground which are met with at every chain's length, and are but just perceptible; these latter obstructions affect the chainage vertically as much as the former do horizontally, and both occur at every chain nearly throughout a survey. Now the same amount of irregularity may be allowed as regards the *exact* position, mathematically speaking, of the arrow. The result, as affecting the true chainage, will be found for all practical purposes inappreciable, more particularly as regards hedges and ditches, and all ordinary boundaries. When we come to the survey of streets and buildings to a scale of thirty or forty feet to one inch, it is another matter; but of this we shall speak in another place. To return to our present subject, and further to illustrate our meaning, let the reader measure over three or four times a line of thirty or forty chains, at first attending *minutely* to the position of the arrows, and so measuring it twice over; and next let him measure again twice over, neglecting within the bounds of common sense this *minutely* correct position of the arrows, and keep notes of all the four measurements; he will find that the differences are utterly inappreciable, and that there will be, in all probability, as much difference between the first and second, and between the third and fourth measurements, as between the first and third, or second and fourth. Possibly the reader may now inquire the motive of this lengthy explanation of a matter apparently trivial; to this the answer is, to prevent the young surveyor wasting his time, for waste it is, since the result of this *over* caution, against which it is now the object to warn him, cannot, in fact, be in the least degree practically appreciable. If, on the other hand, he chooses to give this advice an extreme meaning, he will have the mortification to find in the end that his survey is of no value for engineering purposes. As we have said before, nothing will better satisfy the mind as to the true sense and extent of our meaning than measuring a line a few times over, and comparing the results.

Never in chaining allow the chain men to cast the chain *over* a hedge, but always through. The only exception that I can think of to this general rule is, where the ground is very abruptly rising on one side or the other of the hedge. Before reaching a hedge, the follower should very carefully note the spot on the line where the chain should pass through the hedge, and, although the brow of the ditch is the boundary of a field or property, and is to be noted as such in the field-book, it is also very advisable to note also the centre of the hedge carefully whilst the chain is held tight; by doing this, and at the same time cutting a little bark with a knife off one or two branches near the ground, we have always the means of finding a well defined spot on the line, both as regards position and measured distance. Such marks as these will remain a long time after "*stations*" proper have been effaced by accident or mischief.

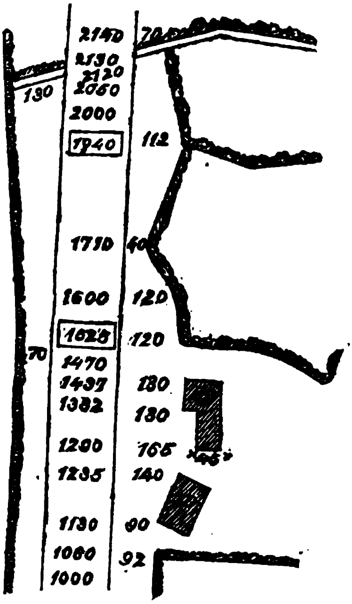
There is another practical hint as regards hedges which we would venture to give the student, and even to some surveyors. It is to avoid cutting down hedges unnecessarily and indiscriminately. We have often seen a gap cut in a hedge wide enough to drive a wagon through—to say the least of this it is sheer waste of time; such acts of mischief enrage the farmers, who consequently turn the surveyor off; and although he may come back again in a day or two, it occasions another loss of time. Again, if it be found necessary to clear the view through a hedge, never cut down any of the main branches, but merely the leaves and twigs which obscure the sight; in a year or so these will all grow again, and the damage disappears; but cut away any of the main branches, and a handsome quickset hedge is disfigured for years afterwards. For every reason, therefore, it is advisable to avoid cutting gaps through hedges, at least as a general rule, though necessarily many exceptions will occur.

In going up or down sloping or "sideling" ground, it will be

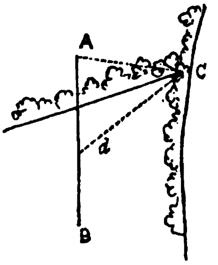


observed that if we measure the *inclined* length we do not obtain the *horizontal* length, which is that which we require for laying down the plan of the survey on paper; for instance, in the annexed figure, A B is the *inclined* length measured along the sloping ground; but A C is the *horizontal* length required for laying down the plan on paper; for many ordinary purposes, and where the inclination is not great, we may obtain the horizontal measurement by holding up the chain at one end, whilst the other is held close down to the ground, as shown in the dotted lines, taking care as much as possible to prevent the chain from bagging, and also that the arrow is placed vertically under the end of the chain which is held up off the ground. When the inclination is sufficiently great for the vertical height at which one end of the chain is held from the ground to amount to three or four feet, and particularly if there are several chains of such ground to measure over, it is well to observe further precautions. All the chainmen should be provided with five or six feet of line, and something in the shape of a plummet, even if it be only a stone at the end of the line. By holding this in his hand instead of the arrow, the leader ascertains in sloping ground where the arrow should be placed, and the follower by the same means, supposing he has to keep *his* end of the chain up-raised by keeping his plumb near the arrow, is enabled to hold his chain-handle exactly over it. By these means the horizontal base of very steep inclinations may be obtained, even if each chain's length has to be measured by the above means in several steps, which is the reason of the technical expression "stepping," which is applied to this method of chaining; and where it does not exceed two or three chains, and these precautions are carefully resorted to, the work will come out very well. When, however, there is any length of sloping ground, and the inclination is considerable, recourse should be had to an "inclinometer," which is an instrument for measuring the angles of elevation and depression, and ascertaining the corrections to be made in order to obtain the true *horizontal* distance. Instruments of this kind are to be obtained from the makers of mathematical instruments. At the end of this volume the reader will find a plate, entitled "inclinometer," illustrating the instrument the author has been in the habit of using, and which is very easily manufactured. At one end, through the brass rim enclosing the semicircle, is an opening of about a quarter of an inch in diameter, with a fine wire stretched horizontally across, coinciding with the diameter A B of the semicircle; at the other end, which is the *eye end*, is a small aperture of about one-twentieth of an inch in diameter, the centre of which is also made to coincide very exactly with the diameter, and therefore also with the wire above-mentioned; this gives the line of sight, and the pendulum points to the degree and allowance in links to be made at each chain's length. Where, for any length, the ascent or descent is regular throughout, it will be sufficient to send a chainman forward to stand on the line, having previously marked off on your offset-staff the height of your eye from the ground. The man starts forward with this offset-staff, and holds it at the required distance, placing his hand on the above-mentioned mark, so that you may ascertain the point to which you are to direct your sight. Immediately you have done this the pendulum points to the allowance to be made at each chain's length, which your assistant carefully observes immediately the pendulum is at rest. This allowance you may as well make on the spot, by drawing the chain *forward* a certain distance equal to the allowance pointed out by the pendulum on the instrument. Suppose, for instance, that the allowance pointed out be six links, then for the chain to measure one chain *horizontal* along the *inclined* ground, you must lay your chain on the ground to its full length, and place the arrow at six links beyond the end of it; and this at every chain. To make this more intelligible, let the reader, as soon as he understands the use of scales, lay down an horizontal line and an inclined line, the two forming an angle of twenty degrees. To a large scale lay off five or six chains along the *horizontal* line, and he will find the inclined chain to measure 106 links. This drawing of the chain *forward* will have to be done going either up or down the side of a hill. This instrument the reader may have very cheaply manufactured, by making a tracing from the plate, pasting it on a bit of drawing-paper, getting a semicircular box made about half an inch deep with the apertures above-mentioned, and with a glass face to the box to prevent injury and also the effect of the wind on the pendulum when the instrument is in use. Messrs. Elliott now manufacture a most complete little card for this purpose.

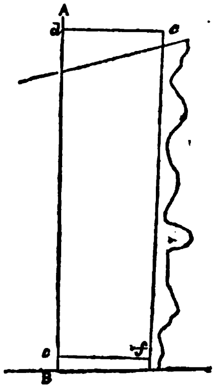
As yet we have only spoken of chaining as regards measurement, without saying anything of the observations that have to be made as the work proceeds, consisting of notes as to where the chain intersects hedges, fences, brooks, &c., &c., on the line, and of such objects, or any others by the side of it, as we go along. These observations to objects by the *side of the line* are made by what is called "taking offsets." They are simply short measurements taken at right angles to the chain; the exact chainage at which these offsets are taken, and their lengths, being all carefully noted. The annexed figure will best explain the nature of offsets, how, where, and for what purpose they are taken, and also how they are afterwards laid down on paper. This figure is a short extract from the field-book, in which you read *from the bottom upwards towards the top*. The extract commences at 1000 links, or 10 chains, showing that the whole number of arrows has changed hands once; at 1060 an offset is taken; it is taken at 1060, because that is the link at which a line at right angles to the chain will intersect the corner of the two fences; the length of this offset measures 92 links, and is entered as such; 1130 again is the chainage from which a perpendicular to the chain will fall on the corner of a house; the same at 1235; at 1525 a similar offset is taken to the corner of a fence; a square has been drawn round this chainage, to show that it has been marked as a "*station;*" that is, a point from or to which another line hereafter will be measured; the same at 1940, from which an offset is taken not only to a bend in the fence, but also to another fence, which there runs into it; at 2000 another change of arrows; at 2050 there is an offset to a junction of fences; at 2120 the chain cuts the brow of a ditch, and at 2130 the quick; the figures 70, 45, and 80 in the buildings note their widths. In taking the boundaries of fields which are surrounded by hedge and ditch, it is the edge of the ditch which is the true boundary, and to which therefore the offsets are to be taken, or an allowance for the width of the ditch should be made in the field-book; six links is often given as the width of the ditch. Taking the offsets to the bends and irregularities of a fence is a very simple matter, but taking offsets to the junctions of fences requires some care, because it is the intersection of the edges of the ditches which has to be noted, and it requires some caution in finding on the chain the exact point from which a *perpendicular* will



fall on the intersection of the edges of the ditch. By neglecting to pay due attention to this, a boundary between two fields may be placed some eight or ten links wrong; in a working, or "permanent" survey, as it is called, this should be carefully attended to.



As regards the lengths of offsets, that is, the greatest distance from a chain line to which offsets may be taken; where merely the bends in a fence have to be determined, a length of offset of two or three chains may very well be taken without further precaution than fixing the perpendicular by the eye, and measuring its length; supposing, for instance, that in chaining through a field, a fence running by the side of it is at about from one to three chains from the chain line; this distance is not at all too great for the correct fixing of the position of the offsets to the bends in the fence; and the lengths of such offsets may be very accurately measured. But where the chain line cuts a fence obliquely, and there is another fence running into it, which is more or less parallel to the chain line, a little more caution is requisite to fix correctly the boundaries; the annexed cut will explain our meaning to the young practitioner; let A B be the chain line; it is required to fix the point of intersection of the ditches at C, which is about two chains off; instead of fixing the position of C merely by a right-angled offset, it will be found better to measure from A to C, and then laying out another chain, measure from *d* to C, and note these dimensions in the field-book, marking in the dotted lines to explain how this has been done. It will be found that there is no loss of time in doing this, that the point is more accurately fixed, and consequently more accurately laid down on paper by means of arcs from A and *d* with the lengths A C and *d* C intersecting at C, the point required. The same observations will hold good as regards the fixing of the positions by offsetting of the corners of buildings; let it be understood that we are now speaking of surveys to be plotted or laid down on paper to a scale of from six to three chains to the inch; where, however, in consequence of the great number of irregularities or bends in a fence, it would be necessary to take a correspondingly great number of offsets of an average of two chains length, it will be found much shorter and more correct to lay a couple of perpendiculars towards the fence and join them by a line parallel or thereabouts to the fence, and so close that the offsets will only be about half a chain long, or even less, as shown in the annexed figure; where A B is the main chain line, *d c* and *e f* the two short perpendiculars, and *c f* the line from which offsets to the fence are to be taken; a little observation will show that this will be the shortest method of proceeding in such a case, and in every way better and more correct than the number of long offsets which would have to be taken from the main line.



There are two instruments in use for measuring offsets; the tape, and the offset staff; the first requires no description, and in using it, we have only to note the spot *to which* we are going to offset, and to find the link on the chain from which the perpendicular is to be measured; this is ascertained by the eye, and very correctly after a little practice. The principal objections are, that it requires two persons to attend to it, that even with care it very quickly frays out, and that in dirty weather, more particularly if much required, it soon gets so dirty as to become quite illegible; it is more desirable to use amongst buildings than in the field. The surveyor, however, should always have one in his pocket. The offset staff which is generally used by surveyors is the tenth of a chain in length, equal therefore to 10 links, or 6 feet 7¼ inches nearly. It may be of hickory, ash, or red pine, requiring to be made of tough springy stuff, as it has a good deal of rough usage to go through; in the centre it should be about one inch and a quarter, or one inch and a half in diameter, round, and slightly tapering towards the ends, which should be about three-quarters of an inch in diameter; one end should be armed with an *obtuse* pointed iron ferrule, with a steel point for sticking in the ground; at the other end should be another ferrule not pointed, with a strong hook fastened to the side, and wide enough to receive the brass handle of the chain; with this hook the offset staff is extremely useful for dragging the chain through a hedge; the links may be marked upon the offset staff by drawing rings round it with a red-hot wire; the centre may be marked with a V, and the other numbers read off by the eye. It is used in the following manner; having noted the spot *to which* the offset is to be taken, stand sideways along the chain with the rod in your right hand and between your legs, and pointing to the spot to which you are going to offset; you will soon see, after a little practice, when it is at right angles to the chain, which will give you the chainage to enter in your field-book; this done, lay the rod pointing to the spot you are offsetting to, and keeping this point in your eye, turn your rod round end for end over and over again along the ground, until you reach the other end of your offset, counting the length as you go along, each length of the rod being 10 links; if you are at all active you will, after a time, do this at a running pace, but of course for accuracy the staff must come flat to the ground each time it is turned over. We may now conclude this portion of our subject with the following hint; that it is unnecessary to offset to fences or other objects one or two chains off, if these have to be picked up upon other chain lines to be set out hereafter.

If the reader has had no experience whatever in surveying, and is anxious to learn, we would now advise him to take a chain, arrows, and an offset staff, go to some quiet place in the fields, hire some youth to drag his chain, lay out a line of some half-mile in length—taking a tree, for instance, as a distant object, being careful that there are hedges along the side of the line to offset to—and then and there put the advice above given into practice; measure this line half-a-dozen times over, taking the offsets every time, and compare the results. With a little care in practising all this he will, in two or three days, be able to chain and take offsets. Writing off the chainage will perhaps puzzle him a little at first, but he will quickly overcome this; for instance, suppose his first offset comes upon the first half chain, then it will read 50 in the field-book, being 50 links; let another come upon 6 chains and 47 links, when he has 6 arrows in his hand, and the chainage will read 647; after the first change, let the cutting of a fence come at 3 chains and 83 links, then it will read 1383; if this had been after the seventh change, then it would have read 7383; again, suppose it to be at 67 links beyond the tenth change of arrows, then it would read 10067; or 5 chains and 79 links after the twentieth change, then it would read 20579, and so on; hence the necessity of carefully entering each time a change of arrows occurs, as it denotes that 1000 links, or 10 chains, have been expended; if this be nēglected, you are very likely to make a 10 chains' mistake. Also with regard to the finger-marks on the chain, remember that 3 or 4 fingers on one side of the round 50 mark in the centre, read 30 and 40 links, but on the other side of the round 50 mark, they read 60 and 70, being respectively 10 and 20 links beyond the 50, and 30 and 40 short of the 100. This must not appear fastidious, for such mistakes do occur even to practised surveyors from oversight and occasional hurry.

We will now say a few words upon the subject of "*stations.*" In making a survey, a line is chained from end to end to ascertain the measure of the total length, the distances at which boundaries are apart, as also the different lengths at which offsets are taken to objects near the line, and more or less parallel to it; and if a survey is only to be four or five chains wide, the whole of it may, in many cases, be obtained upon that one chain line; but instead of this, it is seldom less than twenty chains wide for engineering purposes, except amongst buildings, and often as broad as long. Hence the necessity of other lines to pick up the features over the breadth of country under survey, and all these lines must be connected and tied together. The points at which they start out of, into, or through each other, or where they begin, and where they end, are called "*stations*." They are marked as such in the field-book, for reference during the survey, and when the work is being laid down on the paper; and they are also marked on the ground in order that they may be found when wanted. In the field-book this is very easily done by drawing a line above and below the figures denoting the chainage at which the station is made. On the ground it is sometimes more troublesome. In grass lands it is easy to cut a triangular mark, or a crow's-foot, just large enough to catch the eye, when revisiting the spot to pick up such station; but in a good deal of arable land such a mark would soon be lost sight of; in such cases, the best thing to be done is to drive in a picket, cut out of a hedge, of which the chainmen should have a few in their pockets, or in bags slung over their backs; one of these can be driven into the ground at the spot required with the bill-hook, which they have also to carry about with them. When, however, a station of this kind comes anywhere near a hedge which the chain line is intersecting, we often prefer marking the quick in it, as a false station, and chain back or forwards from it, when the station is required at a future time; we are then certain that our mark will not be lost or destroyed.

When, however, circumstances are such that the station can be conveniently marked it should always be done, taking first due care to look about you to see how a line running through it, right and left if necessary, will lay to pick up the work that will be by the side of it. In doing this, keep your eyes also open to any opportunity there may be of laying your station so that you may be able to take any natural object for the line to be chained upon instead of having to send forward to stick up a pole. It can never be too carefully or constantly kept in mind, that by attending to all these things much valuable time is often saved. If your fences, &c., lay so that you are likely to have two or three lines in different directions from about the same place, endeavour to place your station so that it may answer for all. Avoid putting a station upon odd links, but put it at a ten, or at least a five if possible. When we come to the subject of Chain Surveying, we shall have an opportunity of saying a few words more on this subject.

The "*field-book*" is a register in which all observations and notes connected with a survey are duly made, and we might almost say that the number of forms for keeping this field-book is as numerous as the number of surveyors; the great desiderata, however, are simplicity, and easy reference throughout its pages; it ought to be so very intelligible, that any person acquainted with surveying can make the plot of the survey from end to end without any uncertainty or difficulty, and although this may at first appear difficult with three or four hundred pages of notes, a good system makes it perfectly simple and easy. The form of field-book given in these pages is one we have been accustomed to use for upwards of twenty years, on every description of survey, and we have never seen occasion to alter it; we have therefore reason to believe it one of the very best—it is convenient in the field, intelligible in the office, and of so easy a reference, that there is no difficulty in plotting a survey from it years even after such survey has been made. This field-book is about four and a-half inches wide, seven or eight long, and opening lengthways, and should be paged from end to end. Down the centre of each page, lengthways, a column about half an inch wide should be ruled for entering the chainage, the offsets being registered right and left of the said lines as they occur.

With the illustration of this field-book, given with Figure 1, it is hoped that a few words will suffice to make it intelligible. For easier and more accurate reference, the chain lines are all numbered, as well as the pages, as L. 1, meaning Line No. 1. The reason for this is, that, in each long double page, there may sometimes be half-a-dozen chain lines, when there are not many offsets to take on them; and it may so happen that two stations, on different lines, may be registered by the same figures; referring, therefore, to the number of the page would not be sufficient, and reference must also be made to the number of the line on which the station occurs. Let the reader now refer to L. 1 in the field-book, and to the plan shown at Figure 1, where L. 1 is shown by the letters A B; and note particularly that, in this field-book, the pages all commence at the bottom of the page, by which means you read onwards in the same manner you are chaining. After taking a cursory view of the ground to be surveyed, this line was adopted as the base of operations. We commenced at A, by the trunk of an old oak tree, and there being a tall chimney about a mile off, in the direction required, we chained for it instead of sending on a pole. Nothing occurs until we come to the third chain, on which, at 40 links, a station is made for running a line to take the curved fence on the left-hand side. The follower having two arrows in hand, this station is registered as 240. On the fourth chain we intersect a fence, the brow of the ditch at 340, and the quick at 348; the direction at which the chain passes by these being sketched in, there is no further note to make until the first ten arrows are expended, which is duly registered. The next observation to make is the intersection of ditch and fence at 1395 and 1405; we have now some offsets to take, and to sketch in the relative position of the fence with the chain line. To do this stand on your chain line, *facing your fore object*, and sketch in the fence as it appears receding from or approaching the chain, and never sketch in more than three or four chains at a time, and as you proceed, or you will find that very often you will be obliged to have recourse to your Indian-rubber to efface that which on close inspection proves to be different to what it appeared seven or eight chains off; look out carefully for the chainage, from which a perpendicular will fall to meet intersections of fences. The habit of accurately sketching in the position and appearance of objects, as they occur along the side of the line, is very valuable as affording material assistance in laying down the plot with the help of the registered lengths of chainage and offsets; it is only to be acquired by practice. No further observations are now required as to this line until we reach the end of it at 5060, from which point it is observed that two lines can be struck out right and left for taking up the fences on both sides. We now come to line 2. This commences with the following sign ↳, which means turn to the right from the last line; the starting point of this second line is 5060 on the last line, and the last station of that line; it is therefore entered as "From 5060 last;" now observe that this sign and the figures of the station give the starting point and the direction taken, this information being for afterwards plotting the work; were the sign left out we should have to trust to memory as to whether we turned to the right or the left. Having chained this line, and taken the offsets, we come to the end of it at 2300 as a station; this point is fixed upon because, standing on the chain, we see that we shall intersect from it line 1 at station 2845, left for this particular purpose, and continuing the straight line on through this station, the end of line 3 at chainage 3500 is a fixed point; its direction being determined by the starting point and the station it passes through; and this direction has been so fixed because it gives opportunity within good offset distance of picking up the fencing on the right and left of it. Similar remarks are made at its commencement as for the last line; still turn to the right, and from the *last* station. Had it been any other station on that line it would have been from station so and so "*last line.*"

As regards the fourth line, we have only given the commencement of it as ↰, meaning turn to the left, instead of to the right as heretofore. Let us now suppose that we are beginning another line from A, but instead of going in the direction A B, that we are chaining to the right, then the sign will be ↳. A few words more will conclude this tedious, but very necessary explanation to the reader who is endeavouring to acquire the art of land surveying unassisted. In chaining along a road always chain in the road itself instead of in the fields by the side of it; in the latter case you would have to offset to the one fence by crossing over the other, which is always very inconvenient; you have besides no opportunity of observing many little details which become concealed from view; such as waste by the roadside, gates, pools, bends, and breaks in the opposite fence, all of which should be shown on a good survey, particularly if the plot is to be to anything like a large scale; always offset also both sides of a brook; buildings require particular care with regard to offsets; where in an ordinary survey these are in numerous detached blocks, it is by no means a bad plan, after having surveyed the fences immediately bounding each block of buildings, and one or two of the principal buildings also, to plot what you have actually surveyed of these, and make a tracing of the work; then at the first convenient opportunity, with a small pocket-scale and a tape, fill in the details on the spot on the tracing; remarkably accurate work may be thus produced and very easily, for the slightest omission or error as to the shape of the buildings will strike the eye when the work is thus examined on the ground.

A wall very often crosses a chain line, and of course totally impedes the view; it becomes necessary to find on the other side of the wall the exact spot where the line crosses; by means of a plumb-line we can transfer this spot, first to the top of the wall, and then dropping the line to the other side obtain what we require.

We have now made every observation we can think of on this subject, and strongly recommend the reader to practise all this until he finds himself familiar with this portion of the work; then, and not till then, it will be time for him to study the laying out of a survey, by chain alone, or with the additional help of instruments, for which he will find instructions in the following pages.

Supposing the advice above given to be followed out, the next thing the reader will require will be the means of laying down his straight lines, his distances, and offsets on paper. To do this a straight-edge, a scale, needle-point, and pencil are required.

A straight-edge is a long flat rule, the sides of which are as perfectly straight as it is possible to make them. This instrument is required to lay down the chain lines on paper, and since we are as careful as possible on the ground to keep this chain line straight, it is equally requisite to do so on paper. Land surveys are most usually plotted to a scale of three, four, five, or six chains to one inch; that is to say, that one inch on paper represents one or the other of those numbers of chains on the ground. Take five chains, for instance; a chain being equal to sixty-six feet, and there being five chains to the inch, the latter will represent three hundred and thirty feet; one foot, therefore, will be represented on paper by the 330th part of one inch. The reader will now perceive the importance of a straight-edge in laying down a long chain line on paper, perhaps two or three miles, and therefore, also, the importance of the instrument with which it is to be done; and we must request of him to understand that there is nothing fastidious about the caution here given, as he will find to his cost when neglected; on the other hand, we must also request of him not to be over cautious in carrying this advice into practice, or he will positively waste much valuable time in endeavouring to obtain that which has no practical results. The following circumstance will illustrate how easy it is to fall into either extreme. Many years ago the writer was assistant to a gentleman who was engaged on a very extensive survey on which one base line measured upwards of seven miles; a *straight-edge* (?) was purposely made for laying down this base, which was on mounted paper, and strained on a large board. When we came to fill in the survey, a great number of lines were run from, to or through this base, and on plotting the work we were very much annoyed and surprised, for great pains had been taken, to find that many of these lines plotted too short or too long; many of them were chained over again without finding any defects to account for the cause of our annoyance. At last the guilty one was suspected; a long piece of fine strong silk was procured, slightly waxed, and strained from end to end of the base, which was then discovered to be in many places as much as ten and fifteen links out of the straight line, and accounted for apparent errors in the field-work; on a mathematical examination of the straight-edge, it was found to be no straight-edge at all, and a great deal of work had to be done over again; the rule was sent to a first-rate instrument maker to set right, and came back a straight-edge. Unfortunately our mishap made us over fastidious, and a very great deal of time was wasted in straightening that which was not straight in fancy only. To ascertain the correctness of a straight-edge, two or three of them are required; two of these should be placed edge to edge, and slided along each other, whilst held between the eye and the light; if the edges exclude the light in some places, and admit it at others, then the edges are not straight, and require correcting; but they may happen to be equally curved, when, although they would fit close, still they would not be straight; then try the other side of the edge, or apply the side of a third one, and if all now be satisfactory, it may be said that they are straight, and should be prized accordingly. But even with a straight-edge it is not so easy as at first sight appears, to draw a perfectly straight line; for instance, in joining two distant points together by means of a straight line, the edge of the rule has to be placed so as to just pass by the side of the two points; the pencil or pen must then be carried from point to point, close along the straight-edge during the whole of its transit, for if it leave the side here and there, then the line is not straight, and even the inclination of the pencil or pen will do this; you must therefore be careful to hold the pencil perfectly steady the whole way; having drawn the line, you may test it by turning the straight-edge end for end.

Chain scales are made of ivory or boxwood, usually six or twelve inches long. The principal divisions are so many to the inch, according to the scale adopted, and are numbered both ways along the side of the scale; these are subdivided into parts of ten links each, which in plotting are subdivided by the eye; when, therefore, one of these is laid along a chain line on paper, the chainage or the different measurements are easily pricked off with a needle-point; these distances being marked along the chain line as they occur, the intersections of fences are laid down, and the offsets plotted right and left at right angles, by means of short two-inch scales, which are called offset scales. An examination of a chain scale bought at a maker's, will easily explain this; a beginner had better begin plotting his work to a large scale, that is, two or three chains to one inch, and always remember that the smaller the scale the more carefully it should be plotted, as in this case, apparently a very small error would be a considerable distance on the ground.

In a scale of 3 chains to the inch, every inch will be divided into three equal parts, each representing one chain, and on a scale 12 inches long, there will be 36 chain divisions; now each of these being subdivided into ten, the same as a chain, it follows that each *subdivision* is equal to 10 links; in a 4 chain scale the inch is divided into four, and again each division into 10, and such a scale a foot long will contain 48 chains, and so on with a 5, 6, or a 10 chain scale. Suppose, now, that from such a scale you have to mark off the following distances in chains, 600, 740, 1257; lay your scale along the pencil line with 0 at the beginning; with a needle-point prick off 600 at the 6th *main division marked* 6; for 740, it will be 4 subdivisions beyond the main division marked 7, four subdivisions being equal to 40 links; for 1257, it will be, first 12 main divisions and 5 subdivisions, plus 7 links, being a little less than a third short of the sixth subdivision. In the same manner these measurements may be taken off with a pair of compasses having the pencil leg at one end, and arcs of circles described from one end and the other of a chain line already laid down; lines drawn from the end of this line to the intersection will give two more chain lines.

CHAPTER II.

Chain Surveying.—Triangulation.—Systems to be pursued under varying circumstances.—Setting out Base Lines.—Poles and Ranging Rods.—Difficulties to be overcome.—Perambulating the Boundaries.—General Bearings of Bases of Operation and Conditions that will alter Systems of Triangulation.—Suggestions as to the Study of Land Surveying.

CHAIN surveying, that is, with the chain only, is that to which the beginner should at first confine himself. When with this he can correctly and easily make a survey of a hundred acres or so, and produce a correct plan of his work, when he no longer finds any difficulty in chaining correctly, in reading the chainage, keeping his field-book, taking the offsets (however numerous), laying down the chain lines and offsets, then it will be time to apply himself to the use of instruments and extensive surveys. In this system of surveying we are obliged to divide the survey into a series of triangles, and again dividing and subdividing these triangles until the work is filled in; as our only means of laying down the chain lines is with three given lines to form a triangle. Some of our best plans have been produced from field-work done with the chain only, and yet circumstances occasionally occur when it is difficult on extensive surveys to produce satisfactory work by this means alone. In the course of this chapter we shall have occasion to point some of these out.

Whether the lands to be surveyed consist of one field only, or of half-a-dozen, or of one or two thousand acres, the system to be followed out is the same; the area must be divided into or inclosed by triangles, or both; the angles as subtended by the sides must be checked by tie-lines, and all the lines must be laid out to the greatest advantage, so as to secure as few stations as possible; triangles as nearly equilateral as may be, and consistently with these two conditions, the offsets as short as possible, practically speaking. It is, however, but seldom we can secure equilateral triangles, and we must be content with having as few very acute or very obtuse angles as we can manage according as the work lays. In Fig. 1, the exterior boundaries might be the enclosures of one field only, instead of a few fields, or they might include a considerable estate. We should still have the same process to go through, or the same system to adopt, for one field as for the two latter cases, only that we should have fewer interior lines for taking the offsets to the fences, or filling in. With *any* three given lines a triangle may be described, and therefore any triangle B D C might be laid down, though neither of the lines D B, B C, C D, had been correctly measured. The only check upon this would be to measure a *tie-line*, from either of the angles to the subtending side; but if the straight line, C D, be produced and measured on to E, then the measurement of E F will also be a check; for if D C or B C be not correctly laid down, E F will not measure on the plan as it is actually on the ground. Also in the triangle A G H, the accuracy of the three sides will be tested by the tie-line G I, as also by the line J K L, which at K and L, stations fixed on the lines C A and G H, must measure on the plan the given length K L on the ground between them, and J K L must be in one straight line on the plot as on the ground. To save time, the surveyor has to lay out these check or tie-lines so as to be able to take up offsets at the same time that he checks the chainage of his triangles; and to do this practice and care are required.

It is not possible to give the reader instructions as to the different systems of triangulation he may have to adopt according to the outline forms of the districts he may have to survey, as he may never perhaps meet two alike. The five figures, 2, 3, 4, 5, and 6, will enable him to form some idea of the different forms that may arise, but in all of which the same principle is carried out. In the first figure, A B, is a tie-line; so is D E, being C D produced, which also checks the measurement of C F. In the second figure it will be seen that A B being produced to C, the whole connexion of the chain lines is checked by the intersection at D, and by C D and E F. In the third figure we have the base A B, and first the triangle A C D, which is checked by the prolongation of D C to E and the measurement of E A. The small triangle B F H being thrown up, it is checked by the measurement of F D, and F B being produced to G, by the measurement of G E. An infinity of such examples might be given, but after a little consideration the reader can lay them down for himself. The subject is deserving of study as one of importance to the engineer and surveyor, for by the production of lines, and by visual as well as by measured intersections, maps may not only be checked, but the position of distant and inaccessible objects may be often laid down on paper with such accuracy as to form a very correct map.

Although in the above figures we have by the dotted lines laid down a system of triangulation suitable to each under certain conditions, it is to be understood that it is perfectly possible that, in consequence of other considerations, the triangulations might have to be laid out in a totally different manner; for instance, it would be by no means desirable that the intersections D or G should fall in the midst of a wood or village; again, a range of hills might so run across that it would be extremely difficult and inadvisable to sight the bases by the eye alone, as we are now working, from low to high ground, and down to low ground again, unless there happened to intervene a considerable space of table-land, so as to be able to make sure of the direction of the line along the high ground. If this were not the case, we should have to lay down a different system of triangles, and probably make one base run along the range of hills, and work right and left of it. It is only necessary to mention all these possible and continually arising contingencies to an intelligent mind to show the necessity of a general study of a locality, and the consequent advantages and disadvantages of assuming one series of triangles instead of another.

We must now say a few words as to the means of sighting a long line, for unless this be carefully done, it is better to make the survey in smaller triangles one after another, a method, however, not to be advocated.

The poles or ranging rods used on surveys should be about about eight or nine feet long, perfectly straight,* and cut from well seasoned stuff; about one inch and a half in diameter at bottom, and tapering at top; shod with iron tipped with steel; instead of making them round, they are better with the corners merely chamfered off, as this gives a better hold, and they should be painted white; fastened to the top should be a small flag. Not less than a dozen or two of these are required on an extensive survey to be carried out on a broad system of triangulation.

With regard to the station poles to be fixed at the ends of the base lines, they will require to be often as much as thirty to forty feet in length; they should be clean, straight, natural spars, of which a more detailed description will be found under the head of "Station Poles," in the chapter on Surveying by Instrumental Observation.

Two of these poles being temporarily set up at the proposed ends of a base—that is a line crossing a district from end to end, or running by one whole side of it—it is sometimes possible to select two stations on rising ground, half or three-quarters of a mile apart, from each of which the main station poles at each end of the proposed base are visible, though *both of them* cannot be seen from either of the intermediate spots selected; in such a case a surveyor and his assistant may, by waving their flags right and left, bring themselves on the line proposed. In such a case, the two main poles may at once be firmly and permanently fixed, as also the two intermediate ones last found; the same operation may now be repeated so as to find intermediate stations to chain upon, when the base is laid out, and the greatest care should be taken that the poles are fixed perfectly upright. It is not always, however, that such favourable circumstances occur, and it is often necessary to find roughly at first where the intermediate poles must come; when the approximation has thus been made, then the situations of the poles may afterwards be accurately fixed.

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* When you find one of your ranging rods warped, break it to prevent your men from using it.

Sometimes it happens that you are obliged to lay out your line from one point or main station pole only, and to guess, as it were, as well as you can from a general map, from some old plan, or only from observation, where a line will take you; if on reaching the other end of the district to be surveyed, you come to such a point that you can from it lay out your other lines, the thing is settled, and you have only to re-examine your work to see that the line be perfectly straight, or to make it so. But, on the other hand, if you are too wide of the mark sought, you have nothing left but to begin over again. This is one of the most tedious operations you have to perform on an extensive survey to be enclosed in as few triangles as possible, and induces many surveyors to give up the system, which, however, is certainly the most minutely correct as to the *whole of the survey*, as well as the details. Without any instruments, such as a compass or theodolite, much time is unavoidably lost in carrying out this principle in all its integrity, when the survey is very large, say three or four thousand acres; but on lesser occasions, say one half of this, it may generally be done even with the chain alone, without meeting with such obstructions as to induce one to lay the system aside. Where, however, you have to deal with a country where there is much wood, you will have to give it up, as, practically speaking, the obstructions are insurmountable. In calm weather you will often receive assistance at first starting the sighting of a line by tying two poles strongly together, but care must be taken that they are *firmly fixed upright* in the ground; I have often tied them in a hedge fast to some stout stem; to make sure they are perfectly upright, try the plumb line upon them in two or three directions, for if a pole leans in sighting a line you would be much better without it, as it is sure to make your work of no avail.

It may very truly be said that these observations are very tedious, and that to many they are unnecessary; but it must be remembered, that we are not writing for the experienced, but the student; strange as it may appear, it is on the carrying out of these details that the correctness of a survey often depends. Most generally speaking, unless some good general map or an old survey be at hand to guide us, we shall have, in order to fix our main stations, to walk twice, or perhaps three times, round the district to be surveyed; this will a good deal depend on the use we make of our eyes during such perambulation. Let us suppose, for instance, that we are starting from A, Fig. 5; the first thing to be done is to look in the direction of B, C, and E, if we can form any idea of their whereabouts on the ground, and we shall generally be assisted in this by making inquiries of some local men accompanying us as to whether such and such objects as may happen to be in sight are within the district to be surveyed, such as a house, a church, a mill, a clump of trees, &c.; then supposing vision is unobstructed before us, see how lines would lay from A; perhaps, however, on account of high ground in every way before us, we can see hardly anything; in this case we must be content with marking any object that may happen to lay about this corner of our survey; then walk on towards E or B, as circumstances may advise. On reaching another marked angle of the survey as E or B, we must endeavour to descry any of the objects we have just noticed about A, though even two or three chains off from A; and look out for what we may see or guess at round about C and B; walk on to C and B, making similar observations, and if we judge it advisable, back to A. Should A, however, lay completely in a hole, it will probably not be of much use, unless to set up a temporary pole there, and this we should not probably do unless our walk along the boundaries between A and E, and between A and B, led us to form some rough idea of the position of the main station thereabouts. Perhaps, however, by walking in the direction, as far as we can guess, of B A or E A produced, we may be able to get a better view, when we should at once do so.

After this first walk we may suppose we have formed some rough idea of the triangles to be laid out, though perhaps we may not know within half-a-dozen chains whereabouts either A, B, C, D, or E will lay. Still, if we have kept our eyes open, and endeavoured by observation and inquiry to form some idea of the position of the main angles of our district, we shall be much better off than when we first started. Now that we have ascertained some of the main features of our district, we shall probably be able to fix, at least temporarily, something like the position of the main stations, when we go round a second time, and perhaps fix them permanently, if we can obtain good views from either E, C, or B. Should there lie high ground anywhere in the centre of the district, and which we perceive is visible from all four points, A, B, C, and E, or even two or three of them, then we should at once proceed to such high ground, after having formed some idea of the position of our main boundary angles. When there, with a couple of assistants and a man or two, we may, by moving about a little, set up rods on one or two lines, until we form some approximation as to where the bases crossing the survey will lay. This may assist us a good deal if we have previously taken care to mark objects at about A, B, C, and E, or even if we have only left a pole at any of these points with a flag on it. Generally, on an extensive survey, a pocket telescope is required for this work. The writer has a small double one of Elliott's, which has often mainly assisted him in laying down base lines. If, however, it should so happen that such difficulties or obstructions lay in the way, that we cannot lay out two such triangles as A B E, and A C B, we may perhaps get one of them, and then, from whichever base happens to cross the survey, we may throw up a couple of triangles over the remaining portion; and if we cannot do this, we must adopt another system of triangulation altogether. Therefore it is desirable to keep our eyes open all the way as we first perambulate the district, for although we may not then be able to fix upon points for main stations, we ought to be able to decide pretty nearly what system of triangulation we shall have to adopt. Generally speaking, it is not wise to be in too great a hurry to commence chaining on such a survey, nor indeed in any case, because we are then liable to lay down lines on which no offsets are to be taken, when, by shifting the line a little at one end or the other, or perhaps at both ends, we may get a line with a good deal of offsetting upon it, thereby saving the necessity for filling in lines, to say nothing of the general rule in surveying to avoid a multiplicity of lines, because the greater the number of lines the greater the chances of error. At the same time the value of a base is not to be endangered for the sake of a few offsets, and sometimes a little shifting will make angles more acute or obtuse than desirable, more particularly when they are to be fixed by intersection of arcs only.

Should the surveyor happen to be provided with a compass, the prismatic compass, for instance, he will find it of considerable assistance in the sighting out his approximate bases; supposing, for instance, that on first starting from A, he proposes a line, A E, bearing 15° east of north, and that he has marked some object in the distance having that bearing; but having walked in that direction for some distance, perhaps half a mile, he sees before him something which makes it desirable to alter this course; keeping the back point A in mind, let him move right or left to get a new line to accommodate the new position of things before him; say this new line bears 10° east of north; let him now turn round and see how a line 10° west of south will suit the ground left behind us, for 10° west of south will be a prolongation backwards of 10° east of north; if he sees reason to approve of this new line, let him fix on some forward object having that bearing, and if he sees that he is likely, from the nature of the ground, to lose sight soon of the greater portion of the back part of the line, let him fix on two objects. Having moved further north he may have to repeat the same operation, and so onwards. This will not fix the position of a base line, but will often be of considerable service in getting an approximation to it.

To guard, however, the reader from falling into the erroneous idea that having an outline of a survey of a particular form, he is at once to assume that one special and particular system of triangulation must be adopted, we have at Fig. 6 repeated a similar outline to that at Fig. 5, with this difference, that there is a long narrow ridge of high ground running east and west, about the centre of the district; this may at once render necessary a system of triangulation entirely different from the former one.

On the first inspection of the ground we see both difficulties and facilities in sighting two bases through this district, over the high ridge A B; moreover, we perceive that from this high ground we get a command of the whole of our district, and that along this ridge we can lay down a base from which we can throw up triangles north and south, and also that we can lay out our lines to such advantage as to get a good deal of offsetting upon them. All this is sufficient to determine us on assuming the line A B as the principal base. The points A and B are not to be fixed of necessity exactly where they are, and on settling their position it will be our business to determine them so that along A C and B D we may be able to pick up offsets in the filling in; the position of C and D must be settled with the same view, keeping, however, in mind, the superior consideration of laying out C D, if possible, so as to produce it both ways to E and F, so as to make it a good and fixed line for tying others into, as H E and G F, thereby including the south side of our survey by as few lines as possible, and making the lines on the east side check those on the west; to settle finally, therefore, on the points E and F, may take a little time, but the firm tying up of this half of our survey makes it worth while. Whilst laying out this line we must not neglect to look out as to the best position of C and D, with regard to the whereabouts of A and B, so as to get on A C and B D the best intersection of fences, and as much offsetting as may be; the points H and G, as regards this south side of our work, are to be fixed with due consideration to the same purposes, though as to G it is of greater importance to fix this point, if possible, so as to make F G I one straight line, as the *direction* of G I will then be a settled matter, and when we come to describe the triangle H I G with the beam compasses, and the line G I is drawn, it must make one straight line with G F, thereby holding a check on this triangle, independent of the intersection at K, and the tie line J I, which unites the summits of the triangles H G L and H I G; if all this comes upon paper as measured on the ground, our triangulation is safe, and we may proceed to fill in by laying down lines from one side of our triangles to another, or by throwing up interior triangles and tie lines.

We believe that we have now given all the information possible upon this description of surveying, and we shall only have to add presently a few words about "chain angles." For the reader to be able fully to understand and apply what we have above said upon the triangulation and chain surveying of an extensive district, it will be necessary that he should have previously obtained a little practice as to chaining, offsetting, the field-book, and other elementary steps; and we have been writing on the supposition that he has followed advice already given as to obtaining a little practice with the chain, at first merely chaining a line, and taking offsets to adjacent objects two or three times over, and having accomplished this satisfactorily, then applying himself to the survey of 200 or 300 acres of land; if this has been done, it is hoped that what has been said above will not be without advantage.

We would also advise the young surveyor who is practising merely for the sake of study and improving himself, immediately he can, with tolerable ease and comfort, manage 200 or 300 acres, no longer to be satisfied with such limited practice, but to set about much larger areas, even if it be only as to the outlines and principal features; this will bring him more speedily than anything else into such practical experience as will most readily make him soonest useful; an extensive common, for instance, with the circumscribing and intersecting roads, with houses and gardens by the way-side, pools, ponds, streams, &c.; if, instead of doing this, he devotes himself entirely to the practice of a few fields here and there, or a short length of road or two, with a few houses, he will acquire the practice of chaining and offsetting, and throwing up a few little triangles, but he will not acquire the habit of that large and comprehensive triangulation required in almost all such surveys as enter into engineering field-work in practice; and when he comes to the reality of this in the actual experience of professional life, he will feel at a great loss; but if, to repeat ourselves once more, he has taught himself to chain a long line, to read his chain as he proceeds, to keep his field-book, to take and enter his offsets, and sketch adjacent objects as he goes along; if he has learnt to do this correctly, and then, as we have said before, applies himself to surveys first of two or three fields, and gradually extends his work until he can manage two or three hundred acres, he will be perfectly able to try his hand upon two or three large tracts of land—say a couple of miles long, and half as broad; if he has also by this time acquired the habit of keeping his field-book, as shown in a former place, in such manner that there is no difficulty in reading and understanding, and therefore in plotting from it, he will be fit for something in engineering field-work, as far as land-surveying is concerned. But to imagine that, having acquired the habit of measuring two or three fields, and a road or two, he can make himself useful in actual work, is to be led into a delusion from which he may be very unpleasantly aroused — a delusion which can only be fostered by those who are amusing themselves at his expense, and one from which he will find a bitter awaking when he finds that he is not even fit to be a chainman in actual work; if, on the other hand, he has acquired the practice by himself of working on large areas in one place, he can do it in another. The only grounds on which we would recommend the student surveyor every now and then to undertake small surveys would be, having the intention of plotting to a very large scale, say 30 or 40 feet to one inch, and then there should be plenty of offsetting and buildings, &c., all the minutiæ of which should be shown, such as gateways, steps, pavements, drain gratings, &c., in short, everything actually on the ground, and all of which must be shown on the plan. Such surveys demand great care and *patience*, for here an error of even six inches, or less, may be a serious matter.

Surveying by chain angles is a method of fixing the position of the chain lines by triangles set off upon portions of them. Fig. 7 is an illustration of this, where it is required to set out the lines A B, B C, C D, and D E, so as to get a survey of the banks of the stream; A B is produced to *a*, and C B to *c*; the small triangles about B are then measured, to plot the position of C B with regard to A B, and the small triangle about C to fix the position of D C with regard to the line B C, and the same with regard to the small triangles about D, to fix the line D E with regard to C D. It will entirely depend on the accuracy of sighting, measuring, and laying down of B*a* and B *b* and *a b*, as also B *c*, B *d*, and C *d*, that the correctness of the angle A B C depends; and therefore the position of B C as to A B; as regards that of C D with regard to B C, matters are still more critical, depending entirely on the little triangle *g* C *f*, whereas there is a check, such as it is, by the measurement of *cd* as against *a b*, and of *a c* as regards them both; but whenever the lines to be fixed are much longer than the sides containing the chain angles, there is no dependence to be placed on such work. In the surveys of long narrow slips of land, such as those made for railways, canals, roads, &c., this method by chain angles is often adopted; but then, when properly done, it amounts to a well-laid system of triangulation on a small scale, because the sides of the chain angles bear some considerable proportion of length, and *because besides* it is so contrived that other important lines shall fully check the chain angles. The subject is merely mentioned here, as relating to chain-surveying, to guard the student against adopting without due caution a system that may lead to very erroneous results; on short lines, which do not exceed three or four times the lengths of the sides about the chain angles, it may do well enough, but nothing more.

We may conclude this portion of our subject by reminding the reader of the following rules in laying out the chain lines of a survey, which is one of the most important parts of the business, for once that he can chain and book correctly, *this* is always the same, whilst the setting out constantly varies.

Carefully explore your boundaries as the first step of all, and as you do this look behind you as well as before; it is not necessary to abide exactly by the boundary side, but rather to take advantage of every adjacent acclivity that commands the ground forward as well as in the rear, as also towards the interior of the proposed survey; do not be anxious to commence operations until you have made yourself master of the leading features of the ground so as to have a rough map of it in the mind's eye.

If possible select your bases so that they shall lay across your survey in the longest directions and command at the same time a good view here and there of your district, more particularly at the extremities of these main lines, so that you may be able not only to see from them, but to them; it is not of the slightest importance that they be exactly on the boundaries of your survey, or exactly in the centre.

Do not let the angles of your triangles be too acute or obtuse if you can help it; be very careful in sighting your lines and fixing your principal poles, and mark most carefully the intersections of the base lines.

In chaining these lines leave sufficient marks so that you may be always able to find their directions when required, and look out carefully for intermediate stations for your secondary triangles, which are only second in importance to your main ones; select them so that you may chain as many lines to or from them as possible, never losing a chance of producing lines when it may be convenient instead of throwing up the sides of fresh triangles.

Never omit *tie-lines*; if it does so happen that there are no offsets they are soon run.

CHAPTER III.

Instruments for Laying Down and Plotting Chain Surveys.—Contraction of paper.—Precautions to be taken in consequence.—Beam Compasses.

Besides a pencil and a pair of full-sized compasses, there are required for laying down and plotting a chain survey a straightedge, a scale, a beam compass for long lines, and a needle-point. With the use of the compasses we suppose every one acquainted, and we can only add here that in describing with these the sides of triangles by means of arcs, it is always desirable to lay down the longest side of the triangle first, and from the ends of this side describe your arcs; and that the legs of the compasses must be kept as upright as possible: with long chain lines we must resort to the use of a beam compass, which will be presently described.

In a former place we have already said something of scales; Fig. 8 shows a short chain scale divided to two chains to one inch; it will be observed that it is numbered from 0 to 12 both ways for convenience of reading; each main division, as 1, 2, 3, is, in fact, a miniature scale of a chain, and each subdivision counts for ten links. In first laying down your scale by the side of your chain line after you have drawn it in, be careful to place zero at the commencement of your chain line, and not at the end of it. Having done this, and ascertained that your chain line has been correctly laid off in length, with your field-book before you, you prick off with your needle-point* all the distances at which you have taken offsets, all your stations, and intersection of fences; round the station marks draw a little half circle in pencil as you prick them off, and mark with a little cross line all your fence intersections; this is of assistance when you come to lay off the offsets. Having thus pricked off all your distances on the chain line, lay a short scale across it at right angles, and one after another plot your offsets as you find them in the book, and at about every ten or twenty chains, before proceeding farther, draw in your fences, &c., as you go along. The reader must now work this out for himself to understand it more completely. The scale shown in the illustration is only six inches long, which is too short for any extent of survey; it should not be less than a foot in length; for extensive work it is well to have two or three, of well-seasoned box-wood, about three feet in length.

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* Ivory needle-points are sold by instrument makers, but a very good one may be made by taking a very fine needle in a pair of pincers and driving the head of it into a pencil top, leaving out about the eighth of an inch, and then pointing your pencil.

Fig. 9 is an illustration of a chain scale, with offset attached, but not very generally used. The scale A B has a dovetailed longitudinal groove nearly from end to end, in which slides an accurately fitting sliding piece, to which, by means of a screw, is fixed the offset scale C; it is used in such a manner that zero moves along the chain line, whilst the scale A B is parallel to it, and its zero exactly opposite and at right angles to the beginning of the chain line; the offsets are thus pricked off right and left as the offset scale is made to slide along the chain scale according to the chainage in the field-book. The scale A B, when adjusted to its position, is fixed by weights.

A scale of 4 chains to 1 inch is a very good and useful scale for taking out quantities by, and is one very generally employed for general working plans; by some engineers a preference is given to 3 chains to the inch. These two scales are those adopted for Tithe maps of parishes.

For parliamentary, as railway and canal work, a scale of 6 chains to 1 inch has now been in general use for some years past; it is the smallest admitted without enlargements of buildings. See Standing Orders. A few years ago the scale more generally adopted was one of 5 chains to 1 inch. Where for parliamentary work we come through town lands and numerous buildings, a scale of 6 chains is rather small; for this purpose a scale of 4 chains to 1 inch is becoming more generally in use.

For working plans among town lands a scale of 30 or 40 feet to 1 inch is very commonly adopted.

There are a great number of other scales mentioned in works on this subject, and sold by makers of mathematical instruments; but they are very rarely used in professional practice, and they are therefore not described in these pages.

The scales required are 3, 4, 5, 6, 8, and 10 main divisions to 1 inch, the two latter being generally used for the vertical heights in sections. The three first may be had in box-wood, but the three latter must be in ivory on account of the small size of the subdivisions.

For general maps the scales most usual are 1, 4, and 6 inches to 1 mile. The survey of London and environs has been published to a scale of 1 foot to 1 mile

Before we entirely dismiss the subject of scales, we must say a few words as to the expansion and contraction that paper is liable to as the atmosphere happens to be damp or dry. Where there is a really extensive survey of great breadth as well as length, this often occasions considerable trouble; we have experienced this more particularly by the sea-side; after laying down a long chain line on a fine dry day, and the scale is laid by the side of it on a damp one, it will be found that it measures too long, and *vice versâ;* the longer, and generally therefore the more important the line, the greater the error; the only way to get over this inconvenience is to lay down two scales of inches divided into tens of chains on your paper by the sides of your work; the one horizontally, and the other vertically; these scales expand and contract equally with the rest of the paper according to the hygrometrical state of the atmosphere, and you can apply your beam compasses to them when you want long distances; for the short lengths the matter is of less importance.

The construction of the beam compass is illustrated at Fig. 10, where A is a long beam, sometimes 5 or 6 feet and more, made of box-wood or well-seasoned mahogany, and may be divided into feet, and inches, and tenths of inches. B and C are two sliding pieces of brass encompassing the beam, on which they are tightened by the clamp screws E and D; to each of them is attached a point, as G and H. When any distance is to be set off with this instrument, the screw E is released, and its sliding piece moved along the beam until the required distance is compassed between the points G and H, when E is clamped, and any slight deviation from the exact measurement required is corrected by the slow motion screw F at one end of the beam where B is always situate. As we have not as yet had an opportunity of mentioning the *slow motion screw,* we may here observe that it is so constructed to give a *slight motion* to the piece B when the screw D is clamped.* In this state the instrument is fit for use to lay off distances by having one point fixed where required whilst the other is steadily swept round to describe an arc in the same manner that an ordinary pair of compasses is used; it is scarcely necessary to add that there is the necessary arrangement for a pencil point.

We have now explained the use of instruments for plotting a chain survey as far, we believe, as it is possible to do so; practice must accomplish the rest.

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* The beam itself is sometimes divided as a scale, but this is rarely trustworthy; it is much safer to take off the distance from a long scale; or from that laid down on the paper, as shown above.

CHAPTER IV.

Geometrical Memoranda.—Paper Protractor.—Inaccessible Distances.—Computation of Areas.—Triangles, Trapeziums, and Reduction of Figures.

A *straight line* lays evenly between its extreme points, as A B, Fig. 11; if we wish to draw as traight line on paper from one point to another, we lay a straight-edge close by the side of these, so that the point of a pencil may be carried from one to the other, and during the whole transit be in close contact with the straight-edge. To do this on the ground, provided the points be not too far apart, we strain a line or a chain from point to point, and to lay down works, a pick traces the line by the side of the cord instead of a pencil. But if the points be too far apart for the cord to reach from one to the other, we must find intermediate points on the line sufficiently near to each other for the line to reach from each to each. To find these points set up ranging rods, so that standing a short distance from one or the other of the extreme points, one rod will cover the whole of those set up; the intermediate points are thus found; in doing this great care is required that the poles be all perfectly upright, which in very many cases can only be tested by the plumb line. By the same method a straight line may be produced, but here even greater caution is required as the length of the line is increased, for if there be a bend two lines containing an angle will have been laid out instead of a straight line.

Straight lines must either be so disposed with regard to each other as to be equidistant from each other throughout their lengths, when they are called parallels, or they must lie in such position that they meet and form an angle, or would do so if sufficiently produced. See Fig. 12.

When two straight lines meet each other, as in Fig. 13, they must do so either in such manner that the one of them does not lean to the other more on one side than on the other, as C D or A B, and C D is then perpendicular or at right angles to A B, and the angles are equal, and the numerical measure of each angle is 90°, or half 180°, or a fourth of 360°; or if the adjacent angles be not equal, then the one line E D meeting the other leans more on one side than on the other, and the angles are unequal, and the one measures more than 90°, and the other less; E D is then said to be oblique or askew to A B.

And any number of lines may meet a given line at a given point, and on the same side of the given line, but the angles contained by the given line and the concurrent lines will be equal to 180°, as in the last figure, where the angles A D F, F D C, C D E, and E D B, are all made up of, and equal to the two right angles A D C and C D B = 180°. One straight line, therefore, meeting another straight line makes the adjacent angles equal to two right angles.*

An angle less than a right angle, as C D E or E D B, is an *acute angle*, and an angle greater than a right angle, as F D B, which is made up of the right angle C D B plus F D C, is obtuse. Also what any angle wants of 90° is called its *complement;* thus C D E is the complement of E D B; and what an angle wants of 180° is called its *supplement;* thus A D F is the supplement of F D B, or F D B is the supplement of A D F. Therefore the adjacent angles formed by one straight line meeting another are the supplements each to each.

The above circumstance is often of considerable use in surveying, as we will endeavour to explain, having still recourse to the same Figure. Let A D B be supposed a straight base of any length, and let A D be supposed to have been run over a range of hills and through broken ground, such as insures difficulty in tracing a straight line along the ground. It is very possible that the line may have become distorted before reaching D, which is supposed to be in a hollow, from which the greater part of the line A D is invisible, but D B visible; now the position of the latter enables us to check the former, for the angles F D B + F D A will together equal 180° if A D be straight. Again, suppose the position of A D to have been lost on the ground, and in the course of the survey wanted again, then the angle F D B being known, its supplement, or what it wants of 180°, enables us to set out D A with correctness.

Any number of angles formed by straight lines intersecting each other at one point are together equal to four right angles or 360°. Produce F D to G; then A D G and G D B together are equal to two right angles in the same manner as A D F, and F D B.

When two straight lines intersect each other, the opposite angles are equal to each other. In Fig. 13 let A B and D F intersect at C; then the angle A C E is equal to the angle D C B, and the angle A C D is equal to the angle E C B. For A C D and D C B are equal to two right angles; and D C A and A C E are also equal to two right angles; from each of these equal quantities subtract the angle A C D; then the remainders A C E and D C B will also be equal. In the same manner it may be shown that A C D is equal to E C B.

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* In mentioning an angle by three given letters, as A D C, the letter *at* the angle is always the middle one, and when there is no likelihood of confusion, an angle is often named by such letter alone.

The circle is a plane surface circumscribed by a curved line, called *circumference*, any number of points on which are all at an equal distance from the point C called centre, Fig. 14; and this distance is called the *radius;* in the circle, therefore, any number of radii are all equal to each other.

The circle is divided into 360°; each *degree* into sixty *minutes* marked thus ′, and each minute is subdivided into *seconds* marked thus ″.

A chord is any line drawn from any one point to any other point on the circumference, as A B, or B D or E A, and the greatest of chords is that which passes through the centre of the circle and is known as the *diameter*, which divides the circle in two equal parts, equal therefore each to 180°. The diameters of a circle can only intersect in the centre, as it is a condition of such a line to pass through it.

An arc is the portion of the circumference cut off by the chord.

A segment is any portion of a circle cut off between a chord and that portion of the circumference which the chord intersects, as B D E B, or A B *a* A, or A D E B.

A quadrant, as E B C, is the quarter of a circle and the half of a semicircle; it is bounded by two halves of the two diameters intersecting each other at right angles, and by the quarter of the circumference cut off by them. This quarter of a circumference, which is equal to 90°, is the measure of the quadrant or angle E C B or B C A. An angle is thus measured by that portion of the circle intercepted between the two sides forming the angle, and subtending it. Thus each of the angles at C being equal by construction to right angles, is subtended by a fourth of the circle circumscribing the whole. An angle is also measured by the chord subtending it, as A B, which cuts off the arc B *a* A, so that an angle is measured by the chord or by the arc, either of which will give the same measure.

A tangent, as F D G, is a line which touches a circle, as at D; a perpendicular to a tangent at the point of tangency will always pass through the centre of the circle if produced, as D B. An explanation of the trigonometrical signs will be found in another part of this work.

The protractor is an instrument by means of which the mechanical operation of laying down angles on paper is performed. A variety of these are sold by the instrument-makers, and will be hereafter described; but a very useful instrument may be easily made, and which will answer a variety of general purposes: this is a paper protractor, constructed in the following manner:—With a radius of from four to six or seven inches describe a circle with your compasses; through this draw a diameter, and being careful that the legs of your compasses have not moved, and commencing at one end of your diameter, run the legs of your compasses right round the circumference. If you have done all this accurately, the third intersection of the circumference will be at one end of your diameter, and the sixth will bring you back exactly to your starting-point, and the circumference will have been divided into arcs of 60° each. Nów take twice the natural sine of 15° and multiply it by the radius, which will give the chord of 30°, with which bisect each arc of 60°, and next twice the natural sine of 10°, multiplied by the radius, which will give the chord of 20°, with which each arc of 60° should be divided into three, whereby the circumference will be divided to 20°. Now, with twice the natural sign of 7° 30′, divide the circumference to 5°. With twice the natural sine of 3° you may now subdivide to single degrees, and with twice the natural sine of 3° 45′ you may divide to half degrees, by working from the points you have already found. Twice the natural sine of 3° 20′, which is equal to the chord of 6° 40′, will enable you to divide the circumference to 10′, if the size or radius of your protractor will admit of it.

This operation, which is rather a lengthy one, demands care and a nicely pointed pair of dividers and a good scale; the work may be checked first from one point and then from another, and if any trifling inaccuracy should be found, divide the error. Every ten degrees requires to be numbered from 0°, round to 360°, and the principal divisional lines should be longer than the rest, the lines for the single degrees should be somewhat less than these, and less again for the divisions of each degree.

Copies of this protractor may be obtained by pricking through the divisional points on a sheet of paper placed underneath. Should the reader be unacquainted with the use of trigonometrical tables, he will find instruction on the subject further on.

On one of the chords subtending any one of the quadrants of this protractor may be laid down "a line of chords," in the following manner:—Let A B, Fig. 15, be the chord to the quadrant or fourth part of the circle, and which is here only divided to every tenth degree. Set one foot of the compasses at B, and describe arcs through the divisions on the arc on to the chord A B, which will then be a line of chords, by which any angle may be measured by making the two sides containing the angle equal to each other and equal to the radius of our protractor, and carrying the length of the third side in a pair of compasses on to the line of chords, which will give the measure of the angle required.

Problem 1.

From a given point on a given line to set off a perpendicular.

Let A B, Fig. 16, be the given line, and C the given point. From C, and on each side of it, set off equal distances as at D and F, and from these points as centres, and with the same radius, describe two arcs intersecting at G; then a line from G to C will be the perpendicular required. Draw the lines G D and G F, which are equal by construction; G is therefore equidistant from D and F, as will also be any other point on the perpendicular G C. When not provided with any angular instruments, our means of setting off a perpendicular on the ground are generally very limited; so much so, that if such line be afterwards produced to any distance, it requires testing, which is readily afforded by the above means, since any point on the perpendicular must be equidistant from two points also equidistant from the point on the line from which the perpendicular has been set off. Let H be the point to which the perpendicular has been produced, and let I and K be the two points made equidistant from C; if C H be perpendicular to C, then I H and K H will be equal. Another means of testing this is merely to measure off C I and C K equal to each other, and produce D G and F G to intersect H I and H K at L and M, when, if H C be perpendicular to A B, H L will measure equal H M.

Of some of the Properties of Rectilineal Triangles.—Any side of any triangle may be taken as its base; the angle opposite to such base will then be the summit, and a perpendicular to the base from the summit will be the height or altitude of the triangle.

The sum of the interior angles of any triangle is equal to the sum of two right angles; therefore the sum of the three internal angles of any triangle is equal to the sum of the three internal angles of any other triangle; therefore, also, if two angles of a triangle are equal to two angles, each to each, of any other triangle, the remaining angles will be equal, and the triangles will be *equiangular.*

The exterior angle of any triangle is the angle contained between any one side and either of the adjacent sides produced, and the adjacent angle of the exterior angle is the angle contained by the side which has been produced and the side falling on it. The exterior angle and its adjacent angle are together equal to two right angles, because one line falling on another line is equal to two right angles.

The exterior angle of any triangle is equal to the sum of the two opposite angles; for let the exterior angle be called E, and the adjacent angle A, and the two other angles of the triangle B and C; $E + A =$ the sum of two right angles; and $A + B + C =$ to the sum of two right angles; if from these two equals the angle A, common to both, be subtracted, the remainders will be equal; since any exterior angle of a triangle is equal to the two interior opposite angles, therefore any such exterior angle is greater than either of the interior opposite angles.

In any triangle the greater angle is subtended by the greater side, and the smaller angle by the smaller side, and therefore the greater side subtends the greater angle, and the smaller side subtends the smaller angle; and the equal angles of a triangle are subtended by equal sides, and the equal sides subtend equal angles.

In any two adjacent triangles, with equal sides containing unequal angles, the greater angle will be subtended by the greater side, and the lesser angle by the lesser side.

Any two triangles having the three sides of the one equal to the three sides of the other, are equal, equilateral, and equiangular.

Any two triangles having each an equal angle contained by equal sides, are equal.

Any two triangles having each two equal angles on equal bases, are equal.

Any two triangles of equal altitude, and on equal bases, are equal.

Equal triangles upon the same base, and on the same side of it, are between the same parallels.

If any two sides of any triangle are intersected by a line parallel to the third side, the intercepted segments will be proportionals; and the one side will be to either of its segments as the other side is to the segment opposite to the segment of the other side; and the parallel sides will be to each other as the segments of either of the other sides are to each other; and the three sides of the larger triangle will be proportional to the three sides of the lesser triangle. It is to be observed that if the parallel to the base, instead of being drawn within the triangle, had been drawn without, so as to intercept the two sides produced, the three sides of the two triangles thus formed would be proportionals, because they would be similar triangles having two opposite angles equal, and two sides parallel.

If any angle of a triangle be divided into equal parts by a straight line intersecting the opposite base, then of the two sides containing the angle, the one side will be to its adjacent segment of the base as the other side is to the other segment, and the segments of the base shall have the same ratio which the other sides of the triangle have to one another.

Any two triangles having an equal angle in each, contained by proportional sides, are similar to each other.

Any two triangles having two angles of the one equal to two angles of the other, are similar triangles, because the three angles of the one triangle are equal to the three angles of the other; therefore two rectangular triangles, having, besides the two right angles, two other equal angles, are similar triangles.

Two triangles having three sides of the one perpendicular to the three sides of the other are similar to each other; the equal angles will be contained by the *homologous* sides, and any homologous side in the one triangle will be perpendicular to its homologous side in the other triangle.

In order to study the nature of similar triangles, and the proportions of homologous sides, the equal angles should be designated by the same letters in different characters.

In triangles, the sides containing equal angles, being proportionals; to divide any line into parts proportional to those of another line, it is only requisite to make the two lines contain an angle between them, and to join their other extremities by a third line; then parallel to this third line draw lines through the points of division of the line already divided, to intersect that which it is required to divide.

Of some of the Properties of the Circle.—In a circle, any ordinate to the diameter, that is, any perpendicular from the diameter to the circumference, is a mean proportional to the segments into which the diameter is divided by the perpendicular; let the perpendicular be O, and the segments into which the diameter is divided be A and B; then

$$A : O :: O : B; \text{ and } \frac{O^2}{A} = B; \text{ or } B : O :: O : A, \text{ and } \frac{O^2}{B} = A.$$

In any circle let a diameter be drawn; at right angles to it, and from one of its extremities let a straight line be drawn, and from the centre of the circle let a third line be drawn to intersect the perpendicular to the diameter; such perpendicular will be a mean proportional between the segments of the line last drawn, one of which is inside the circle and the other outside. If any straight line touch a circle, without cutting it, it shall be perpendicular to the radius.

In a circle, a perpendicular bisecting a chord and produced through the circle will be the diameter, and the middle of the diameter will be the centre of the circle.

Equal straight lines in a circle are equally distant from the centre; and those equally distant are equal to one another; the greatest line is the diameter, and of all others, that which is nearer the centre is always greater than that more remote.

The angle at the centre of a circle is double the angle at the circumference, *upon the same base*, or upon the same arc of the circle.

The angles in the same segment of a circle are all equal to one another.

The opposite angles of any quadrilateral inscribed in a circle are together equal to two right angles.

In equal circles, equal arcs are subtended by equal chords; and equal chords cut off equal arcs; and equal angles stand upon equal chords, whether they be at the circumferences or at the centres, and any arc is equally bisected by equal chords.

In a circle, the angle contained by a semicircle is a right angle; and an angle in a segment greater than the semicircle is less than a right angle; and an angle in a segment less than a semicircle is greater than a right angle.

If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle.

On a given line to describe a segment which shall contain an angle equal to a given angle. If the given angle is a right angle, the required segment will be a semicircle; and it is only requisite to bisect the given line, and from the point of bisection to describe a semicircle with half the line as radius. But if the given angle be less or greater than a right angle at either end of the given line, which we will call A, make an angle equal to the given angle, and call the second line B; from the point where A and B meet, draw a line perpendicular to B, which call C; bisect A, and from the point of bisection raise a perpendicular to intersect C; take this point as a centre, and take for radius the distance on C between this centre and B, and describe a circle; the circle will be divided into two segments by the line A; if the given angle be less than a right angle, it will be contained by the greater segment, and if less than a right angle, it will be contained by the lesser segment.

From a given circle to cut off a segment which shall contain an angle equal to a given angle. Draw any line tangent to the given circle; at the point of contact describe an angle equal to a given angle, making the tangent one side of it; produce the other side to cut through the circle, which will thus be divided into two segments, the greater of which will contain the given angle if less than a right angle, and the lesser segment if the given angle is greater than a right angle.

If two straight lines cut one another within a circle, the rectangle contained by the segments of one line will be equal to the rectangle of the segments of the other line.

In any circle the square of the diameter is equal to the square of the chords drawn from the two extremities of the diameter to the circumference.

In any quadrilateral inscribed in a circle, the rectangle contained by the diagonals is equal to the rectangles contained by the opposite sides of the quadrilateral.

If from any point outside of a circle two straight lines be drawn, one of which cuts the circle and the other touches it, then the rectangle under the line which cuts the circle, and that part of it outside the circle, is equal to the rectangle under the line which touches the circle.

Of Rectangles.

In any right angled triangle, the square of the side subtending the right angle is equal to the square under the sides containing the right angle.

If there be two straight lines, one of which is divided into any number of parts, the rectangle contained by the two lines is equal to the rectangles contained by the undivided line, and the several parts of the divided line.

If a straight line be divided into any two parts, the rectangle contained by the whole and one part is equal to the rectangle contained by the two parts, together with the square of the aforesaid part.

If a straight line be divided into any two parts, four times the rectangle contained by the whole line, and one of the parts, together with the square of the other part, is equal to the square of the straight line, which is made up of the whole line and that part.

In obtuse angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square of the side subtending the obtuse angle is greater than the squares of the sides containing the obtuse angle, by twice the rectangle contained by the side upon which, when produced, the perpendicular falls, and the straight line intercepted without the triangle between the perpendicular and the obtuse angle.

In every triangle the square of the side subtending an acute angle is less than the squares of the sides containing that angle by twice the rectangle contained by either of these sides, and the straight line intercepted between the acute angle and the perpendicular let fall upon it from the opposite angle.

If four straight lines be proportionals, the rectangle contained by the extremes is equal to the rectangle contained by the means; and if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines are proportionals.

If three straight lines be proportionals, the rectangle contained by the extremes is equal to the square of the mean, and if the rectangle contained by the extremes be equal to the square of the mean, the three straight lines are proportionals.

PROBLEM 2. Fig. 16.

On the ground from a given point on a given line to set off a perpendicular.

Double a long, light cord, fasten the ends at D and F, the middle of the cord held towards G with an equal strain on both ends of the line, will give the perpendicular G C; the chain may be used for doing this, or in the following manner: make D C equal to 30 links, fix one end of the chain at C, and the 90 at D, and fix 40 (not 60) towards G; G C will be the perpendicular required; G D will be equal to 50, as will also G F, if the work is correctly done.

PROBLEM 3. Fig. 17.

To raise a perpendicular from a point near the end of a line.

Let A B be the given line, and P the given point. From any convenient point C, with the distance C P, describe a circle; through A and E draw A E; E P will be the perpendicular required. To do this on the ground, from the point P set off 50 feet or links at D, just as you may happen to be using Gunter's chain, or a chain or tape of 100 *feet*, and 50 more to E; stick an arrow at E and P, pass the loops of a line one 100 long over each of them, as E and P, and take the middle of the line towards G, which fix exactly with another arrow, holding the line quite tight; E G P will be an equilateral triangle. Now set up a rod exactly in line with E G, in the direction of H, and make G F equal to E G; F P will be the perpendicular required. If you have a line equal to two chains in length instead of only one, so much the better, only then the triangle E G P will be isosceles instead of equilateral. Also you will have the following proportion:—E D : D G : : E P : P F.

PROBLEM 4. Fig. 19.

From a given point to let fall a perpendicular to a given line.

From P as centre with any radius greater than P C, describe the arc D E, and from D and E, with the same radius, describe the arcs intersecting at R; join P R, and P C will be the perpendicular required.

PROBLEM 5. Fig. 20.

On the ground, from a given point to let fall a perpendicular to a given line.

Let P be the given point, and A B the given line; from P towards A B lay out any two lines, as P C and P D; set up two rods at C and D, taking care that they are exactly on the line A B; make P E and P F each equal to one chain, and stick a couple of pegs at E and F, taking care that they are exactly on the lines P C and P D respectively; measure P C, and make P G equal to P C; measure C G, E F, and G D, and take the following proportion:—

C G : E F : : G D : F H.

From F to H set out this fourth proportional just found, being careful that H is exactly on the line P D. P E being equal to one chain, take a line exactly equal to it, fasten one end to an arrow, or peg, at P, and bring the line round so that it shall exactly intersect E H at I. This can be done by a person standing at E sighting the one holding the end of the line at I until it is exactly on the line E H. Now measure E I, and bisect it at K; a line produced through P K will fall on L, and be perpendicular to A B.

PROBLEM 6. Fig. 21.

On the ground, through a given point to set out a straight line parallel to another straight line.

Let P be the given point, and A B the given straight line. From any convenient point, C on the line A B, set out through the point P the line C D, making P D one or two chains in length. From D set out the line D E, carefully placing E on the line A B; make D F equal to P D, and D G equal to D C. Now measure C G and G E, and take the same proportion as that given above, though in other letters; that is,

A G : P F :: G E : F H.

Set out the length of F H thus found. P H is parallel to A B. If the required parallel is to be of any length, set up a rod as T in line with P H. If the prolongation of P H is of any considerable length the work may be tested; produce C G to intersect P T at I; measure off I L and I K equal to two or three chains each, and measure L K. Then proceeding to C, make C M and C N each equal to I K and I L, and N M should be equal to L K.

Or, use the following means. Let P, Fig. 22, be the given point through which it is required to set out a parallel to the straight line A B. With rods only set up the line A P; make A C two or three chains in length, and set off by some of the foregoing rules the line C D perpendicular to A P, and produce it towards G. Make P E equal to A C, set up the perpendicular E F, and make E F equal to C D, through F sight P F G, which will be parallel to A B. Measure E H and C G, which should be equal to it. Should there be found any slight inaccuracy, correct it.

Should A B, Fig. 23, be inaccessible, sight the line with ranging rods to any convenient distance, as to C, and with ranging rods set out C P; measure off two or three chains along C P as C D, and from D set out D E perpendicular to C P, and measure D E. Make P F equal to C D, and the perpendicular F G equal to D E. Through P set off G P I, which will be parallel to A B. Produce the perpendicular D E towards H, and take the following proportion,

C D : D E :: P D : D H.

Measure off D H accordingly, when H I will be in line with the points I, P, and G.

By similar means we may often obtain the length of inaccessible distances, as in the following problems:—

PROBLEM 7. Fig. 24.

To ascertain the length of the inaccessible distance A B through a sheet of water.

Sight A E on any convenient object, and measure on to C, where you observe you can run clear of the obstruction up to B, and leave a mark at C; make C E equal to A C, and leave a mark at E; now measure B C, and make C D, being B C produced, equal to B C. The parallel line D E will be equal to the inaccessible length A B.

Now let there be another obstruction, as in Fig. 25, such that you cannot make C E equal to A C. Measure out C E as far as you can conveniently, and having measured B C, take the following proportion,

A C : C E :: B C : C D.

Then make C D equal to this fourth proportion thus found. Now measure D E, and take the following proportion,

E C : A C :: E D : A B;

which fourth proportional gives the length required.

Again, let there be such an obstruction at C, Fig. 26, that we cannot proceed beyond. Produce C B to D, and make B D equal to B C; now from C B cut off C F, and from C A cut off C E, so that F E shall not exceed one chain in length. Make D G equal to C F, and on G D set off the triangle D H G equal to the triangle C E F; this may be done with the chain and a tape, making the length of one equal to C E as D H, and the other equal to F E as G H; their intersection will give the point H. Through this point produce D H to I on the main line A B, when I B will be equal to A B. This may be tested by measuring D I, which should be equal to C A.

It may also happen that there is another obstruction on the other side of the main line, as at F, Fig. 27, so that what we require cannot be accomplished by the above means; then proceed in the following manner:—Make B F equal to C D, and in the same manner as before make the triangle F G B equal to the triangle C E D; produce F G on to I on A B produced, when

F B : C B : I B : B A,

which fourth proportion, therefore, gives the length required.

Or, at the other end of A B produce C A, and make A K equal to A C; at K set off the triangle equal and similar to C D E, and produce K L on to M; M A is equal to A B.

The above are based on the 4th, 15th, 27th, and 28th propositions of the first book, and 4th, 5th, and 6th of the sixth book of Euclid.

PROBLEM 8. Fig. 28.

To bisect a given angle on the ground.

Let A B C be the given angle; make B D and B E equal, and measure D E, and make D F equal to half D E, B F will bisect the angle.

Problem 9. Fig. 29.

To lay out on the ground, from a given point, an angle equal to a given inaccessible angle.

Let P be the given point, and A C B the given angle. With rods sight A C to E, and B C to F, and from any convenient points, as E and F on these lines, sight E P and F P; then, by the second part of Problem 6, set out the parallels P *d* and P *e*; *e* P *d* will be the angle required.

Problem 10. Fig. 30.

To lay out on the ground two lines parallel to two given inaccessible lines containing any angle, and at a given distance from the two given lines.

Let A B and C B be the two given lines; with the ranging rods produce A B to any point D, and C B to any point E. At any convenient point on B E, as F, set up the perpendicular F G, which make equal to the given distance, and leave a mark midway at *c*; leave marks also at *a* and *b*, making *a* F equal to F *b*; also make G *d* equal to *a* F or F *b*. At any other convenient point, I on B, repeat this operation; K G produced will be parallel to B C, and at the given distance from it, and if the work is correct, *a c*, *b c*, and *c d* will all be equal to each other. Exactly the same operation repeated on A B produced, D will give the parallel as required to A B.

Problem 11. Fig. 31.

To lay down on the ground an angle equal to a given angle.

Let A B C be the given angle; from B to C measure off any convenient distance as B C, and from C set up the perpendicular C D, intersecting B A at D, and measure C D, as also B D. To lay down the required angle, set off E F equal to B C, and perpendicular to it F G, which make equal to C D, and lay off the line E G; F E G will be equal to the angle C B A as required, and E G will be equal to B D.

PROBLEM 12. Fig. 32.

To bisect on the ground a given inaccessible angle, as, for instance, to find the capital of a bastion.

Let A B C be the given angle which it is required to bisect; produce C B indefinitely as to D, and A B to E; parallel to these, by the second part of Problem 6, set out the parallels F O and G O; a line through O B will bisect the angle as required. To check this, make O I and O K equal, and from I and K set off the perpendiculars I L and K L; their point of intersection at L will be in line with O B.

Amongst the many propositions of Euclid which are of the greatest service in engineering field-work, the four following are of everyday use in surveying:—

The 13th, Book I. The angles which one straight line makes with another upon one side of it, are either two right angles, or are together equal to two right angles.

The 14th, Book I. If at a point in a straight line, two other straight lines, upon the opposite sides of it, make the adjacent angles together equal to two right angles, these two straight lines shall be in one and the same straight line.

The 15th, Book I. If two straight lines cut one another, the vertical or opposite angles shall be equal.

The 17th, Book I. Any two angles of a triangle are together less than two right angles.

The 18th, Book I. The greater side of every triangle is opposite to the greater angle.

The 32nd, Book I. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are equal to two right angles.

Reference to Fig. 33 will illustrate the value of the above in triangular surveying. The line D C falls upon the straight line A B at C; then if the angles A C D and D C B measure equal to two right angles, then A B is a straight line. And if the angle A C D measure equal to the angle E C B, then A B and D E are straight lines. Now where, in triangulating a district, it so happens that such lines intersect on a narrow ridge of high ground, over which it has been difficult to sight them, and to make certain of the exact position of intermediate stations along them, this circumstance enables us, in the first place, to ascertain that C is the true intersection of the bases, and also to fix any number of points on the straight lines down in the valleys below. From such a point we may also fix the direction relative to the bases of any other points, as *a*, *b*, *c*, &c., as they may happen to be hereafter required. Also the measure of the angle E C A, being the exterior angle of a triangle, checks the measurement of the angles at A and D, taken together; the same of D C B as to the angles at E and B. Should it so happen, however, that the intersection C falls in a narrow valley, then there will be no opportunity from such a point to fix such stations as *a*, *b*, *c*, &c., as above mentioned; but if the opposite angles at C prove to be equal, and if A C D equal to C A D, plus C D A, and A C E equal to A E C, plus E A C, and the same of the others, we may consider C as safe; so much so that if circumstances, such as obstructions, for instance, had hitherto prevented such a line as C B from being set out, so that if it were purely imaginary the position of A C, relative to D E, combined with the measurements of the angles as above mentioned, would enable us to sight out C B, and if correctly done it will fall in at the point B, and no other.

Amongst the many methods for obtaining the length of inaccessible distances without instruments, the following will, I think, be found the most useful.

Problem 13. Fig. 34.

On the line A B to obtain the inaccessible length C D across a river.

On the line A B take any point E, say two chains from C, and sight the line D F, making any angle, not too acute, with the line A B; from C and E set up the perpendiculars C G and E F, to intersect D F at G and F; and from G let fall the perpendicular G H; then, because of the similar triangles

H F : H G : : F E : E D.

and D E minus C E equal to D C, the length required.

Problem 14. Fig 35.

Another method of obtaining inaccessible distances.

Let A B be the required inaccessible distance. Produce A B to any convenient point C; set out any two lines, B D and E C, making F D equal to B F, and E F equal to C F; through the point of intersection F, lay out the line A F G to intersect the line E D produced to G; G E will be equal to C A, and C A minus C B equal to B A, which was required.

PROBLEM 15. Fig. 36.

To find the intersection of two lines meeting in an inaccessible point.

Let B A and C A be the two lines meeting at the inaccessible point A. From any two convenient points, as D E on B A, set out any two lines, D F and E G intersecting each other on the line C A, as at I; make I G equal to E I, and I F equal to D I. Produce F G to intersect C A, as at K; then K L equal to I A, and K equal to D A.

But whenever there is a theodolite or box-sextant at hand, the best way of obtaining inaccessible distances is given in the two following problems.

PROBLEM 16. Fig. 37.

To find an inaccessible distance on a chain line with the box-sextant.

Let the chaining on the line A B be interrupted by the coming across a block of buildings. From the point C set out an angle of 60°, as B C D, being careful to observe that the line C D will clear the obstruction. Measure on to D, from whence you can perceive you can return to the line A B clear of the obstruction; set out again the angle of 60°, C D E, and make D E equal to C D, when E will be on the line as required, and C D or D E will be equal to the distance required, C E. Over a sheet of water some point on C B would probably be visible from C, whereby to set out the angle of 60°, but with a block of buildings or a wood this would not be the case; we must, therefore, to make E C D equal to 60°, set out A C D equal to 120°, and with the sextant it will be better to do this by setting out the angle of 60° twice, which will give the angle of 120°. It may also happen, that on reaching the point E, all forward points to guide us are invisible; then produce D E to F, and at F set out another angle of 60°, and make F G equal to E F, which will again place us on the base or main chain line A B, and E G will be equal to E F or F G.

PROBLEM 17. Fig. 38.

To find an inaccessible distance, as the width of a river, with the box-sextant.

Let A B be the length required; set out B C at right angles to A B, and make B C any convenient length; measure the angle B C A, from the natural trigonometrical tables, take out its natural tangent and multiply it by B C, which will give the required length, A B; in this way any number of inaccessible distances on the same line may be obtained as *a*, *b*, *c*, &c. In the latter cases a boat would be used with two persons in it, the boatman to guide it, and an assistant to direct him; the assistant keeping himself in line by means of two points, as B and D on the main line.

COMPUTATION OF AREAS.

In the article on "Gunter's Chain," it has been shown how the product of one side of a figure by another side is treated to obtain acres, roods, perches, and odd links, as also the principles of division of this chain; in the following remarks, therefore, in order to keep the subject within limits, we shall only give one further example, and confine our remarks to the division and treatment of irregular figures mostly, to the practice of ascertaining areas in the office, and at the end of this work to the description and use of an instrument lately introduced by Messrs. Elliott, and called the planimeter, by means of which the labour of this kind of work is very considerably reduced.

The area of any plane figure is the measure of the space contained within its boundaries. The most simple of these is the square or parallelogram composed of two pairs of parallel boundaries and four right angles; and if one side of such figure be multiplied by one of its *adjacent* sides, the product will be the included area.

The next most simple figure for admeasurement is the triangle, in which one side, being multiplied by *half* the perpendicular, let fall thereon from the opposite angle, the product will be the area. Half the perpendicular is used because the area of every triangle is equal to half a parallelogram upon the same base, and of equal altitude, or between the same parallels; and "if a parallelogram and a triangle be upon the same base, and between the same parallels, the parallelogram shall be double of the triangle."

A trapezium is an irregular four-sided figure, of which the sides and angles may all be unequal; and the area of a trapezium is equal to the product of the diagonal drawn from one angle to another, by half the perpendiculars let fall thereon from the other two opposite angles, by which operation the figure is divided into two triangles with one side common to both.

A polygon is a figure bounded by more than four sides. In a regular polygon the sides and angles are all equal, and in an irregular polygon they may be *all* unequal. The interior angles of a polygon are together equal to twice as many right angles as the figure has sides, less four. On this is based the theory of the traverse, of which further explanation will be given in another place.

The area of any polygon may be ascertained by dividing the figure into trapezia and triangles, as in Fig. 39, where the polygon A B, C D.... G A, is so divided, and the whole area is equal to

$$CE \times \frac{DK}{2} + CF \times \frac{EM+BL}{2} + AF \times \frac{BN+OG}{2} =$$

$$\begin{array}{rl}
900 \times 150 = 135{,}000 & \\
1{,}200 \times 334 = 400{,}800 & \\
1{,}280 \times 250 = 320{,}000 & \\
\hline
8{\cdot}55800 & \\
4 & \\
\hline
2{\cdot}23200 & \\
40 & \\
\hline
 & \text{A. R. P.} \\
9{\cdot}28000, & \text{or } 8\text{—}2\text{—}9{\cdot}28. \quad \text{(See Page 3.)}
\end{array}$$

The above division of a polygon into trapeziums and triangles is very commonly adopted in most offices; and where the figure is carefully divided, and the lengths carefully scaled or read off, it is very accurate. When the area to be computed consists of several fields, then the whole should be computed first, and then the fields separately. If the computation has been correctly made, the areas of the fields taken together will be equal to the area of the whole.

A little scaling, adding, and dividing may be saved in computing the trapeziums in the following manner:—With a pair of parallel rulers draw E *a* B *b*, and with a scale *twice the size* of that of your plan measure *a b*, with which to multiply the diagonal C F. The same operation may be repeated at B *c* and G *d*.

To reduce a trapezium to a triangle of equal area.

In Figs. 40 and 41, set the edge of the parallels on A C, and through B draw the parallel line B E; join A E. The triangles D A E are equal to the trapeziums A C.

To reduce a pentagon, or five-sided figure, to a triangle of equal area.

In Fig. 42 produce A B both ways; set the parallels on the points D and A, and make E G parallel to D A; join D G; set the parallels on the points D and B, and make C F parallel to D B; join D F. The triangle F D G is equal to the pentagon A B C D E.

To reduce hexagons, or six-sided figures, to triangles of equal area.

In Figs. 43 and 44 produce A B both ways; join two sides containing an angle, as A F and F E, by the line E A, and draw F *e* parallel to E A; draw the diagonal E *e*. The hexagon will be thus reduced to an equal pentagon, the side E *e* being substituted for the sides A F and E F. Now make E F parallel to D *e*, and join D *f*; proceed in a similar manner on the other side of the figure, and the triangle *g* D *f* will be the triangle required, and equal to A B C D E F.

To reduce a figure bounded by any number of sides to a triangle of equal area, Fig. 45.

Let A B C D E F G H I K L A be such a figure; produce A B on both sides; join the two sides K L and L A containing an angle, and make L *a* parallel to K A; join K *a*, thus for the present substituting K *a* for the sides K L and L A. Now draw K *b* parallel to I *a*, and join I *b*; I *b* is now substituted for I K and K *a*; proceed thus throughout on both sides until the figure is reduced to the triangle *g* G *h*, which will be the triangle of equal area required.

This method of computing is by far the easiest and most accurate; it is not necessary to draw all the dotted lines we have shown to explain the process; it is merely requisite to find the intersections on the line A B produced. After a very little practice with a good pair of parallel rulers any figure is very quickly reduced, and only one scaling and one computation are required. We recommend the system not only from our own experience, but with the knowledge that one of the principal computers in one of the Government offices, who has passed a large portion of twenty years in doing such work, finds it the most accurate and simple, after having tried every method that has been brought out.

As to the many "succedaneums" so often proposed, we will not refer to them, as we know that they are not practised even by those who have advocated them.

It has been already observed that when the plan consists of more than one field, the computation of the whole should be compared with the computation of the enclosures taken separately, and then added together. The computing book should be kept in such manner that the calculations may be referred to at a future time.

Where an extensive area has been triangulated, the whole should be first computed from the triangles, making such additions and deductions as may be rendered necessary by certain portions of the area under survey being cut off by the sides of the triangles, or by other portions outside of the survey proper being included within the triangles.

To calculate the area of a triangle by means of the lengths of the sides: take *half the sum* of the three sides added together; from such half sum subtract the three sides separately; the square root of the three remainders and the half multiplied consecutively together, is equal to the area of the triangle; let A, B, and C represent the three sides of a triangle, and S the sum of the three sides; and therefore $\frac{1}{2}$ S, half the sum; the area of the triangle will be equal to

$$\sqrt{\{\tfrac{1}{2} S \times (\tfrac{1}{2} S - A) \times (\tfrac{1}{2} S - B) \times (\tfrac{1}{2} S - C)}.$$

The fences of fields are generally more or less crooked or uneven, and one of the first operations, before commencing the computations, is to reduce them to straight lines, which is done by a system of "give and take;" to effect this, draw a fine line on a very clear bit of tracing-paper, some six inches long, and lay this on the fence so that you can see when the line takes as much in on one side as it leaves out on the other; then prick through at both ends, and draw in the line in pencil, which will equalize the fence; or lay down the edge of rule, guessing at the equalization, and draw in a pencil line; if, when the rule is removed, it does not appear quite correct, rub it out, and draw in another, until there appears to be as much left out as taken in, so as to balance; a little practice will soon make this right.

CHAPTER IV.

SURVEYING INSTRUMENTS.

The Cross Staff and Optical Square.—The Prismatic Compass.—The Sextant and Box-Sextant.—The Transit Theodolite.—The Ordinary Theodolite.—Colonel Everest's Theodolite.

The Cross Staff and Optical Square are used for setting out lines perpendicular to others, and which seldom exceed five or six chains in length. The cross-staff, though rather old-fashioned and cumbersome, is still very commonly used for this purpose, and it is made two different ways. One shape consists of four sights, fixed at right angles upon a brass cross, which, when in use, is fixed to the top of a staff. To set out a perpendicular to a given line with this instrument, and at a given point, thrust the staff into the ground at this point, so that through one pair of sights you can see both ends of your main line by moving from one side to the opposite one, which will ensure your being exactly on the line, and your instrument in the proper position; now move round to the cross-sights, and in line with them stick up a pole, you will thus have set off your perpendicular.

The second shape consists of a hollow brass cylinder, about three or four inches in diameter, and about as many in depth; through this are pierced sights at right angles to each other, and it is adapted to a staff. This is to be used exactly in the same manner as the other.

The *optical square* is very superior to the above for convenience, dispatch, and portability. It is a shallow, circular, brass box, containing two mirrors fixed at an angle of 45° to each other, so as to reflect a right angle or of 90.° Fig. 46 explains the construction and use of this little instrument. A and B are two mirrors, fixed at an angle of 45° to each other; E is supposed to be the place of the eye, and the line F E is perpendicular to the line D E. A ray of light from the object F falling upon the mirror A at the above angle will be reflected at the same angle on to B, from whence it will be reflected at the same angle again to the observer's eye at E. The mirror B has only the upper half of its surface silvered, so that the eye looking along the line E D through an aperture in the box, sees D by direct vision through the lower and unsilvered portion of the mirror B; and any object seen in the upper or silvered portion of it, and coinciding so as it were to make one with the object D, is on a line perpendicular to E D at the position E of the observer. The reader will see how rapidly this instrument is used by standing at any point on a line, and holding it in the hand, looking towards an object at the end of the line, and sighting an assistant with a rod until it is seen to coincide with the first object; a perpendicular is then set off from the point where the observer is standing.

In the same manner, if a perpendicular is to be made to fall from a given point on to a given line, it is only necessary to walk along the line until you come to a point from which you see one end of your line and an object at the given point coinciding together in the mirror D, when you will be standing at the spot on which the perpendicular will fall.

It is on a similar principle that the box-sextant, one of our most valuable surveying instruments, is constructed, to which we shall come presently.

THE PRISMATIC COMPASS.

This pocket instrument, shown at Fig. 47, is contained in a shallow brass box about two inches in diameter; it is used for measuring horizontal angles, which it will do with a considerable degree of accuracy considering the nature and size of the instrument, and as this may be done by merely holding the instrument in the hand, the work is carried on with much rapidity; for an approximate survey, or for filling in where the principal lines and stations have been previously fixed, and for taking the angles on lines not exceeding fifteen or twenty chains in length, it is very useful, more particularly where there are not many angles to take before we chain into a point previously fixed; for this purpose of filling in it is daily used to a very considerable extent.

B is the brass box containing the compass card C, to the under side of which is attached a magnetic needle, turning upon an agate centre fixed to the bottom of the box. The compass card is usually divided to thirty minutes, according to the size of the instrument, and therefore to half a degree; a more minute division must be appreciated by the eye. The sight vane V is mounted upon a hinge, so that when the instrument is not in use it may be turned down flat on the glass. Along the opening in the sight vane is stretched a fine thread, which is made to intersect the object on which the observation is taken. P is a prism attached to a plate sliding up and down in a socket S, so that it may be raised or lowered to admit of distinct view of the divisions on the compass card; this also is supplied with a hinge joint, by which means it can be turned down into the box. In the plate attached to the prism is a narrow aperture, to which the eye of the observer is applied when making an observation; there is a button attached to a spring, and which on being slightly pressed checks the vibrations of the compass card so as to bring it sooner to rest when an observation is being taken. There is a stop in the side of the box B, by touching which the compass is thrown off its centre when the instrument is not in use; particular care should always be taken to do this after making observations, or the constant vibrations of the needle will soon wear off the delicate finish of the agate point and sensibly impair the value of the instrument. A brass cover fits on to the top of the box, so that it may be carried in the pocket, or packed in a leather case, which by means of a belt may be strapped to the side, which is much the safest plan. Besides the above, Messrs. Elliott make a larger instrument of the same description with a silver compass ring, and which is a superior instrument; this as well as the above is provided, if desired, with a stand for the instrument to be placed upon when observations are being taken; as this, however, is rather inconvenient to carry about in the fields, the prismatic compass when in use is generally held in the hand.

On looking through the aperture above mentioned, and raising or lowering the prism in its socket, distinct vision of the graduations on the compass card is soon obtained, as they vibrate under the sight vane; seen through the prism they appear as prolongations of the thread as each successively comes into coincidence with it, the latter being seen by direct vision through the part of the aperture projecting beyond the prism.

The following shows the method of using the instrument: turn up the sight vane V and the prism P on their hinges, and hold the instrument as nearly horizontal as possible, so that the needle may play freely; raise the prism in its socket until the divisions on the compass card are distinctly seen through the prism. Turn the instrument round until through the portion of the aperture projecting beyond the prism the object to be observed is seen to coincide with the thread of the sight vane; now bring the card to rest by touching the spring; the reading at the graduation on the compass card which appears as a prolongation of the thread gives the angle made by the line on which you are observing with the *magnetic* meridian. The bearing of a second object from the same place being made in the same manner, the difference between the two bearings gives the angle subtended by the two objects observed. When vision is obstructed by the sun, a dark glass G is interposed.

By repeating an observation two or three times, and taking the mean, the bearing will be obtained more accurately, and it will be still more so if the bearing is again similarly taken at the other end of the line, and the mean taken again between the bearing at one end and at the other. This will also check any local influence on the needle.

M is a mirror which is made to slide on or off the sight vane, and may if required have its face turned downwards, or it may be inclined at an angle by means of its joint *h;* it will remain stationary on any part of the sight vane by the friction of its slides. This is applied for the purpose of taking the bearing of objects much above or below the level of the point of observation; it is put on with its face upward or downwards accordingly as the objects observed are much above or below the level of the observer.

The circle being divided to 360°, the compass card is also so divided, and, as we have said before, each degree is again subdivided. In some instruments the degrees are numbered from the North point round by East, South, West, and to the North again up to 360° consecutively; in others, and as we think in a better way, the instruments are numbered from North round by East to the South Pole as 180° and from the South round by West to North again as 180° more, completing the 360°; the readings in this manner are much simplified, for if we have a reading so many degrees N.E. we know it will be the same number of degrees S.W.; for instance, in Fig. 48, where the circle is divided into twice 180°, the one reading from North round by East to South, and the other from South round by West to North, a bearing is taken to the left of the North, and reads 135° West from the South Pole; then we know at once that it will read 135° East reading from the North Pole. This renders unnecessary any mention of North and South in reading a bearing, as we know at once that all bearings reading East go progressively from North to South, and all bearings reading West go progressively from South to North.

The extent to which this instrument may be used will depend on many circumstances, such as the ability of the observer, the quality and object of the survey, &c. As there are no means of reading correctly to within fifteen minutes, it of course only gives approximations; but then something of the same kind may be said of larger instruments, which divide to minutes, and not to seconds. The reader must ascertain for himself the value and capabilities of the instrument, and thereby judge of the extent to which it may be used; much, also, must inevitably depend on the manner; a person with a steady hand will manage this instrument better than another; much, also, depends on the diameter of the instrument used.

In Fig. 49, let A B and C D be portions of two main chain lines of a survey; it is required to fill in the fencing along *a*, *b*, *c*, *d*, *e*, *f;* from the point *a* take the bearing of A B, and of *a f*, *f*, being a station on the line C D, and previously fixed for this purpose; fix on some object, in the fence, for instance, near *d*, in line with *a f;* take also the bearing of *a b*, and chain to *b*, offsetting as you go along, and at *b* take the bearing of *b a*; if *a b* bear 125° East, then *b a* shall bear 125° West; if there is a difference of a few minutes between the readings, take the mean for the correct reading; at *b* and at *c* repeat the same work; on reaching *d*, find this station on the line *a f*, and take the bearings of *d a* and *a f;* whatever the degree the one is West, the other will be East; in the four-sided figure *a b c d*, the four angles will be equal to four right angles, or 360°; and as, having the bearings, we can obtain the angles from them, we can ascertain how far our bearings give us these four right angles; if, as there is sure to be with this instrument, we find a difference of some few minutes, being over or under the four right angles, divide this difference over the bearings. It is also quite possible that from *d* we may be able to get a bearing on some known point on the line A B, by which, when we come to plot, we may check the position of *b*, for if *d g* reads 130° West, then plotting from *h* on the plan a line bearing 130° East, will run into station *d*, when this also has been plotted. For the continuation of our work *d*, *e*, *f*, we have only a triangle, the three internal angles of which should be equal to two right angles, by which we may check the angles at *d*, *e*, *f*. It cannot fail to be observed that all these bearings may and should be plotted from the station *a* without once moving the protractor; and as we consider it very desirable that the student should make himself thoroughly master of the prismatic compass before he engages with the principal surveying instrument, the theodolite, we will go a little further into this matter, premising, however, that it will be more as an introduction than anything else to what is termed surveying by traverse, to which we shall come at a future page; probably, also, it will make the subject more familiar to the reader when he comes to it.

At *a* let the bearing of *a d* be 91° 5′ East, and the bearing of *a b* 136°; note this at the commencement of your line *a b* before you commence chaining; on reaching the point b, take the bearing of ba, and let this be 136° 5′, note this at the end of the line, and take 136° 2½′ for the mean or corrected bearing East and West of the line, ab; in the same manner, let the mean bearing of bc equal 89°, and that of cd equal 30°; at d take the bearing of da, and let this be 91° 10′; then the mean or corrected bearing of ad or df equal to 91° 7½′.

Now the angle dab, equal to 136° 2½′, minus 91° 7½′, equal therefore to 44° 55′, the angle abc is made up of the angle abN plus the angle Nbc; and abN equal to Sab; then abN equal to 180° minus 136° 2½′, equal to 43° 57½′; and Nbc equal to 89°, the angle abc equal to 43° 57½′ plus 89°, equal to 132° 57½′. The angle bcd is made up of the angles bcN and Ncd, and because NbS and NcS, the two magnetic meridians, are parallel, and Nbc equal to 89°, then bcN equal to 180° minus 89° equal to 91°, and Ncd equal to 30°; then bcd equal to 121°, and cda equal to Sda minus Sdc equal to Ncd equal to 30°, and cda equal 91° 7½,* then cda equal to 61° 7½′.

And $44^\circ\,55' + 132^\circ\,57\tfrac{1}{2}' + 121^\circ + 61^\circ\,7\tfrac{1}{2}' = 360^\circ$.

In the same manner we may check the triangle dcf, the three angles of a triangle being equal to two right angles.

Observe that three more lines for filling in have been run from g, on AB to h on CD, and that a tie line has been chained from d to i, being a further check upon the work, for if in plotting d or i lean one way or the other out of position, then the line di will not plot true.

For thus filling in, the large 5-inch prismatic compass with silver ring is very useful, and when once the surveyor has become familiar and handy with the use of the instrument it will give very close approximations.

To plot these bearings, place your protractor at a, so that AB shall have its due bearing with regard to the magnetic meridian NS, the North and South being each represented on the protractor by 180°; prick off all the bearings, as af, ab, bc, cd, &c., draw af and ab, making the latter the length of the measured chain line; lay your parallel ruler from a to the point pricked off, as the bearing of bc; parallel to this rule in bc make bc.

With due attention this instrument may also be used for a road survey, as at Fig. 50; the bearings along the bends of the road are taken at the same time the lengths are chained, care being taken at the same time to get bearings from several stations to one or two objects likely to be visible from different parts of the survey. The work is plotted as before, by laying off the bearings all from one point. Fig. 51 illustrates a similar method of getting a plan of a stream; but here a *visual line*, A B C, has been set off, for which purpose any two objects are selected to lay down the line; as the bearings of the chain lines along the banks of the stream are measured, a few bearings are taken here and there at different stations upon the two prominent objects, and, if convenient, one or two back sights upon the starting point; if the chain lines have been correctly measured, and the bearings correctly taken, these observations upon the prominent objects will intersect and prove the degree of accuracy of the work, for it must be remembered we cannot expect the same accuracy as with a theodolite; if, however, the prismatic compass is applied in the manner we have been pointing out, errors will be inappreciable. In both Figures 41 and 42 the measurement of the bases A B and A C will completely check the work.

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* Because a straight line meeting two parallel lines makes the alternate angles equal, then S dv equal to N ad.

There is yet another service to which the prismatic compass is well adapted, which is, from the bearings to ascertain the angles which fences make with chain lines, more particularly those straight fences which do not require offsetting. If in the course of filling in, such a fence be intersected by two chain lines, the intersections of course fix the direction of the fence upon the plan; a check upon the dependence on the chaining only is afforded by the use of the prismatic compass in the manner here mentioned.

It may also be rendered very useful in testing the accuracy of a plan the correctness of which we have any reason to doubt. By drawing two or three lines across the plan, and marking on it the angles they make with each other, and then in the field with the plan set out such lines with rods, take their bearings, and see if the angles agree with those measured upon the plan; this may also be done without rods by drawing any line on the plan through two or three junctions of fences, setting off such line accordingly on the ground, and taking the angles which adjacent fences make with the above line, and then comparing these angles with those similarly made on the paper.

Supposing that the reader has had sufficient confidence in our advice to follow it with regard to the *two former practical lessons* we recommended, and that he has satisfied himself of his ability on those points, we now further advise him to practise what we have just observed in this section on the prismatic compass; at first let him content himself with laying out triangles and taking the bearings of the sides; then lay out a series of these three-sided figures, take all the angles, remembering that the three angles of a triangle are equal to two right angles, and let him see how near he can bring this out in practice; let him also chain his lines, and plot the work. In the next place, lay out four and five-sided figures, and so proceed gradually; he may also, by degrees, extend the length of the lines circumscribing the above figures. By this means he will not only make himself perfectly familiar with the use of this instrument, but he will ascertain of his own knowledge its capabilities, and to what extent he may trust himself to use it. This is only to be acquired by one's own practice; and this being acquired, we know what reliance we may place on our own exertions.

Before proceeding to a description of the box sextant and the theodolite, we must make a few observations on a particular member of these instruments, which is the following.

THE VERNIER.

In our most complete and perfect instruments for measuring angles, the limb is divided into degrees, and these are again subdivided into parts of a degree; the number of these subdivisions depends on the size of the instrument, and the degree of accuracy to be desired in the instruments; in some the degrees are halved, which gives a division to 30 minutes; in others the degree is trisected, giving 20′, or divided into four, giving 15′, whilst in others the subdivision extends to 10′. Suppose, now, that the index of a theodolite, which corresponds to a visual line, should point to an angle between 21° 30′, and 22°; whatever the difference may be, we can only give a guess at it; this would be very unsatisfactory with an instrument professed almost to attain perfection; to supply this defect an additional scale, called "a vernier," has been contrived to measure parts of space otherwise undefined between any equidistant divisions of a graduated measure.

The length of this vernier scale is exactly equal to a certain number of the divisions *on the limb*, and the *number* of divisions graduated on the vernier is *one more* than the number upon the same length on the limb. The length taken on the limb for that of the vernier depends on the degree of minuteness of subdivision to be attained; whatever this may be, the difference of a division upon the vernier from a division on the limb is equal to the *n*th part of a division upon the limb, *n* representing the number of divisions on the vernier.

If we take two straight lines, or arcs, of exactly equal dimensions, calling one A and the other B, and let one be divided into *one more* equal parts than the other, *then the difference of any* *two of the equal parts of the two lines will be a fraction, the numerator of which is the length common to both lines, and the denominator the product of the numbers of parts into which each is divided.* If we put A for the *length* of each of the equal lines, as also n and $n+1$ for the numbers into which each line is respectively divided, the length of the *divisions* of each will be,

$$\frac{A}{n} \text{ and } \frac{A}{n+1}, \text{ and their difference;}$$

$$\frac{A}{n} - \frac{A}{n+1} = \frac{A}{n \times (n+1)}$$

In figures let A = 14° 30′, or 29 half degrees, or 870′; then,

$$\frac{870}{29} = 30;\ \frac{870}{30} = 29;\ \frac{870}{870} = 1'; \text{ and}$$

$$30 - 29 = 1.$$

Suppose, in the limb of an instrument, each degree is subdivided into three parts, or to 20′; it is required that the vernier shall read to 1′; then an arc of 19 parts on the limb divided into 20 parts on the vernier, will give the scale required; the 19 parts on the limb divided to 20′, will be 6° 20′, or 380′; then

$$\frac{6^\circ\ 20'}{19 \times 20} = \frac{380'}{380'} = 1'.$$

This *difference* becomes the index for subdividing the smallest division on the limb, and the amount of this subdivision is ascertained by the coincidence of a stroke on the vernier beyond zero, with a stroke on the limb beyond the division last past over by the zero of the vernier; if this coinciding stroke on the vernier be the fifth, then it will be 5′, if the ninth, it will be 9′, &c., which must be added to the number of degrees, or of degrees and minutes passed over by the zero of the vernier on the limb. As there can be no error as to the point of coincidence of a stroke on the vernier with one on the limb, there can be no mistake as to how many minutes will have to be added, as the measure of a fractional portion of the division on the limb that the zero of the vernier has entered into. A rough judgment will easily be formed as to whether the zero of the vernier has passed over a ½ or a ⅓ of a division on the limb, and this will readily guide the eye as to the part of the vernier where we must look for the coincidence of one of its strokes with one on the limb; should there, however, happen to be any uncertainty about the exactitude of coincidence, then 20″, 30″, or 40″, may be taken instead of the last minute, accordingly as the eye can best judge by the magnifying-glass of the small quantity defective of perfect coincidence; in this the observer will be assisted by examining the places of the preceding and following strokes.

Many of the instruments now made by the best makers have the degree divided into 20, 15, and 10 minutes, by means of a vernier with divisions and subdivisions acting with divisions and subdivisions on the limb. On examining the limb of an instrument, as, for instance, a sextant, it will be observed that it is divided at every 5 degrees by a longer divisional line than the others; these spaces contain each 5 degrees, all of which are again subdivided by shorter lines representing minutes; if there be four spaces, each will be 15 minutes, and if six, then each will be 10 minutes. On the vernier, likewise, the divisions marked by longer lines are subdivided by shorter ones, the divisions being minutes and the subdivisions being seconds. If the spaces between each long division be divided into four, each will be 15″, and if into six, then 10″; the number of subdivisions between the minutes of the vernier are generally, though not necessarily so, the same as the subdivisions between the degrees on the limb, so that if the limb is divided into 20′, then the vernier is divided into 20″.

With regard to reading off an angle on the limb of an instrument provided with a vernier; if the zero of the vernier coincide with a division on the limb, such division gives the angle measured; thus, if it coincide with 30°, then 30° is the measure of the angle; if with the first or second long division beyond the thirty, then the reading will be 31° or 32°, as the case may be; but supposing that the degree on the limb is subdivided into three parts, each, of course, indicating 20′, and that the zero of the vernier coincides with the first of these subdivisions beyond 31°, then the reading will be 31° 20″, and if with the second, the reading will be 31° 40′. In the next place let us consider that, on a limb divided as above, and with a vernier having also three short divisions between the long ones, and therefore reading to 20″, the zero of the vernier has passed by 31° 20′, but not reached the 40′; we must now look for a division on the vernier coinciding with one on the limb; let it be the first *long* division, then the reading will be 31° 21′; now let us suppose that it is the second *short* division beyond the first long one beyond zero, then the reading will be 31° 21′ 40″.

The Figures 52, 53, 54, 55, and 56, have been introduced to illustrate our observations on the vernier; in Fig. 52, the degrees from 35° to 40° are shown on the limb, each degree being divided into four divisions of 15′ each; in the vernier each division between the long strokes being also divided into four, gives 15″ for each subdivision; in our example the zero has passed by 35° 15′, and the second subdivision counting 30″ beyond the two main divisions counting 2′ *on the vernier*, coincides with a stroke on the limb; then this quantity of 2′ and 30″ is to be added to 35° 15′, making together 35° 17′ 30″. In example 53, each degree on the limb is divided into six parts of 10′ each, and each minute on the vernier is divided into four spaces of 15′ each, and the vernier reads 76° 52′ 45″. In example 54 we read 131° 44′ 40″. In example 55, each degree on the limb is divided into two spaces, and the vernier reads to minutes, and we have 22° 44′ for the reading of the instrument. In Fig. 56, supposed to give a scale of a mountain barometer, every inch on the limb is divided into ten parts, and each tenth part is divided into two, each of which will be five-hundredths of an inch; the scale of the vernier is nine-tenths of an inch divided into ten; the long divisions give one-hundredth of an inch, the subdivisions reading thousandths; in our example 56 the instrument reads 30·420 inches. With any instrument provided with a vernier before him, the reader will have but little difficulty in understanding this with the help of the magnifying-glass, for it must be remarked that, for the sake of plainness, our scales in the examples are much exaggerated; the first thing to be looked at is, into how many divisions each degree on the limb is divided, each division being a certain number of minutes accordingly; and in the next place, into how many divisions, if any, the minutes of the vernier are divided; this ascertained and remembered, there will be no difficulty *after a little practice* in reading off the instrument.

It will no doubt be readily perceived that the accuracy of observations thus made, of necessity must depend on the correct division of the limb of the instrument, and still more so where it occupies the whole circumference of the circle; another source of error may be that the centre of the arm of the index may not be exactly coincident with that of the instrument itself. To correct this, two opposite verniers are generally applied to theodolites, and by taking the mean of the vernier readings this correction is made; the best proof of the wisdom of the introduction of this double vernier is, that in most cases very minute observation will show that they do not read exactly alike. Two opposite indices will correct such errors as those just mentioned, as will also three verniers placed equally distant from each other; these three are often placed on theodolites with a large limb.

If we have the limb of a theodolite slightly compressed on one side, and elongated on the other, but with the index revolving round the point of intersection of the long and the short diameters; now, although the end and side divisions are altered by the compression, the readings of two verniers placed diametrically opposite to each other will not correct the error, inasmuch as the alternations correspond at the opposite ends of the diameter; but *three* verniers equidistant from each other, although they do not give an exactly mathematical correction, approximate sufficiently near for all practical purposes; and quite as well as four, with which some instruments are provided, though not for this purpose.

At the ends of the limbs of sextants are small additional portions of arc, which are called the arcs of excess, the nature of which will be explained under the head of "*the sextant.*" When reading off on this arc, we first observe what quantity on the limb is passed over from the zero *by the zero of the vernier;* in the next place we proceed to read backwards on the vernier counting from the last division which we now consider as the zero.

THE SEXTANT AND BOX-SEXTANT.

The principle of construction of the sextant, as well as of the quadrant, is that the total deviation of a ray of light after successive reflection in the index and the horizon glasses is double the angle of inclination of those two glasses or mirrors.

That the angle of incidence is equal to the angle of reflection needs no demonstration; of this mention has been made in the article on the optical square; the fact being admitted, let A M, Fig. 57, be a mirror, and let S be an object from which the radiant S B falls on A M at B; the angle M B C is equal to the angle A B S, equal to 45°; let D B be perpendicular to A M; now let the mirror revolve on B into the position *a m*, S B remaining fixed as before, and let A B *a*, D B *d*, and M B *m* be equal to 10°; A B S being equal to 45°, *a* B S will be equal to 35°, and the angle of incidence being equal to the angle of reflection M B *c*, will also be equal to 35°, and it will at once be seen that C has moved to *c* in an arc equal to twice A *a* or D *d*.

In Fig. 58 let I *i* and H *h* be two glasses, which are known as the *Index glass* and the Horizon glass, let them be placed with regard to each other at an angle I A H, and let S I be a radiant from S making with the mirror I *i* the angle S I *i*; the reflected ray I H will make with I *i* and with H *h* the angles H I A and I H E, each equal to the angle of incidence S I *i*. The angle

A H V = the exterior angle I H h, and = the two interior angles H I A and H A I of the triangle I H A; and H V A = I V E, therefore V I E + I E V = V H A + H A V, and therefore to H I A + 2 H A I, but V I E = S I i is also = H I A; taking away these equals, the remainder S E H = the remainder twice the angle H A I.

The above is the principle of construction of quadrants, sextants, and reflecting circles; for I is the *index* mirror fixed on and revolving with the index bar, I A, by the vernier of which the angle is measured or read off along the limb L M; H is the horizon glass through the *unsilvered* portion of which any object is seen by *direct* vision from the point E, where the eye is applied, whilst the *silvered* part receives the reflection from the index mirror I of an object, as at S, as this mirror is made to revolve until the image of contact is effected at H. It will be seen that whilst the angle formed by two objects with the eye is thus obtained, the angle really measured is that formed by the bearings of the two glasses relatively to each other; but the limb L M is so figured as to give twice the angle between the glasses; thus, if the spaces on the limb measure 15′, they are figured 30′; therefore the reading of the instrument when in adjustment, gives the angle between the bearing of the object from the centre of the index mirror and the bearing of the reflected image from the place of the eye.

Fig. 59 is a representation of one of Elliott's sextants; L L′ is the limb, and V the vernier, graduated according to the size of the radius of the instrument; I is the index mirror fixed perpendicular to the plane of the instrument, and over the centre of motion of the index I V; M is a microscope connected with the index by an arm, and by the aid of which the whole of the vernier may be examined. The limb is also divided from zero towards L′, forming the arc of excess by which the index error of the instrument is determined, as will be seen hereafter. In reading off on the arc of excess, the vernier must be read backwards; that is, the division on the limb next *to the left of the zero of the vernier* being read, the divisions of the vernier *to be added* must be read from the other end, marked 30, to a division coinciding with a division upon the limb. A clamp C is attached to the index to fasten it to the limb; T is the tangent screw by which the index may be moved any small quantity after it is clamped; this tangent screw having a most delicate motion, enables the observer to perfect the contact of the objects observed in a manner which could not be done by the hand acting on the index merely.

H is the horizon glass, the lower half of which only is silvered; this glass, also, must be perpendicular to the plane of the instrument, and in such a position that its plane must be parallel to the plane of the index mirror when the vernier is set at zero; a deviation from this constitutes the *index error*; for adjusting the horizon glass there is a small screw at the bottom of the frame in which it is set.

O is a telescope carried in a ring, attached to a stem called the *up and down piece;* this may be raised or lowered by means of a milled headed screw, till the objects seen in the telescope directly, and by reflection, appear of the same brightness, which will be the case when the field of view in the telescope is bisected by the line on the horizon glass that separates the silvered from the unsilvered part. A plain tube and two telescopes, one showing objects erect, and the other inverted, are supplied with the sextant; the inverting telescope having a higher magnifying power than the other, shows the contact of objects better; the adjustment for distinct vision is obtained by sliding the tube at the eye end of the telescope in the inside of the other.

Four dark glasses of different shades and colours are placed at G, any one of which may be turned down between the index mirror and the horizon glass, when the light seen from any object by reflection is too intense for the eye; similarly, there are three more at B, any one of which may be brought before the horizon glass when too bright a light proceeds from the object seen by direct vision. For the same purpose, to the eye end of the telescope a dark glass may be fixed, one or two of which are generally in the box containing the sextant; they are, however, principally used in observing the sun's altitude with an artificial horizon, or for ascertaining the index error, as the use, for this purpose, of the shades attached to the instrument would involve any error they might possess.

In the instrument case is a key for turning the adjusting screws.

When in use, the instrument is held by a handle placed underneath.

By means of the *up and down piece* above mentioned, the axis of the telescope is brought to the same height above the plane of the instrument as the height of the line of separation between the silvered and unsilvered portions of the horizon glass, to prevent parallax of the reflected rays; by this adjustment, also, the *image* of one object may be made as luminous as the *real body* of the other, by increasing the light of one and decreasing that of the other; that is, by making more of the silvered or of the unsilvered part of the horizon glass fall before the object glass of the telescope, when this is necessary.

When the sun is the body observed, the light is generally too intense for the eye to bear with the telescope; to render the rays tolerable to the sight, the coloured glasses above mentioned are interposed, and the blank *tube* instead of the telescope is applied, by which means the sun may be viewed without inconvenience.

The adjustments required in the sextant are the following:—

The index and horizon glasses must be perpendicular to the plane of the instrument.

The planes of these glasses must be parallel to each other when the zero of the vernier is set to zero on the limb.

The optical axis of the telescope must be parallel to the plane of the instrument.

To test the perpendicularity of the index glass.

Slide the index to about 40° or 45° on the limb, and holding the instrument nearly parallel to the horizon, that is, about level, look obliquely into the index glass to see whether the sharp edge of the limb seen by reflection is an exact and straight continuation of the same edge of the limb seen by direct vision. If the edge and its image form one continued arc, the index glass is perpendicular to the plane of the instrument; but if otherwise, the instrument must be sent to the maker to adjust.

To adjust the horizon glass.

Fix the zero of the vernier to zero on the limb, hold the instrument level, and looking through the eye end steadily at some convenient object, sweep the index slowly along the limb; if the reflected image do not pass so as to coincide with the direct object, but the one projects beyond the other, then the glasses are not perpendicular to the plane of the sextant, and supposing we have already tested the index glass, then the horizon glass is at fault; this is adjusted by turning a small screw at the bottom of the frame in which it is set until the reflected image passes exactly over the direct object; the horizon, or any straight distant line, may be used for this purpose.

To examine the parallelism of the planes of the two glasses when the index is at zero, or to determine the index error.

The best way to do this is to measure the sun's diameter both to the left and to the right of zero; that is, first on the arc of the limb, properly so called, and next on the arc of excess. First clamp the index at about 30′ to the left of zero, and with the sun for object, bring, by means of the tangent screw, the reflected image of the upper limb into contact with the direct view of the lower limb, and set down the minutes and seconds marked by the vernier. Now clamp the index at about 30′ on the arc of excess to the right of zero, and bring the reflected image of the lower limb into contact with the direct image of the upper limb, and take off the reading from the vernier. Set down this reading beneath the last observed; then half the sum of the two is the correct diameter of the sun, and half their difference is the index error. When the reading on the arc of excess is the greater of the two, then the index error must be *added* to all the readings made with the instrument; when less, then the index error must be subtracted; it is as well to repeat this two or three times, and the *difference* of the sums of the readings on *the arc*, and on the arc of excess divided by the number of times the operation has been repeated, will be the index error; and the *sum* of all the readings divided by the number will be the sun's diameter.

To examine the optical axis or line of collimation.

When the sun and moon, having a distance of 90° or more, are brought into contact just at the wire of the telescope nearest to the plane of the instrument, the index fixed, and the position of the instrument altered to make the objects appear on the other wire, then if the contact still remains perfect, the axis of the telescope is in proper adjustment; but if otherwise, alter it by means of the two screws which fasten the collar into which the telescope screws to the *up and down piece*, tightening one screw and turning back the other, till after a few trials the contact remains perfect at both wires.

To examine the error from the imperfection of the dark glasses.

With the dark glass only at the eye end of the telescope, first bring the *reflected* image of the sun into contact with his image seen directly through the unsilvered half of the horizon glass. In the next place, having removed this dark glass, set up separately, and then in combination, the various coloured glasses; the angle through which the index must be moved to perfect the contact is the error of the dark glasses, or of any combination of them, used in making the observations. This error should be recorded for each glass, and for each combination of glasses, and so as to be easily referred to.

To examine the limb and the vernier.

When all these observations and adjustments have been made, the instrument may be applied in practice, if we may rely on the skill and reputation of the maker; but if otherwise, it is as well to examine the materials of which the instrument is made. The accuracy of the divisions on the limb and vernier may be tested by sliding the latter slowly along the former, and examining how they agree, or as it were *stepping* the arc of the limb with the vernier, remembering what we have said about the total length and the divisions of the latter in another place, under the head of "Vernier;" the intermediate strokes on the vernier itself may be examined, lastly, by the strokes on the limb taken in various places. Of course, in doing this, the microscope is to be used; the divisions should all be fine and distinct, and the inlaid plates, on which they are engraved, should be perfectly level with the surface of the instrument. If all this be not found perfectly correct, the instrument should be rejected, or at least fresh divided by a good maker.

To examine the glasses of the mirrors: first, to try the quality of the glass, whether the surfaces be perfect planes, and whether the faces of each glass are parallel.

Bring two distinct and distant objects into good contact, and let them be seen at the upper edge of the silvered part; now move the instrument in its own plane, and move the image and body along the line of separation or edge of the silvered part. If the coincidence is not disturbed, the plane is perfect. Or look separately into each reflector in an oblique direction, and observe the image of some object; if it appears clear and distinct, the glass is of good quality, but if it appears notched or irregular, the glass is bad. Again, if the image appears singly and well-defined about the edges, then the two surfaces of the glass are parallel, but if misty, or appearing like two images, the two faces of the glass are not parallel, and consequently it is not fit for its purpose. This examination may be performed with the flame of a candle.

Very possibly, on reading these instructions as to the examination of all the parts successively of the sextant, they will be considered very tedious, but it must be remembered that when they are once performed the work will not have to be repeated if the instrument is taken proper care of; in the next place, nothing will tend more to make the reader thoroughly acquainted with the principle and construction of the instrument; to this we must add, that it is not one of ordinary use, being principally employed for astronomical observation, where these are required, for maritime surveying, and a few more important matters. A little attention to the nature and construction of the instrument will show of what it is capable; but there being no outward case to protect the various parts, it will not bear the slightest approach at ill usage.

When an observation is to be made with the sextant, it is evident that the plane of the instrument must be held in the same place as the two objects of which the angle is required; vertically, therefore, if altitudes are to be taken, and horizontally or oblique where horizontal or oblique angles are to be taken. The reader will at first find some little difficulty in thus adjusting the plane of the instrument, and his first attempts will very likely be failures, but as, indeed, in the use of all instruments, after a few further trials, facility will gradually come round. The instrument is to be held in the right hand by the handle underneath, though this will, of course, be sideways when the plane of the instrument is held vertically; and it should not be *grasped tight*, but held with ease, though with safety, for by holding it too hard the hand is apt to grow unsteady.

Supposing now *an altitude*[*] of an object—as of the sun, for instance—is to be taken, with the horizon of the sea before the observer; one or two of the dark glasses must be used according to the brilliancy of the sun; the eye is applied to the eye-piece, and the sight directed to that part of the horizon immediately beneath the sun, and, at the same time, the left hand gently glides the index forward, until the image of the sun, reflected from the *index glass*, appears in contact with the horizon at the line of demarcation between the silvered and unsilvered portions of the *horizon glass;* clamp the index, and with a gentle motion of the tangent screw, perfect the contact of the *upper* or *lower* limb (or part) of the sun with the horizon, when it will appear a tangent to his circular disc.[†] To the angle read off apply the *index error* (*see. ante*); then add or subtract the sun's semi-diameter,[‡] as given in the *Nautical Almanac*, according as the lower or upper limb has been observed to obtain the apparent altitude of the sun's centre. Before, however, this observation can be used for determining the time, the latitude, &c., it must be further corrected for parallax[*] and refraction,[†] to obtain the true altitude, *adding* the former, and *deducting* the latter. A quantity must also be deducted for the dip when the *sea horizon* is used, which is not the case when an artificial horizon is employed; but this will be mentioned hereafter.

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[*] The altitude or elevation of any point of the sphere is an arc of a vertical circle intercepted between it and the sensible horizon; this is peculiarly denominated *sensible* or *apparent* horizon, to distinguish it from the rational or true horizon; the higher the eye the further is the visible horizon extended.

[†] If the observer knows his latitude approximately he may find the miridional latitude nearly, to which he may at once set his instrument: he will then have only a small quantity to move his *index* to perfect the observation. Take from the *Nautical Almanac* the declination of the object, and if it be the same name with the latitude, add it to the co-latitude; if of a different name, subtract it; the sum or difference will be the meridian latitude.

[‡] The apparent diameter of the sun or moon is the angle under which they appear to the observer; the amount of this depends upon the real magnitude

EXAMPLE.

	°	′	″	°	′	″
Observed altitude of sun's lower limb. . .				60	45	5
Index Error				—	0	15
Apparent altitude				60	44	50
Sun's semi-diameter				+	15	46
Parallax				+	0	4
Refraction . .	— 0	0	34	61	0	40
Dip for an elevation of 18 feet	— 0	4	3	—	4	37
True altitude				60	56	3
	— 0	4	37			

When the angle subtended by any two objects more or less in an horizontal plane is to be measured, it is best to direct the

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of the object and its distance from the observer. The sun's semidiameter is entered in the *Nautical Almanac.* Its semi-diameter is 16′ which is often used in practice, as it does not vary more than half a minute from that amount.

[*] The situation of a celestial body as viewed from the *surface* of the earth, is called its *apparent place*, and that part of the heavens where it would be viewed, if observed at the same moment from the *centre* of the earth, is called its *true place.* The difference between the two is called the *parallax* of the object. This parallax is greatest at the horizon, gradually diminishes as it rises above the horizon, and vanishes when the object reaches the zenith. As the altitude of an object, as observed from the earth's surface, is less than if observed from the centre, the parallax is to be added to the apparent altitude to obtain the true altitude.

[†] The rays of light from a celestial body on entering the atmosphere obliquely are diverted from their rectilinear course, and are bent more and more towards the centre of the earth as they dive deeper into the atmosphere; hence they fall upon the eye in a different direction from that of the object, and make it appear higher than its real place. The difference thus caused between the object's real and apparent places is called the *refraction.* The more obliquely the rays fall, the greater the diversion from their rectilinear course, and the greater the refraction; therefore it is greatest at the horizon, and vanishes at the zenith. It is always to be deducted from an object's apparent altitude, because the effect of refraction is to make bodies appear higher than they really are; so much so, that celestial objects may appear above the horizon when they are actually below it. Tables of refraction are usually found in nautical works.

telescope to the fainter object, so as to get the more visible one by reflection; to do this, when the brighter object is to the left, the sextant must be held *face downwards*.

"The enlightened limb of the moon is always to be brought into contact with the sun or star, even though the moon's image is made to pass beyond the sun or star before the desired contact can be obtained.

"Perhaps the best method of taking a lunar distance is, not to attempt to make the contact perfect by the tangent screw, but when the nearest limbs are observed, make the objects overlap each other a little when they are receding, or leave a small space between them when they are approaching, and wait till the contact is perfect, and the reverse when the furthest limbs are observed.

"The altitudes of the two objects should be observed at the same instant as the distance, and the time noted by a chronometer, or watch; this would require several observers; but one person may take them all, by having recourse to the following method. First, observe the altitude of the sun or star; secondly, the altitude of the moon; then any number of distances; next the altitude of the moon, and lastly, the altitude of the sun or star, noting the times of each by a watch. Now add together the distances and times when they were observed, and take the mean of each; and in order to reduce the altitudes to the mean time, making the following proportion:—As the difference of the times between the observations is to the difference of their altitudes, so is the difference between the time that the first altitude was taken and the mean of the times at which the distances were observed to a fourth number; which added to or subtracted from the first altitude, according as it is increasing or decreasing, will give the altitude reduced to the mean time."

Artificial horizon.

In making the above remarks we have been supposing the observer as having the sea horizon before him; but this may not be the case in taking altitudes on land; he must under such circumstances have recourse to what is called an artificial horizon. Such an horizon may be obtained by pouring a quantity of tar, oil, or other viscous fluid into a shallow vessel, and covering it over with a piece of thin gauze muslin to prevent the wind from imparting a tremulous motion to the liquid. A very portable artificial horizon consists also of plate-glass of two or three inches in diameter, fitted in a brass frame standing upon three adjusting foot screws, by means of which and a small spirit level placed on the glass in various positions, its surface may be placed in a perfectly horizontal plane; the foot screws are of course turned until the bubble of the spirit level settles in the middle of the tube, or, as it is termed, of its run. The surfaces of the glass should be perfectly parallel and true planes; the under surface should be unpolished and blackened,* so that the sun can only be reflected from the upper surface, which should be perfectly polished.

Another kind of artificial horizon consists of a quantity of quicksilver poured into a shallow wooden trough, over which is placed a roof to protect the wind from disturbing the quicksilver; the sides of the roof are made of two plates of glass, the surfaces of which are perfect planes and parallel to each other. The glasses pack in a small box, with a small vessel containing a quantity of quicksilver ready for use.

When the artificial horizon is used, the observer must place himself at such a distance that he may see the object itself, and also its reflected image; supposing he is taking an altitude of the sun, the image of the upper or lower limb of this object, reflected from the *index* glass, must be brought in contact with the *opposite* limb reflected from artificial horizon and seen from the *horizon* glass. If the *inverting* telescope is employed, it must be remembered that the lower limb will appear as the upper, and *vice versâ*, but in taking an observation of the sun this inverting telescope is unnecessary. The angle shown will be double the observed altitude of the sun's limb above the horizon; the half of this altitude with the corrections for index error, semidiameter, parallax, and refraction will give the true altitude. The correction for dip is here unnecessary.

Fig. 60 illustrates the method of taking altitudes with the artificial horizon; A B is the quicksilver in the trough covered by its glass roof; S the object; the angle S E R between the object and the reflected image of it seen in the mercury is double the altitude or angle of elevation SEH^1; for by parallels $S^1RH = SEH^1 = ERB$; and $SER = SRS^2 = 2SEH^1$; the half, therefore, of the measured angle will be the apparent altitude to which the corrections last-mentioned will have to be applied.

THE BOX-SEXTANT.

The box-sextant is one of the most useful in the whole list of surveying instruments; it is only about three inches in diameter, and scarcely two inches in depth when shut up, and yet it measures an angle to one minute, and if rather larger in size, to thirty seconds; its construction and adjustments are precisely similar to those of the sextant above described, but being much smaller is more portable; it possesses besides the advantage of an outward shell, by which its constructive parts are more readily protected from injury.

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* It may be necessary to observe that every silvered glass of any sensible thickness has two reflections; the one on the anterior side and the other on the posterior or silvered side; errors may be occasioned by these double reflections, to prevent which the silvered side should be blackened.

This instrument is illustrated at Fig. 61. A, is an outward box into which the instrument proper is screwed when in use, and which serves as a cover when the instrument is packed away in the pocket, or in a case attached to a belt; it is not to be confounded with the "*pocket sextant*," which is literally a sextant in every sense of the word, but of only about four inches radius or less, so as to be carried about in one's pocket. I is the index, which may be moved along the limb L by a rack and pinion motion concealed within the case surrounding the constructive part of the instrument, and which is turned by the milled head B, answering the same purpose as the tangent screw of the large sextant illustrated at Fig. 59. To the index is fixed the index glass G, acting as the index glass of the sextant; H is the half-silvered horizon glass; the adjustments of these are so protected by the outer case that, when made perfect, it requires ill-usage to disturb them. For the adjustment of the horizon glass H, there is a contrivance at *a* and *d*, the latter being concealed in the drawing, to make it perpendicular to the plane of the instrument, and also to correct what has been called the index error, or defect of parallelism between the planes of the two glasses; this parallelism in the box-sextant had better be kept perfect, as, when once the adjustment has been made, it is not likely to get out of order. The key K, fits both squares at *a* and *d*, but is kept where shown in the drawing, so as to be always at hand when required. T is a telescope fitting to the instrument by means of shoulder and screw, to be used or not at pleasure, as there is a plain sight attached, which, by means of a little stud, may be slided into the place of the telescope and used instead; indeed, nine times out of ten the instrument is so used. There are two dark glasses within the box, either or both of which may be applied by means of little levers as required, and there is also a small dark glass attached to the eye end of the telescope, to be used or not as may be desired. M is a magnifying glass, mounted on a joint to move along the vernier, and by the help of which the angle may be read off; the construction and the adjustments of this instrument are so precisely alike those of the sextant that we should have to repeat what has been said before if we were to describe them all; so indeed is the use of the box-sextant; we shall therefore only make a few observations such as relate to the common use of the instrument in surveying, which is very general.

The dark glasses are within the box, and situated between the horizon glass and the plain sight or place of the telescope; when used, therefore, these glasses are left so, but when otherwise, they are turned down by means of their levers through a slit into the box A, so as to be out of the way; when the instrument is to be shut up they are returned to their place.

The adjustments.

To examine if the instrument be in perfect adjustment, look at the sun through the plain sight, leaving one or both of the dark glasses before the object, as may be required, according to his brilliancy; move the index a little right and left of zero on the limb, so as to bring the sun's image *reflected* from the index glass to pass over the disc as seen *directly* through the *horizon* glass; if in doing this the *reflected* image coincides with the *direct* image, so that one unbroken orb is seen, or the upper and lower limbs form one unbroken circumference, then *the horizon glass is perpendicular to the plane of the instrument;* otherwise make it so by means of the screw at *a*, with the key K. In the next place, with this adjustment perfect, *to examine the parallelism of the two glasses*, pursue the course directed at page 71; if, with this instrument, there be an error, it had better be at once corrected, and this should be done by turning the screw at *b*, by means of the key K, above-mentioned. When this adjustment is made perfect, and the zero on the *vernier* perfectly coincides with the zero on the limb so as to form one straight line, the *reflected* image of the sun will exactly coincide with its direct image, and the two images will form one perfect and complete orb.

In using the instrument the bottom of the box is held in the left hand, and the index worked with the right, by turning the milled head; the instrument being thus held, an angle subtended by two objects is measured by looking at one of them directly through the unsilvered half of the horizon glass whilst the index is carried round to the right hand until the object reflected from the index glass is made to coincide with the image seen by direct vision. This presupposes that the object seen directly is on the observer's left, and that seen by reflection to the right hand. Should the reverse of this occur, as, for instance, in the case of setting out an angle to the left of the object seen directly, then hold the instrument upside down.

This is the best way of observing an angle between two objects, the one of which is near and the other distant, as by so doing parallax is eliminated. Should both objects be near, select some third one in line with that to the right or left, or set up a pole in direct line with one of them.

Should the angle required be a vertical one, then the sextant is held vertically, so that the index may be worked similarly instead of sideways; the sextant is thus held for taking the altitude of a celestial object, for which see what has been said under the article "*sextant.*"

The principal objections to the sextant are that it will not measure angles greater than about 120°, and only *horizontal* ones, or rather, in the same plane; consequently, if an angle be measured with this instrument between two objects, the one of which is much above or below the other, the true horizontal angle is not obtained, for to measure such angle the observer will have to hold the sextant, not horizontally, but in the plane formed by the two objects, the one of which is much above the other. An angle so taken requires *reducing* to the horizontal, which is a troublesome operation. As much as possible it is better to avoid having to measure angles between such objects, that is, much out of the horizontal level; when they do occur, set up an object in line with one of them so as to get an horizontal angle; in mere *sketching* this may be avoided by selecting some object already in line; but in actual *surveying*, this is not always so easily done as may be supposed, even with sufficient accuracy for many practical purposes, and the observer will never have any faith in the measurement of an angle so taken, particularly if the higher or lower object of the two be much out of level; under such circumstances, it is much better, safer, and more practical, to set up a pole carefully in line, and by means of a plumb-line this is very soon done; we may then measure the angle by holding the sextant quite or nearly in a horizontal plane.

This will be the greatest objection the inexperienced practitioner will find with the sextant, but a few days' practice will entirely overcome it, and prove to him that it is one of those instruments without which he should never be in the field; it is besides as useful in *setting out works* as it is in surveying; it may, indeed, be considered indispensable in engineering field-work, and when the reader has made himself familiar with the use of the sextant and of the prismatic compass, he will soon learn how to use the theodolite.

THE TRANSIT THEODOLITE.

This is the chief instrument used in surveying, in setting out curves, and in all instances where important angular observations are taken; it may, indeed, be resorted to for many astronomical purposes. Fig. 62 is a drawing of one of Elliott's improved transit theodolites, and the most perfect and complete instrument of the kind yet produced; when in use it is fixed, by means of a female screw under the lower part, to a three-legged triangular stand, which affords a firm support even on the most uneven ground; this tripod is so constructed that it shuts up in a round staff for carriage with a couple of rings.

L and V are two circular plates, L being the horizontal graduated "*limb*," and V the "*vernier plate;*" both turn horizontally upon the vertical axis immediately underneath them; this axis consists of two parts, external and internal; the external belongs to the graduated *limb*, and the internal to the *vernier plate;* the edge of the limb is chamfered off and silvered to receive the graduations; on *diametrically opposite sides* of the *vernier* plate, that is, 180° apart, are the verniers V, and the chamfered inclinations of the verniers and that of the limb form one continued inclined plane. The graduations on the limb and vernier depend on the diameter of the plates; on this subject the reader is referred to what has been said under the head of "Vernier."

At P and P are the "*parallel plates,*" held firm and set parallel to each other by four mill-headed screws, *p*, *p*, pairs of which are opposite to each other, so that there are only three seen in the drawing; these, by being turned, act opposite to each other, and by their means the instrument is set up level for taking either horizontal or vertical angles.

There is a "*clamping screw,*" hidden from view in the drawing, by means of which a collar, embracing the axis *between the parallel plates and the horizontal limb*, may be tightened; this clamping screw serves to fix the horizontal limb in any direction that may be desired; but as this motion is only effected by hand, and as it is requisite that such direction should be fixed with minute exactitude, there is a "*slow motion screw,*" S, by means of which the whole instrument may be turned any small quantity after having been fixed by the clamping screw; this slow motion screw is attached to the upper parallel plate. Similar contrivances are applied to the *vernier plate*, which has also *a clamp* C, and a screw for slow motion, T, called the "*tangent screw;*" only the best instruments are supplied with this "slow motion" movement, which is far superior to any other, as by its means the most gentle and steady motion possible is imparted to the instrument.

In the centre of the vernier plate is a compass, and there are also two small spirit-levels *l*, *l*, on this plate, by means of which the observer is guided in setting the instrument level; these two spirit-levels are supplied with adjusting screws at the ends. A small microscope is attached to the limb for reading the vernier, and so as to slide round the instrument.

We must now call the reader's attention to the two frames, Y, Y, which support the remaining portion of the instrument, viz., the telescopé and its spirit-level above, and the vertical circle M, called the "*vertical limb*," with its two verniers *v*, with their microscopes, for measuring altitudes or vertical angles; these are supported on the frames by the pivots of the horizontal axis resting in the "*y's*," so called from a rough resemblance to the shape of that letter. It is this system of construction which gives the transit theodolite one of its very great advantages over the old style of instrument, as when it is required to reverse the ends of the telescope, it is merely necessary to make the telescope revolve vertically on its axis, instead of having to perform the troublesome operation of opening clips by the removal of pins every time it is done.

At either end of the spirit-level are capstan-headed screws, to adjust it parallel to the optical axis of the telescope, or line of collimation, by tightening the one and loosening the other. The vertical limb is also provided with clamp and tangent screw for slow motion, as at O.

For the purpose of adjusting to distinct vision, the object glass is thrust outwards by means of a milled-head at the side of the telescope, but which is concealed in our drawing from its being on the other side; this is to adapt the glass to the sight of the observer, and to his distance from the object observed; distinct vision, or the proper focus, is obtained when, by moving the milled-head the object observed appears clear and sharply defined; when it does so happen that the milled-head alone does not give this, move the eye-piece in the eye-piece tube.

When looking through the telescope the sight embraces a considerable field of view; but when making observations we only seek for one point in it, which is considered as the centre; it becomes, consequently, necessary to place some object in the focus of the eye-piece, and when making observations this object must coincide, or "bisect" that on which we are observing. The fixed point required in the telescope is obtained by the intersection of lines of spider's web, attached to a diaphragm, as seen at Fig. 63, where these threads are shown as attached to a ring of smaller diameter than that of the telescope, and which is fixed to it by four small screws acting in opposite pairs, so that if one is eased and the other tightened, the ring is moved right or left, or up and down accordingly as one pair or the other is acted upon, and with these of course the intersection of the threads; when this portion of the instrument is in adjustment, it coincides with the axis of the telescope, and the line of collimation is then correct. In our drawing of the theodolite these screws are lettered *d, d.* Sometimes there are only three such screws.

A plummet and line are supplied with theodolites; this being attached to a hook placed under the centre of the instrument, guides the observer so that he may plant the centre of the instrument exactly over the centre of his station, which is a matter always to be carefully attended to. There are also two extra eyepieces, the one called the "diagonal eye-piece," for astronomical observations, by the use of which the observer avoids an uncomfortable position in making such observations; the other has fewer glasses and a higher magnifying power, but inverts the object, so that in looking through it, objects appear turned upside down, and an object to the right appears to the left, but this one becomes very soon accustomed to, and I would advise the beginner to use this eye-piece from the first, and condemn the other; there will also be found in the box a small magnifying-glass, a turn-screw, and a small pin for turning the capstan-headed screws.

THE ADJUSTMENTS.

When the instrument comes from a good maker, it will very rarely happen that anything is out of adjustment, but as derangements of the parts may accidentally occur, it is indispensable that the reader should know how to correct them; and under any circumstances it is desirable to examine a new instrument, and also from time to time afterwards.

The line of collimation is the first adjustment to attend to; it consists in making the intersection of the wires of the diaphragm coincide with the axis of the telescope. Set up the instrument on its tripod, and level it by means of the screws *p, p,* being guided by the bubbles of the spirit-levels on the vernier plates, which should occupy the middle of the tubes. Set the telescope on some well-defined portion of any object in the distance, and adjusting to distinct vision, turn the mill-head in the side of the telescope until it appears clear and sharp, and perfectly steady, whilst the eye is moved up and down, or right and left, before the eye-piece. In the mind's eye carefully mark the position of that on which the intersection of the wires bears. Loosen the two screws just visible in the drawing below the clamp and tangent screw O, of the *vertical limb,* open the clips at the top of the "*y's*," lift out the telescope, *and turn the axis end for end*, when the bubble will be downwards. Now examine, through the telescope, if the intersection of the wires still coincides with exactly the same object as before; if so, the line of collimation is in adjustment; if not, then it must be made so; if too high or too low, the two screws, *d, d*, must be turned to correct *half* the deviation, whilst the other half is corrected by depressing or elevating the telescope itself; if the observer has been careful in making his first observation, he will be able to estimate correctly this half of the deviation, or very nearly so; now restore the telescope to its former position, and if the coincidence with the object remain as corrected, then the line of collimation in altitude is correct; if not, exactly the same operation must be repeated until it is so; a similar proceeding will set this line of collimation in adjustment laterally, by acting upon the little side screws instead of the vertical ones.

To adjust the level over the telescope.

The former adjustment having been made with the vertical arc clamped, and the clips over the "*y's*" still open, bring the air-bubble of the level to the centre of its run, or of the glass tube, by turning the tangent screw O; now reverse the telescope as before, and very carefully, so as not to disturb the vertical limb; if the bubble now again settles in the centre of its run, then the level is in adjustment, that is, parallel to the corrected line of collimation; but if this be not so, correct half the deviation by the capstan-headed screws at the end of the level, and the other half by means of the tangent screw O; the same as above, this operation must be repeated until this adjustment is made correct.

To make axis of the horizontal limb truly vertical.

Set the instrument as level as you can by means of the tripod legs, being guided by the spirit-levels on the vernier plate and that above the telescope; clamp the collar round the axis under the horizontal limb by means of the clamping screw, which, as before noticed, is concealed in the drawing, and the clamp C of the vernier plate being free, turn the vernier plate round, so as to bring the telescope over two of the parallel plate screws *p, p*, and bring the bubble of the level over the telescope to the centre of its run, by means of the tangent screw O. Now turn the vernier plate half way round, that is 180°, so that the telescope shall still be over the same pair of parallel plate screws, *p, p*, only turned end for end; if the bubble does not now continue in the centre of its tube, correct half the deviation by means of the parallel plate screws, and the other half by means of the tangent screw O; this operation must be repeated until in both these positions of the telescope the bubble of the level remains in the centre of its run. Now turn the vernier plate one-quarter, or 90°, round, so that the telescope may be over the other pair of parallel plate screws, and by their means bring the bubble again to the centre of its run, and now see that it remains so whilst the vernier plate is carried right round the limb. This being so, and this level being the most sensible of the three the instrument is supplied with, the other two, that is, those on the vernier plate, may now be adjusted by means of their screws, that is, should they be out of adjustment, for the bubbles of these levels should now be in the middle of their tubes. To test the construction of the instrument itself on this point, tighten the clamp C, loosen the collar round the axis, and turn the instrument slowly round; if the bubble maintains the centre of its run during an entire revolution all is well, but if otherwise, there is a defect which can only be remedied by the maker.

The vertical limb.

All these adjustments being made, the zero on the vernier should coincide with the zero of the limb; if not, make it so by means of the screws.

THE ORDINARY THEODOLITE.

By the expression "ordinary theodolite" we designate the instrument constantly in use up to a very late period, and still employed by a great many. Its capabilities are perhaps equal to the transit theodolite above described, but it is certainly much less convenient, and involves loss of time. At Fig. 64 is a drawing of this instrument, in which similar parts are denoted by the same letters as those in Fig. 62; it will at once be perceived that the great difference is in the vertical limb M, the attachments of the telescope to the rest of the instrument, and the position of the telescope level, which is here underneath instead of above; O is the tangent screw of the vertical limb, but the clamp of this limb is at N. When it is desired to "reverse the telescope," that is, turn it end for end, the pins *a*, *a*, are taken out, the clips *b*, *b*, are opened, and the telescope is taken out of its "*y's*" (so called from their somewhat resembling the form of that letter), when the telescope may be reversed. Q is a microscope suspended to an arm moveable between the bars of the frame for reading off altitudes or depressions on the vertical limb. Any one who has paid attention to the directions given for adjusting the transit theodolite will easily understand the method of adjusting the instrument now in question.

To adjust the line of collimation.

"First make the intersection of the horizontal wires coincide with some well-defined part of a distant object, then turn the telescope half round in its '*y's*,' till the level lies above it, and observe if the same point is again cut by the centre of the wires; if not, move it one-half the quantity of deviation by turning the two screws at *d*, *d*, releasing one before tightening the other, and correct the other half by elevating or depressing the telescope; now if the coincidence of the wire and object remain perfect in both positions of the telescope, the line of collimation in altitude or depression is correct, but if not, the operation must be repeated carefully until the adjustment is satisfactory. A similar proceeding will set the line of collimation correct laterally." With regard to the other adjustments we have only to repeat what we have said already about reversing the telescope, as to the manner of taking the instrument out of its "*y's*."

To make the telescope level parallel to the line of collimation.

The clips *b*, *b*, being open, and the vertical arc clamped, bring the bubble to the centre of its run by means of the tangent screw O; this done, carefully lift the telescope out of its "*y's*," and turn it end for end. If in this reversed position the bubble resumes its centre, all is right; but if otherwise, it will be evident that the axis of the level is not parallel to that of the telescope. By means of one of the capstan-headed screws correct one-half of the deviation, and the other half by turning the tangent screw O; by repeating this operation the adjustment will be made perfect. But to complete the setting of this level, that it may maintain the centre of its run when inclined a little on either side from its usual position under the telescope, it requires adjusting laterally, which is done by turning, if necessary, the two screws *k*. Should the former adjustment be disturbed by this operation, then the whole must be carefully repeated.

With *good instruments, duly cared for*, these adjustments are but very rarely required.

COLONEL EVEREST'S THEODOLITE.

This instrument has important distinctive features, which are illustrated at Fig. 65. The horizontal limb L consists of one plate only, on which the degrees are graduated; and the verniers V are at the end of bars radiating from the centre, and another bar carries the clamp C and the tangent screw for the verniers. At S are shown the clamp and slow-motion screw of the horizontal limb. The vernier bars are connected with the upper portion of the instrument carrying the telescope, vertical limb M, &c., and turning upon the same centre they show the angle traversed by the telescope.

The tripod support B is provided with parallel plate screws, *p*, *p*, *p*. One advantage connected with this instrument is, that it may be used with a three-legged stand, the same as those already described, or be disengaged from the top of this staff and placed upon the top of a wall, or in other situations where the ordinary tripod could not be used.

It will be seen by the drawing that the telescope and vertical limb are supported in a manner very similar to the transit theodolite already described, the horizontal axis connected with the telescope resting at Y, on two supports only, one of which is seen in the drawing. These are supported by a flat horizontal bar, E *l*, to which is attached a spirit-level, only one end of which is seen in the drawing; this level is for adjusting the axis to the horizontal plane. The vertical arcs, M M, attached to the telescope, move with it in the same vertical plane.

MEASURING ANGLES WITH THE THEODOLITE.

We must now suppose that the instrument is in adjustment,* and fixed to its three-legged support ready for use. The first thing to be then done is to plant the instrument somewhat about level, exactly over the station from which you are going to observe, giving the three legs a wide spread, and fixing them as firmly in the ground as circumstances will admit; the length of this tripod is made such that when a sufficient spread is given, the height of the eye-piece of the telescope will be such as to be convenient to the observer; the position of the centre of the instrument being exactly over the centre of the station, is ensured by the plummet and line hanging from the centre of the instrument being close over it, if it be a peg in the ground, or in the hole itself, when a flag pole has been removed to make place for the theodolite; now set it level by means of the parallel plate screws, *p*, *p*, until the bubbles of the two spirit-levels on the vernier plate keep the centre of their run, whilst the telescope is turned steadily quite round on its centre; the instrument is now ready for measuring angles.

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* It may be as well to observe here to the beginner that but little anxiety need be felt as to the adjustments of an instrument turned out of the workshop of a good maker, though the student may make an examination of the value and condition of the instrument as soon as he feels confident he has mastered the principles and details of its construction; that which he has really to apprehend is, the danger the instrument is liable to when brought into use, and this not from his own hands but from those of others, particularly chainmen, whose hands it is difficult to keep off the instrument when his back is turned for a short time; let him therefore be advised never to leave an instrument on the ground without its being under the care of some person on whom he can thoroughly depend, for the curiosity of an ignorant man, and his desire to handle that of which he has no conception, is often unrestrainable where there is, as he thinks, no fear of detection, and has often led to serious errors. When the day's work is over restore your instrument to its case, and keep the key; the sense of security thus ensured more than repays the few minutes of trouble incurred.

Let A, Fig. 66, be the station; it is required to measure the angle subtended between B and C; clamp the zero of the vernier plate V, to zero, or 360° on the limb, and make the coincidence perfect by means of the tangent screw T; examine the zero of the other vernier, and see that it coincides with 180°; move the instrument bodily round towards B, until the intersection of the wires in the telescope is in contact with the object at B; clamp the instrument by means of the screw attached to the collar round the axis, and make the "bisection" of the wires with the object at B perfect, by means of the slow motion screw S; this exact bisection can only be ensured when clear and distinct vision has been obtained by means of the mill-head in the side of the telescope for thrusting out the object glass. The line of sight now coincides with zero, or 360° on the limb, and both bear exactly on the centre of the object at B. Now release the vernier plate by loosening its clamp, and turn it round until the intersection of the wires is in contact with the object at C; now tighten the clamp of the vernier plate, and by means of its tangent screw T make perfect the bisection of the centre of the wires with the centre of the object at C; if there is any considerable difference of distance between B and C from A, the mill-head of the telescope will have again to be turned to ensure distinct vision; the angle is now measured, and there only remains to read it off from *both* verniers; let the one pointing to C read 62° 37′, and let the other vernier show 36′ 30″ beyond a degree; then the mean of the minutes will be 36′ 45″, and the measure of the angle will be 62° 36′ 45″.

In ordinary cases, and supposing a 5-inch theodolite used, the above measurement of an angle, read off with *both* verniers, will be found sufficient; but where a considerable area is triangulated, or where distant observations are taken from a base line,* as well as in *traverses* of several miles circuit, the angles should be repeated, and the mean of all observations taken as the real measure.

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* Let it, however, be understood that we are not alluding to geodætical operations.

The operation of repeating an angle is performed in the following manner:—Having, as above, taken the first measurement, loosen the clamp of the *lower plate*, whereby you release the whole body of the instrument, and turn it bodily round upon B: clamp the lower plate, and perfect the bisection, by means of the slow motion screw S; and observe that in doing this you have carried the 360° from its bearing on B, round to *c*, and the vernier bearing on C, round also so as to bear on B. Now release the upper or vernier plate, and turn the telescope towards C; clamp it, and perfect the bisection by means of the tangent screw T; read off both verniers again; the difference between this reading and the first will be twice the angle B A C, and the mean between this and the first reading is to be taken as the actual measurement of the angle; this one repetition will generally be found sufficient, unless it should so happen that the difference is considerable, say, for instance, 45″, or a whole minute, in which case we should repeat the angle a second time, by carrying the circuit further round, in the manner above described; this will not occur with a good instrument.

In taking an angle as above, many persons do not think it necessary to begin by bringing the zero on the vernier to coincide with the 360° on the limb; they simply clamp the two plates together, and take off the reading, whatever it may happen to be; the telescope is then brought to bear on B, and the limb is clamped; the vernier plate is then loosened, and the telescope carried round towards C; the vernier plate is clamped, and bisection perfected by the tangent screw; the reading is again taken off (*always* by both verniers, and we shall, therefore, not repeat this), and the difference between the first reading and the second will give the actual measurement of the angle. For this mode of acting they allege the errors arising from graduation and eccentricity, and the probabilities that the verniers cannot in consequence be brought exactly to 360° and 180°; but this is one of the first things that should be examined in an instrument; and we should be disposed to reject an instrument in which we could not effect this. Again, the difficulty, under any circumstances, to effect the exact coincidence, is asserted; although, in fact, in a well-made instrument, of sufficient size, say five inches, there is no more difficulty in reading this off, and, therefore, making it perfect, than there is in any other reading. Adopting, however, either way, the angle may be correctly measured; usually, where one angle merely has to be measured without repetition, we adopt the second method, above described;

but where several angles have to be taken at one station, and these have to be repeated, we prefer making the coincidence perfect between the zero of the vernier and the 360° and the 180° on the limb.

The theodolite is the only instrument used in engineering field-work for setting out long, straight lines, such as the basis of a system of triangulation, or those of a railway or canal survey; and here the advantages of the transit theodolite are made very evident. To do this, we clamp fast the verniers at 180° and 360°, and observe, with minute care, that the coincidence is perfect; then we plant the instrument at the station through which the base line is to be ranged, and bring the instrument bodily round on the starting-point, and clamp it; perfect the bisection with the back object by means of the slow motion screw S, making this bisection as near the ground as possible, and reverse the telescope by making it revolve on its axis, if a transit, or by reversing the telescope in the clips if the ordinary theodolite; now examine that your verniers remain in perfect coincidence with 180° and 360° on the limb, when you may proceed to set out ranging-rods as far as you can see over the ground; but always make the bisection of the wires as low down on the rod as possible. Probably it is unnecessary to observe, that in doing this, the first operation after fixing the instrument by means of the plummet and line is to level it by means of the parallel plate screws; without which, indeed, nothing is to be done. Having ranged, as far as the theodolite commands the ground, move on to the next commanding situation on the base, and repeat what has just been described. Miles of straight line may be thus set out.

There is another kind of transit theodolite made by Messrs. Elliott, from a pattern, we believe, of the late Mr. Rendel, in which there is no vertical limb, and which, on most of the occasions arising in practice, we should consider an instrument to be preferred to any we have seen.

CHAPTER VI.

SURVEYING BY TRAVERSE.

A TRAVERSE is a polygonal or many-sided figure assumed by a number of straight lines lying in various directions, and which, by going North, South, East, or West, we follow from a given point, until we return to it; in tracing this circuitous route, it is necessary to measure the lengths of the sides, and also the angles, before we can make a plan of it. We shall best explain this method of surveying by referring to the lines in Fig. 67. Let a survey be required of the area contained between the stream and the road; set out the lines A B, B C, C D . . . . N A; measure the lengths of the sides and the angles, and it will be evident that the figure may be laid down on paper. The method of procedure in the field is as follows:—

Starting from a point at which you can make observations on surrounding objects, as at A, set out the lines A *a*, A O, A B, and plant the theodolite at A, in the manner above described; clamp the limb and vernier plates at 360°, or zero, and turn the whole instrument round until the magnetic needle points steadily on the North and South line N S, and clamp it firm in that position by means of the clamping screw which tightens the collar round the axis. Having perfected this, release the vernier plate, and bring the telescope to bear consecutively on A *a*, A O, A H, A P, A E, and lastly on A B, clamping the vernier plate each time, and perfecting the bisection of the cross wires with the object observed by means of the tangent screw T; having carefully entered at station A all the angles made by the above lines with the magnetic meridian, and leaving *both* clamps firmly fixed, the last reading being with the telescope bearing on A B, remove the instrument to B; plant it at this station, and carefully level it. Release *only the clamp screw of the limb*, that is the clamp connected with the collar round the axis of the instrument; the vernier plate must not be in the least disturbed, or you will have to begin over again; turn the theodolite bodily round, so that the telescope *reversed* may bear on A, where a man must be left to hold a flag pole, clamp the limb and perfect the contact by means of the slow motion screw S, and examine the readings of the verniers to see that no disturbance has taken place. A few words are now required with regard to the telescope being reversed; this operation places the horizontal limb and the verniers in precisely the same position with regard to the magnetic meridian as that which is occupied at A; if, for instance, at A, the bearing of A B was 31° East of North, by doing as directed you have similarly placed the theodolite at B, with the *same* vernier still pointing 31° East of North, and it is the *second vernier lying diametrically opposite*, which now points towards A, and reading on the limb 31° + 180° = 211°. The new position of the theodolite being now understood relative to the limb and the verniers, we may proceed with our work. Release the upper or vernier plate, and turn the telescope round to bear on C; clamp and perfect the contact with the tangent screw T; read both verniers to get the mean of the bearing B C, which is here 96° 5′; now move the theodolite to C, release the horizontal limb, turn the instrument bodily round to B, clamp and perfect the contact, release the vernier plate, turn the telescope round to D, clamp and perfect the contact with the screw S; by reading off your instrument you get the bearing of CD, which here is 30° 5′; proceed in the same manner at D, E ... M, O, at which station, when you have fixed the back sight on M, and turned the telescope round towards A, the verniers should give exactly the same angle as was read off at A, with the telescope bearing on O; and this because O A makes the same angles with the meridian N′ S′ that A O does with the magnetic meridian N S.* If it be so, then the angles have been correctly taken; if otherwise, then the difference is the error committed. Besides taking at A the bearings of A O and A B, we have taken those of A *a*, A H, A P, and A E; the bearing of A *a* was taken in order to lay down the bit of road beyond the bridge, and to show the position of such road in connexion with the lands surveyed; and also in case the survey has to be extended in that direction, as in such a case the instrument would be planted at station *a*, and the back sight fixed on A, in the same manner as directions have just been given for doing at B, C, D, &c., until we had made another circuit; with regard to the bearings of A H and A E, they are *here* only as checks on the work as it proceeds, for it will be observed that E A makes with the meridian N″ S″ at E the same angle West, that A E, at A, makes East with N S; these observations have equal weight with regard to the bearings A H and H A; we have here taken H and E as points at the other end of the survey, and of which full view could be had from A; otherwise any other points as F, G, or I, would have been taken if convenient; as regards the various bearings on P, let it be observed that these also are checks, for if there is any error, these bearings will not all intersect at P, when the work is plotted and the bearings A P, E P, G P, &c., are laid off with the protractor. It is of course necessary that they be taken on some object in a commanding situation, such as we may depend on seeing, if not all round the traverse, at least at several points. Judgment is required in selecting such points, as they may often be very useful to chain upon in order to fill in interior work, as fences, buildings, &c. On an extensive traverse this should particularly be kept in sight, as it prevents the necessity of having again recourse to the instrument when we come to fill in. It will now be observed that not only care, but judgment also is required, not only in setting out the traverse itself, planting the instrument properly, and taking the angles or bearings, but also in laying out the interior work to the best advantage at the same time, and so as not to be obliged to bring out the instrument again; which not only saves time, but also chances of error, for it must be remembered that the greater the number of observations we can take from one station, the fewer stations for angular observations will be required, at least as a general rule. All these matters considered, a day will often be occupied in taking all the bearings that may be required on a traverse of any extent. As a general rule, lay out your lines as long as possible, as thereby you avoid an excess in the number of your stations. It is also to be observed that in our diagram the stations in the road are all shown as in the centre of it; but this has merely been done to avoid confusion, and not to be followed as a rule; on the contrary, it is to be avoided, inasmuch as all these stations require to be carefully marked, either by driving a picket, or making some other mark, which is often very awkward to do in the centre of a road; place your stations, therefore, as conveniently as you can somewhere near the roadside, but so that you can there plant your theodolite. This is merely mentioned that the subject may not be altogether lost sight of, as many ways of settling these minor matters will suggest themselves to any intelligent mind as circumstances arise, it being merely requisite that the stations be marked so that they may be found to a certainty when wanted, and so that the instrument may be planted over them, and observations conveniently taken. The traverse being thus set out, the sides are chained and the offsets taken in the usual manner, which has already been explained under the head of chain surveying. With regard to the magnetic needle care is required lest it be affected by any local attraction; by pursuing the above method there will be opportunity to observe this at each succeeding station, as the back angles with the meridian are equal to the forward angles.

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* For all purposes of engineering field work we may consider the meridians as parallel lines.

There is a considerable advantage in taking from the starting-point A such bearings as A H and A E, for it subdivides the larger polygon into smaller ones, as in our example, where the figure A B, B C . . . O A, is subdivided by the above bearings into the smaller polygons A B, B C . . . E A, and A B, B C . . . H A; we are, therefore, enabled to check the work as we proceed, for in the same manner then the three angles of a triangle are equal to two right angles, and the four angles of a four-sided figure are equal to four right angles; so all the interior angles of a polygon are equal to *twice* as many right angles, less four, as the figure has sides. The proof of this may be seen in figures 68 and 69; in the first, let the polygon *a b*, *b c* . . . *k o*, be divided into triangles, by drawing lines from each angle of the polygon to any point *o*, in the interior of the figure; then because the three angles of a triangle are equal to two right angles, we shall have twice as many right angles as the figure has sides, for there is a triangle for every side; and all the angles formed by the lines intersecting at *o*, are together equal to four right angles; subtracting these, we shall have for remainder twice as many right angles, less four, as the figure has sides. In Fig. 69, from any point *a* in the polygon draw lines to each of the remaining angles, as *a c*, *a d*, &c.; the polygon will thus be divided into as many triangles as the figure has sides, less two, for there is only one triangle for each of the two sides *a b*, *b c*, and *a i*, *i k*; and one triangle for each of the remaining sides. In any case, therefore, multiplying 180° by the number of sides of the polygon, minus two, will give the interior angles of the polygon.

Thus, in Fig. 67, where the polygon has thirteen sides, all the interior angles will be equal to 180° × 11; or to 90° × 26 = 360°. In the same manner, the angles of the polygon A B, B C . . . E A = 180° × 3, because the figure has five sides; and the angles of the polygon A B, B C . . . H A = 180° × 6, the figure having eight sides.

The rules often given to find the interior angles of the polygon, lead rather to confusion than anything else; the simplest way is to carry a small semicircular protractor, about 3 inches in diameter, in the pocket, and plot the bearings in the field-work, merely writing in the degrees and minutes, &c., inside the several angles; or even to sketch in two lines, at right angles to each other, for the magnetic meridian and the East-West line; and sketch in your bearings as you go along. Many surveyors sketch or roughly plot a traverse in a field-book, quarto size, instead of that which we have formerly mentioned under the head of "Field-book;" a very good way where the traverse is small, and the weather is such as to admit of the use of such a book in the field; but it is very inconvenient in wet, stormy weather, when time presses, and the work must go on.

After reading the observations that will now be made on the subject of "plotting," or laying down the field-work on paper, the student is advised to lay down any irregular polygon on a large sheet of paper with a protractor, setting off also parallel lines to represent the meridians; by this means he will find considerable assistance to the better understanding of this very important subject in the matter of surveying; for want of anything more perfect, he may construct a paper protractor in the manner shown at p. 39, which will answer his purpose perfectly well for a traverse of small extent. He will thus be able to study how the theodolite is worked, before going into the field, and study some of the features of a traverse.

CHAPTER VII.

PLOTTING AND PROTRACTING.

Circular Protractors.—Howlett's Semicircular Protractor.—Metcalfe's Protractor.—Laying Down Angles by Chords.—Protracting by Northings and Southings, Eastings and Westings: Sines and Cosines.

THERE are several ways of laying down a survey on paper, which is called plotting; the term protracting being applied to setting off angles on paper with an instrument called a protractor.

If the survey has been made entirely by triangulation, we may plot the chain lines by laying down their lengths either with the ordinary or with the beam compasses, or protracting the angles of the triangles, if these have been taken. Where the sides of the triangles are of considerable length, the beam compasses will have to be used to lay down measured lines, or those of which the lengths have been calculated; we have already given some explanation of this under the head of "Chain Surveying," and we shall have an opportunity of saying more upon the subject further on.

To plot a traverse a protractor is required, unless we resort to the system of "Northings and Southings, Eastings and Westings," or of "Latitudes and Departures;" we shall explain the method by the protractor first, being that most usually applied.

Under any circumstances, however, the first thing is to be careful that the sheet of paper is large enough to contain the work; if there be no other means of ascertaining this, plot the survey first to a small scale, which will give the size of the sheet of paper required; when this point is settled, be careful to begin plotting so as to be able to get the whole of the survey into the map without having to make any joints.

PROTRACTORS.

There are numerous kinds of protractors, all constructed differently, but for the same purpose of protracting angles.

The circular protractor is illustrated at Fig. 70, and consists of an entire graduated circle or limb, and four radial bars, *b*, *b* and *c;* a piece of the brass metal is removed from the centre, and a circular piece of glass is inserted in its place, on which is marked the centre of the instrument; round the centre is a collar carrying the indices I, I, with verniers; at the end of the index bars are two branches B, B′, both of which, when the instrument is in use, are open, as at B, but when packed away are closed, as at B′; these branches carry at their extremities fine steel prickers, the exact centres of which must form one straight line with the centre of the instrument, if correct; by means of the springs *s*, *s*, &c., these points are kept off the surface of the paper when the arms are moved round; and therefore to prick off an angle when the instrument is in use, a slight pressure upon the buttons, as at B, is required, which leaves a fine puncture on the paper; a line drawn through one of these points and the point on which the centre of the instrument is set, will give a bearing or an angle with any other line previously determined.

Observe that with the circular protractor, if the centre has been correctly set over the angular point from which an angle is being set off, the two punctures from B and B′ will form one straight line with the angular point, and not otherwise.

This instrument is provided with clamp and tangent screw motion, T, by means of which the verniers are set to minutes, and as there are two verniers, we have the means of checking the one by the other; either with this instrument or the theodolite we have always the means of ascertaining the reading of the back angle vernier, by adding 180° to the angle read off by the first. The face of the glass centrepiece is placed as nearly even as possible with the under surface of the instrument, so that no parallax may arise from too great a space between the surface of the paper and that of the glass carrying the centre of the instrument.

This instrument, as constructed by Messrs. Elliott, is the most complete we have, and by the fulfilled requirements assimilates the nearest on paper to the conditions of the theodolite in the field.

It is steadied to any position on the paper by means of short points attached to the under surface.

From the description of the theodolite, and of the method of using it, the nicety required in the observations, as the planting over the exact station point, the readings of both verniers, and the repeating of the angles under many circumstances, it will at once be perceived that to make the precautions thus taken in the field of any value in the office, some special means must be taken to lay down the observations on paper with anything like equal fidelity; for otherwise time would be thrown away on the field-work.

To lay off the lengths of long lines, great pains are taken in the construction of beam compasses. Similarly with protractors much thought has been devoted at different times to devising an instrument that shall lay down angles with the greatest possible accuracy, when a proper method is applied.

That the reader may appreciate all the difficulties that may arise in protracting the angles of a survey, we will endeavour to go methodically through the process of plotting the traverse at Fig. 67. On the left hand side of the paper draw a straight line, N S, representing the magnetic meridian, and with a fine needle point prick off the station A, at which the work commences; over this set the centre of the protractor with the 360° and 180° divisions coinciding with the meridian; set the zeroes of the verniers to these divisions, and press the buttons at the ends of the arms B B′; the punctures thus made will be *exactly* on the meridian N S, if the centre of the instrument is correctly placed; if this is so, gently press the protractor on to the paper to secure it. Loosen the clamp, and move the index to the angle N A B, which is the first bearing; clamp and perfect the reading by the tangent screw T, attending to both verniers, and make a puncture, which mark as No. 1; next set off the bearing C B N‴, and mark it No. 2; then D C N⁗, which mark No. 3; and so on round the figure; set off at the same time the bearings of A *a*, A H, A B, and any others in the field-book; lastly, bring the zeroes of the verniers again to 360° and 180°, and press the buttons; if the instrument has not moved, the points will press into the same punctures as were made at first.

Now with a straight rule draw in the bearing A B, through the point A, and that marked No. 1; on this line set off the length of the chain line; next place the edge of a parallel rule on A and No. 2, and parallel to this line through the point B last marked off as the length of the chain line, draw the line B C, and placing zero of your scale at B, make B C equal to the second chain line; now place the rule on A and No. 3, and parallel to it draw C D, and make this also equal to the length of the chain line; proceed thus until you have completed the sides of your traverse; if all is correct, not only the last chain line, O A, will measure on paper as in the field, but the bearings of A H and A E will pass directly through the stations H and E.

We may now observe, that the degree of accuracy to be attained will much depend on the extent of the traverse; if small, and the work is fairly done, it will "close" easily enough; but the difficulties will increase with long lines, when parallel rulers become next to useless, and another system has to be adopted; as also with a great number of angles, when we must endeavour to subdivide minutes into seconds by means of the two verniers; that is if seconds be entered in the field-book. It will be evident that if the chain lines are long the traverse will be of corresponding extent, and that the difficulties of working the parallel rulers over it will be in proportion; in the same manner, if the angles are numerous and carefully taken, seconds in several instances will probably have been read off, which, if repeatedly neglected, and on long chain lines, may involve an actually practical error. One way of overcoming these difficulties, may be by carefully drawing a parallel to the first meridian line through every third or fourth angle, and protracting the angles from the last angular bearing found; but considerable care must be observed in drawing the parallels; we shall, however, presently describe a more easily practicable method, known as Howlett's, and which, on extensive traverses, may be resorted to jointly with the method just described, the one checking the other. On the other hand, the traverse may be so small and so easily plotted, that even the paper protractor we have described in another place may be sufficient to close the work correctly; between such a simple case and one with numerous difficulties, the reader must apply a discerning intelligence according to the circumstances and magnitude of the case.

It is now some years since Mr. Howlett, chief draughtsman to the Ordnance, made the following observations on the circular protractor, which are deserving of the greatest attention, but which he himself probably never intended to be followed blindfold.

"The circular protractor, at the price of from four to eight guineas,* is generally considered the most perfect kind of instrument for plotting the angles of a survey; but against this instrument there are the five following objections:—

"It is only steadied by being attached to the paper by pins; and in moving the arm it is liable to shift.†

"As the vernier has to be set while the protractor is fixed on the paper,‡ and cannot be held to the light, it is next to impossible in some positions to see the divisions;§ or if the protractor be taken from the paper, time is lost and error is caused by having to replace it on the working meridian.

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* Elliott's *eight*-inch circular protractor is 7*l.* 17*s.* 6*d.*

† We have had this instrument in use for many years without meeting with this accident, which must arise from not pressing down the instrument when fixed in its place, or from the map laying loose and unevenly, in which condition it is not fit to be plotted on.

‡ When several angles are protracted from the same point this must be the case either with the circular or semicircular protractor.

§ This inconvenience will not occur with a seven or eight-inch instrument and such defect applies equally to the semicircular instrument.

"When the whole set of angles required are set off and numbered, they have to be transferred to the station, one after the other, with parallel rulers, in doing which much error creeps in, both while setting the edge of the ruler against the points, and then in shifting the ruler along to a distant part of the paper.*

"It is a very delicate instrument, liable to be soon strained and rendered unfit for use.†

"Lastly, the general inaccuracy of the method which this instrument implies—for the sources of error are so many that the work cannot be brought to a close in a satisfactory manner—and when done, it is little better than a survey plotted by a common protractor, where the degrees and half-degrees only are marked."‡

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"In making surveys of estates and parishes, while on half-pay, I suffered much for want of a better system of plotting than any I could find after making every inquiry, and searching books on the subject. Such methods do not meet the exigencies of practical men, and hence it is usual to employ the chain alone. The theodolite is very little used among private surveyors: they reject the system altogether;§ and, indeed, many use the chain so skilfully that, under ordinary circumstances, it leaves nothing better to be desired. The theodolite is, however, an invaluable instrument; and if the proper use of it, together with a more satisfactory method of plotting, were more generally known among private surveyors, much of this prejudice would certainly give way."

Mr. Howlett then goes on to mention his pattern semicircular instrument, which is well known in the profession as "Howlett's Protractor," and of which an illustration will be found at Fig. 71; he then goes on to make the following very just observations:—

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* This defect entirely applies to the method of using, and not to the construction of the instrument itself.

† The branches of the arms, as will be seen in the drawing, are liable to be strained if the instrument is not taken care of; but there is no reason whatever why the arms should not be constructed the same as those of the semicircular protractor.

‡ This is a very strong and sweeping charge, and must be taken not only *cùm grano*, but *cùm granis*.

§ This will not by any means be found the case at present, and there are now few surveyors without a theodolite, and cases constantly occur where it is next to impossible to produce a good survey with the chain alone; for engineering field-work a theodolite is constantly in requisition.

"As when away from home, it seldom happens that the surveyor can obtain a good drawing board, or even a table, with a good straight-edge, I fix a flat ruler, A, Fig. 72 to the table, B B, by means of a pair of clamps, C D, and against this ruler I work the pattern square, E, one side of which has the stock flush with the blade; or if a straight edged board be at hand, then the square may be turned over, and used against that edge instead of the ruler A. Here, then, is the most perfect kind of parallel ruler that art can produce, capable of carrying the protractor over the whole of a sheet of plotting-paper of any size, and may be used upon a table of any form. It is convenient to suppose the north on the left hand, and the upper edge of the blade to represent the meridian of the station.

"This protractor is held in the hand while the vernier is set, which is an immense comfort to the sight; and it will be seen that as both sides of the arm are parallel with the zero and centre, the angle may be drawn on the paper on either side, as the light or other circumstances may render desirable.

"From this description, and a mere glance at the plate, it is clear that angles taken with the theodolite can be transferred to the plot as accurately as the protractor can be set, namely, to a single minute, and that, too, in a rapid and pleasant manner.*

"By means of the notch at the end of the arm, this instrument may be used in the manner of a circular protractor, should a square not be at hand."

The above method of Mr. Howlett's is generally a very valuable one, and leaves nothing to be desired in ordinary cases.

There is another kind of semicircular protractor, a modification of "Howlett's," and known as Metcalfe's, illustrated at Fig. 73. The principal addition to this protractor is a contrivance at A, by means of which the radius is turned into a scale, so that distances may be set off at the same time that an angle is protracted; there is a scale on each side, and the vernier is *double* so as to apply to each. For extensive general surveys, plotted to scales of three or four inches to a mile, this is a decided addition: when the instrument is used for this purpose, first adjust the vernier to the scale according to distance, then adjust the vernier according to the given angle, and, laying the instrument along the T square, adjust the zero of the scale to the station point on the plan. Diagonal scales, corresponding to those on the arm of the instrument, are engraved upon it.

It need scarcely be observed that in using either of the above protractors for plotting a traverse, the first bearing or angle being protracted, the line is drawn in, and by a scale made the length of the chain line; the blade of the T square is then pushed to the station thus found, the next bearing set off, &c.; and this operation is repeated until the whole of the lines are laid down, when the corresponding offsets are afterwards plotted also, and this part of the survey is complete.

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* With a seven or eight-inch protractor we can read much closer than this.

Although for most ordinary cases these semicircular protractors, used in conjunction with the T square, comply with most requirements, it will be observed that there is no check as to the instrument being exactly over the centre of the station, such as is afforded by the circular protractor when both arms are used. Also, when a long line has to be laid down, we have only two points to do so by, instead of three, which is a decided advantage in the circular instrument; where, for engineering purposes, plans of townlands are often required to scales of 30 and 40 feet to an inch, lines of great length on paper have often to be laid down, in doing which, unless great care is observed, a mistake, amounting to six inches or a foot on the ground, will soon creep in, and occasion considerable trouble. Much the same observations hold good with regard to the bases for a working survey for a canal or railway.

Protracting by Chords.—For laying down the sides of triangles, the protraction of angles by chords is often very convenient where angles have been taken, but some of the sides not measured. An example will best illustrate the manner of doing this:—From the base A B, Fig. 74, measuring 210 chains, to fix on paper the objects C, D and E, the angles at A being C A B = 41; D A B = 75°—10′; E A B = 110° 12′; and C B A = 43°; D B A = 48°; E B A = 20°.

Tabulate your angles in the following manner in pairs:—

Sine.

$$CAB=\frac{41^\circ}{2}=20^\circ\ 30'=\cdot3502\times2=\cdot7004\times210\cdot00=147\cdot08$$

$$CBA=\frac{43^\circ}{2}=21^\circ\ 30'=\cdot3665\times2=\cdot7330\times210\cdot00=153\cdot93$$

$$DAB=\frac{75^\circ\ 10'}{2}=37^\circ\ 35'=\cdot6099\times2=1\cdot2198\times210\cdot00=256\cdot16$$

$$DBA=\frac{48^\circ}{2}=24^\circ\ 0'=\cdot4067\times2=\cdot8134\times210\cdot00=170\cdot81$$

$${}^{*}EAB=\frac{20^\circ\ 12'}{2}=10^\circ\ 6'=\cdot1754\times2=\cdot3508\times210\cdot00=73\cdot67$$

$$EBA=20^\circ=10^\circ\ 0'=\cdot1736\times2=\cdot3472\times210\cdot00=72\cdot91$$

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* Observe that 110° 12′ = 90° + 20° 12.

By means of any protractor whatever, or even by guess, set out lines approximating to the angles required; set the beam-compasses to 210 chains, and from A and B, as centres, describe portions of arcs opposite the lines set off as the approximate angles. Next set your compasses to 147·08, and from B, as a centre, set off this first chord, and mark the intersection as No. 1; next set your compasses to 153·93, and from A make the intersection No. 2; draw in the lines A 1 and B 2, and their intersection will give the point C, and the angles C A B and C B A.

Next set the compasses to 256·16, and from B find intersection No. 3; and with 170·81 in the compasses describe from A the intersection No. 4; draw in A 3 and B 4, produced so as to get the intersection D, and the angles D A B and D B A. With regard to the triangle A E B, observe that the obtuse angle 110° 12′ is equal to 90° + 20° 12, and that we have only to set out the chord of 20° 12′ beyond 90°, to obtain the desired intersection; twice the sine of 45° × radius = 296·98; from B, with this distance in the compasses, set off the intersection 90°, and from it, with the length 73·67, mark off the intersection No. 5; also from A, with the distance 72·91, set off intersection No. 6, to find the point E, and the angles E A B and E B A.

This method will appear tedious, but where the angles are subtended by lines of three and four feet, and sometimes even of twice and three times this length, ordinary protractors are of no use, and this method by chords is extremely useful.

Plotting a Traverse by Northings and Southings, Eastings and Westings, or by Difference of Latitude and Departure.

Reference to Figs. 68 and 69, or a glance at Fig. 75, will show that in going round the perimeter of either of the above figures, it will be necessary to travel as much north as south, and as much east as west; that is, that starting northwards from any point on the perimeter or outline of the figure, and then returning to it, the distance travelled south must be equal to that travelled north before the starting point can be again reached, and the same as regards east and west; thus in Fig. 66, going north from A to *b* = going south from *b* to A; and going east from *b* to B = going west from B to *b*; so that in going from A to B, and returning to A, the distance travelled north = that travelled south, and the distance east = distance west; or in other words, the northings = the southings, and the eastings =

the westings; similarly as to all the other sides of the polygon, so that all the northings = all the southings, and all the eastings = all the westings.

As regards the angles formed by the bearings with the meridians* we may consider the northings and southings as the *cosines* of the angles, and the eastings and westings as their *sines;* thus A *b* is the cosine, and B *b* is the sine of the angle *b* A B, and B *c* is the cosine, and C *c* the sine of the angle N′ B C, or its supplement C B *c;*† and *d* C is the cosine, and D *d* the sine of the angle D C *d;* and D *e* is the cosine, and F *e* the sine of the angle N″ D F, or its supplement F D *e*.

Taking, therefore, every bearing formed with meridian as the angle, and each side of the polygon or "traverse," as radius, we can by means of the tables of sines and cosines ascertain the northings or southings, and the eastings or westings.

When the bearing is not above 90°, enter the reading as N.E. or N.W., in the column allotted to bearings, in the following form; if greater than 90°, subtract it from 180°, and enter the remainder in the column of bearings; if greater than 180°, and less than 270°, subtract from 270°, and enter the remainder; if greater than 270°, and less than 360°, then subtract from 360°, to find the proper entry. From the tables of sines and cosines enter in the column of signs the cosine and sine for each angle.

No. of Station.	Bearings.	Chainage.	Signs.	Northings.	Southings.	Eastings.	Westings.
1	N 40 E	6·00	Cos. ·7660 Sine ·6427	4·59	„	3·85	„
2	S 60 E	6·05	Cos. ·5000 Sine ·8660	„	3·02	5·24	„
3	N 30 W	4·00	Cos. ·8660 Sine ·5000	3·46	„	„	2·00
	&c.	&c.	&c.				

The northings and southings are thus the products of the chainage by the *cosines* of the angles, and the eastings and westings are the products by the *sines*. By means of these a most accurate method is obtained for plotting a traverse. On the assumed meridian, as N A W, plot the lengths due to the northings and southings; through the points thus found, draw in lines perpendicular to the meridian, as through *b*, *c′*, *d′*, *e′* and *g*, on which set off the eastings and westings, as *b* B; *c* C, both eastings; *c* C – *d* D, westing; *e′* F, easting; *e′* F – *g* G westing; observing that whilst the eastings are positive, the westings are negative, or quantities to be subtracted from the eastings; for accuracy this method leaves nothing to be desired, and is undeniably the best for an extensive traverse, for instance, a mile or two across; but unless one is in the constant practice, it takes up a little time, and requires the work to be first plotted roughly to a small scale; a mere sketch in fact, for mistakes are not possible, as in adding up the columns of northings and southings, the totals must be equal, as also those of eastings and westings.

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* These meridians we may, in practice, assume to be parallel as far as all proceedings in engineering field-work are concerned.

† Should the reader be totally unacquainted with trigonometry, we must refer him to the beginning of the chapter on "Setting out Curves."

Some years since, before protractors were brought to the perfection they have since attained, this method of plotting a traverse was very usual amongst the best surveyors; but it is less commonly resorted to at present. Accurary, however, is seldom to be attained by applying one universal method to everything under any circumstances; the reader is by this time supposed to have acquired considerable insight into the practice of land-surveying, for, as far as the writer is aware, no detail of practice has been neglected in the foregoing pages, even at the risk of appearing tedious; the student should, therefore, now be able to lay down two or three traverses on a large sheet of plan paper, and judge for himself how far one system is preferable to another under a variety of conditions.

Setting aside the plotting of a traverse in the above manner, considerable advantage may be derived from the calculation of northings and southings, eastings and westings, in exploring a wild country, in which a general, and, perhaps, very indifferent map is the only guide at hand. After being obliged, by the nature of the country, to abandon any direction first taken, and to traverse round before it is practicable to return to it, it is easy, after a time, to ascertain from a certain point how far we have deviated from our first original route, and therefore how far we have to steer east or west, north or south, in order to return to it.

Our meaning will be best illustrated by means of Fig. 74, where A B is the direction of a line pursued from some distance. On reaching the point A the nature of the country is found to be such that it is no longer possible to do so, and a deviation becomes necessary; let this commence at A, in the direction of the lines A *c*, *c d*, *d e*, *e f*, the bearings of which are all carefully taken, and the lines chained. On reaching the point *f*, we find the country clear in a direction towards the original route, and we desire to return to it; find the northings and eastings made by the above lines to the point *f*, by using the theodolite in the manner already described, and sketch the work in the field-book as you go on, and enter the bearing opposite each station.

No. of Station.	Bearings.	Chainage.	Signs.	Northings.	Southings.	Eastings.	Westings.
1	N 70° E	63·00	Cos. ·3420 Sine ·9396	21·54	„	59·19	„
2	N 35° E	59·00	Cos. ·8191 Sine ·5735	48·32	„	33·83	„
3	N 21° E	81·00	Cos. ·9355 Sine ·3583	75·77	„	29·02	„
4	N 33° E	79·00	Cos. ·8386 Sine ·5446	66·24	„	43·02	„
				211·87		165·06	

The above table having been constructed, take out from the tables the sines and cosines of the angles formed by the bearings with the magnetic meridian, and multiply them by the lengths of the chain lines; in the column of northings, enter the products of the *cosines*, and in the column of eastings enter the products of the *sines*. The latter added up = 165·06, will give the distance travelled eastward from the point A, and the total of the northings = 211·87, will give the distance travelled north of the point A. If now we were to set out in the direction due west, the length of the eastings = 165·06, we should return to a point due north of A, but the bearing of our line A B = angle of 15° with the magnetic meridian; so that whilst we have been travelling to *f*, a distance of 165·06 east, the line A B may be said to have travelled a certain distance east also, according to the angle N A B, and the length of N A. This latter easting will be equal to N *g*, and if we can find the point *g*, we shall have returned exactly to the required position on the line A B.

Now observe that in the triangle N A *g*, the *cosine* A *h* of the angle N A *g*, is to the *sine h s*, as the northing A N is to the easting N G, or,

·9559 : 211·87 : : ·2588 : 57·36;

therefore 165·06 – 57·36 = 107·70 = *f g*, = the distance due west from the point *f*; and a line N 15° E from the point *g*, thus found, will restore us upon our original route in the direction A B. This bearing will have to be set out by making an angle B *g f* = 75°; for 75° + 15° = 90°, and the direction of A B produced will be determined by the line *g f*, instead of depending entirely on the needle in the compass box. In the same manner, when setting out the line *f g*, due west, instead of depending entirely on the magnetic needle, make the angle *g f e* equal to 57°, for the angles at *e* and *f* are together equal to 90°, and 90° – 33° = 57°. Should there be any reason to fear any variation in the needle from *local attraction*, fix on any object as at O, and before leaving *f*, measure the angle *g f* O; leave some mark at *f*, and on reaching the point B measure the angle *g* B *f*, and the angle *f* B B′, which, together with the angle N′ B B′ formed by B B′ with the meridian, should be equal to 180° + 15° = 195°; A *g* = A N × secant N A *g* = secant 15°, 211·87 × 1·0365 = 219·60 = A *g*.

The application, however, of this method, and of many others constantly in use in engineering field-work, presupposes a table of sines, cosines, &c., ready at hand, and which it is not always convenient to carry about; for the purpose of meeting this, which might be considered an objection, a table, consisting of a very few pages, may be readily copied and kept in the field-book, and which will be found to answer all general purposes for which trigonometrical tables may be required.*

Example.—Required the natural sine of 40° 2′; from the table of sines take the sines of 40° and of 40° 10′; subtract the less from the greater, the remainder will be 00223, which divide by 10; this being done by prefixing zero on the left hand, multiply by 2 for the two minutes, and add the product to the sine of 40°, which will give the sine of 40° 2′, sufficiently near for all practical purposes of field-work. Had the minutes been any other submultiple of 10, as for instance 7′ instead of 2′, then 7 would have been the multiplier of the difference divided by 10. Had we required the sine of 40° 25′, then the first figures sought in the tables would have been the sines of 40° 20′ and 40° 30′.† The writer has for years carried a copy of these tables in his pocket-book, and has never required any others for *general field purposes*. With regard to using logarithms for such matters, there is only to observe that the divisions and multiplications effected are so short, that they are done in less time than a table of logarithms can be referred to. There are many persons who carry about with them a volume of logarithms of numbers, and logarithmic sines and cosines, and who might save themselves the inconvenience. With rare exceptions, such tables are only for the study and the office.

We close our remarks on traverses and bearings by observing that, in the case of circumstances obliging us to travel westwards instead of towards the east, as heretofore, we still take the bearing from the reading on the limb of the theodolite. In Fig. 76, let the east and west line be substituted for the former magnetic meridian, so that the former north point shall be west, and the south shall be east; the angle N A c, instead of giving a bearing of 70° with the meridian, will now give one of 15°, being the angle N′ A c, and the reading on the limb of the theodolite will be 345°; and 360° − 345° = 15° = the bearing N 15° W. When the theodolite is moved to station c, the bearing of the *back* angle vernier will be 345° − 180° = 165°, before the bearing of $c\,d$ is taken. The former bearing of $c\,d$ was 35°, and its bearing with the present meridian will be 55°, for 35′ + 55° = 90°; the reading on taking the bearing on the limb of the theodolite, will now be 305°; 360° − 305° = 55° = N 55° W. Similarly the bearing of $d\,e$, as taken at d, will be 291°, which subtracted from 360°, leaves N 69 W.

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* The reader totally unacquainted with the nature of sines and cosines will be so good as to refer to the chapter on "Setting out Curves," where he will find a full explanation of their meaning.

† It is to be observed and remembered that the difference divided by 10, and multiplied by the number of odd minutes *is to be added* for sines, tangents, and secants, and *subtracted* for the cosines, cotangents, and cosecants, as will be more fully shown in the explanation to the table in question.

A little attention to this in the manner we have already described will fully explain it.

CHAPTER VIII.

Railway, Canal, and Road Surveys.—Setting out Bases, and General Management.—Exploring Surveys.—Canal Survey, and Tracing Levels.—General Features.

LAND surveys required for the above purposes generally consist of long but narrow ranges of land, twisting about in various directions, according to the route adopted ; the width of survey required will generally also depend on local circumstances fixing the position of the line in one or more portions to some particular spots ; where this is the case a width of about ten chains may be sufficient ; at other times some uncertainty or other may render a much greater width necessary, as for instance, half-a-mile ; the average width, however, generally surveyed does not exceed ten chains on each side of the centre line, excepting in town lands, marked on some general map, and which in England is usually now a sheet from the Ordnance map, by which the surveyor is guided in his route. Where such a map does not exist, then the best general map that can be procured will be used ; at other times certain rivers, streams, roads, and villages will be named, and certain distances from these will be prescribed or certain bearings by compass combined with landmarks, but in these latter cases a much greater width of survey will be adopted, and a smaller scale than now used at home.

Whatever these circumstances may be, as soon as the surveyor reaches the scene of his labours, the first thing to be done will be to get two or three men as chainmen, who know something of the locality, and who will be able to point to some of the neighbouring objects, such as churches, villages, mills, or homesteads ; anything, in short, as guiding points. Having satisfied himself on the ground as to the direction of a portion of his survey, it will be necessary to set out the base lines, whether these be long or short ; if the direction of the proposed line of railway be pretty straight, then one or two long base lines, or more, according to requirements, may be made to run through the entire extent of the survey ; whilst on the other hand, there may be such short and sharp deflections that the bases will be proportionately short also. Where the bases are thus of inconsiderable lengths, as for instance half or three-quarters of a mile, they are generally very easily set out, and often do not require any setting out whatever, it being merely necessary to select some well-defined and prominent object to chain on, and to leave a flag-pole firmly fixed at the starting point;* in such cases the chaining may commence at once.† But the base line may be of much greater extent, when, on account of undulating ground or tall vegetation, it will require setting out, an operation termed "boning." Under any circumstances, select some such well-defined object as we have already mentioned, if there be one, for the mark to chain *on;* it is not necessary that it be *in* the survey;—rather the better if it be some distance beyond; neither is it necessary that the base should run exactly or quite through the centre of the survey; it is merely requisite that the field-work be based upon it; neither is it to be expected that it will be possible to bone such a base from either end of it; but with a theodolite it may often be done from some commanding point, from which the *far object* may be seen. We shall best illustrate our meaning from Fig. 77, which is a portion of a railway survey in which A B is a base line nearly three miles and a half long. A rough idea of the direction this base would take was ascertained from the commencement, but A, laying in a hollow, it was necessary to find some better spot for defining more accurately the direction of the base, and for boning it out. In the far end, beyond B, the centre of three tall poplar trees was selected as the object to chain upon; the point A was at first quite undetermined, and might fall up or down the road either way. It was only at some distance that high ground could be perceived. Taking our theodolite and poles we walked towards it across the fields, examining the enclosures right and left, and noticing as we went along how cross lines to take up the work would fall into what we supposed generally would be the direction of the base. On reaching the village, and observing the roads, knowing there was to be a junction of a branch with the main line at about this spot, it was determined to traverse these roads; passing onwards we only reached sufficiently high ground to command the view both ways, from the field numbered 42; from here we could perceive that a line A B would intersect our survey from end to end, would be free from obstructions, and that here and there we should pick up some offsetting upon it; the instrument was therefore planted, and the line boned out, one man going towards A, and another towards B, so as to bone both ways at once; we have elsewhere spoken of the marks called "whites," and of ranging-rods; also of the necessity for these being set perfectly upright or vertical, often by means of plummet and line; also of where on a base the mark should be placed so as to prove most useful. All these matters will be found fully described under the head of "Chain Surveying," and the use of the theodolite will be found in our description of that instrument, so that we have no further observations to make here with regard to setting out or boning the base A B; this done, and whilst the man was returning from the far point A, we "traversed"* the roads already mentioned, took the bearings of the base, of some of the interior lines of the traverse, also of the secondary line *a b*, the direction of which was fixed by the position of the fences adjoining it. Whilst doing this an opportunity was afforded of noticing the fields through which the junction of the proposed branch would run, and the theodolite being out, the line *c d* was boned, and the angle A *c d* measured, the commencement of a new base C D was boned out, and its intersection, *by direction only*, with a particular point on the traverse carefully ascertained as one means of checking the direction of C D, besides taking the angle B C D.

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* It will, of course, be understood from what has been already observed in these pages, that, long or short, these bases run through the survey.

† The surveyor is supposed to have examined the length of his chain, and to have marked the correct length of it in some convenient place near his quarters.

The next operation is to chain the base A B, which may, of course, be commenced from either end, as happens to be most convenient; commencing, however, at A, the dispatch with which this operation is performed will depend in a great measure on the number of marks fixed during the boning; and as many of these may be merely "whites" planted on a bank in a hedge, there is no reason why there should not be one nearly, if not quite, for every field; for it will be observed that with the theodolite, when once planted, levelled, and securely clamped, these marks can be set in as fast as the men can be induced to run. Where this can be done from any eminence, so as to get the telescope to plunge as it were into the fields over the hedges, much time is afterwards saved in not having to cut gaps to clear the line of sight. With regard to fixing the stations, it may be observed in railway surveying, or in other cases where the side lines are short, that they may generally be fixed permanently at once; if we can observe from the base line, supposing vision unobstructed, how these side lines are to run, when a picket may be driven in, or if in grass lands a little triangle may be cut out, or a "crow's foot" mark may be made; but if the lines extend such a length that a tall hedge or two obstruct the view, then it is better to make a "false station" or temporary mark only, which may generally be done by notching some stake in the hedge, and of course making an entry accordingly in the field-book. In every case mark on the ground as well as in the field-book every change of arrows, that is, every ten chains; in intersecting straight fences, stations are not required, as side lines will not be wanted, it being merely necessary to note both ends of such fences by offsets, and to take the angle they make with the base line; this is best done with the box-sextant or prismatic compass, as formerly described; these fences are marked *S* in the field-book, and are thus known to be straight when we come to plotting. As regards stations for road-crossings, it will be found that to drive a picket at the foot of one bank, entering it as a "false station," and making some mark on the hedge on the opposite side, such as cutting out a few branches or tying a piece of twine round a stake, will be as safe as anything; driving spikes into the surface of the road, or breaking up a small hole and filling it up with stones, is a highly dangerous practice. There is but one further point to remark on as to the chaining of such a base line, and that is as to whether such chaining shall be done in portions or from end to end before commencing any other portion of the work. Supposing that we commence chaining at A in the first part of the day, the whole line measuring nearly 3 miles and a half, it will bring us very nearly to the close of the day's work on reaching the point B; and supposing we find an hour or two yet to spare, and begin some of the cross-lines at B, we shall then have to return a long distance to get back to quarters at the other end, unless we can find them near B, which, however, may after all be inconvenient. To add to this inconvenience, supposing we begin the cross lines at B, we shall have to return there in the morning to resume operations, or leave this part of the work unfinished, and commence at the more convenient point A, thus preventing the field-book from being as clear as it otherwise might be. If, on the other hand, due care has been taken to make the marks boned out on the base safe from disturbance, and commencing at A we chain on as far as the first cross-road, and leave a blank page for the continuation of the base; then turn to the left down the road as far as *a*, and then to the left again along the line *a b*, which has been fixed by theodolite; in the next place lay out and chain the triangles *g f* A and A *h i*, and the lines *h k* and *k l*, the latter of which has been fixed by the theodolite when taking the angles of the traverse; to complete this we have only short cross lines to run, so that we may soon clear off the work between A and the first cross-road; we may now run the line *c d*, which is produced both ways or at both ends, to fix lines outside the triangle *c d e;* next the line *e d* produced forward for ulterior purposes; the object in producing both these lines is that being two sides of a large triangle on the base line, tied into the traverse by numerous other lines, these two lines will be very useful if carefully produced to fix the base C D which was carried two miles beyond D, and also the base E F, of which we shall speak presently; the sides of this triangle were, moreover, checked by the line *m l n;* there will be now no misunderstanding about filling in and taking up the remainder of the work west of the line *c d*. With regard to the base C D, we have already pointed out that it was fixed by the angle being measured at C, and by being *sighted* on to a station on the traverse, so that if C D had been wrong in position this would have shown it; it was further tied by measuring the lines C B and *o p q* from the fixed point *o* on the line *c d* produced, chained through *p* and *q* on the bases A B and C D; it was only necessary to take a little trouble in boning the line *o p q*, so as to make it a good tie line at the same time that offsets are taken on it all the way. It has already been noticed that the base C D was produced two miles beyond D, and if such a line be out of position, so will also be the whole of the work upon it; it will also be seen that it is quite useless to employ a theodolite to set out a base, unless pains are afterwards taken to fix its position accurately.

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* See article, "Surveying by Traverse."

We now come to the base E F; from E, a full unobstructed view of the country was obtained, and the theodolite was not got out; this base being set out to F, where the direction of the survey turned northwards, another base, G H, was at once set out, only the commencement of which is shown in our plan; it will be readily seen how these bases are connected to the east of the plan by means of two small triangles; this was not considered a sufficient tie for determining on paper the direction of one base line with another; the line G H was therefore sighted backwards to I, so as to connect it securely with some back lines. It may now be observed that we have been fixing the base G H by tie lines to E F, before tying this last line which was first set out; this was because we were in that direction, and therefore determined these points at once when observed, and to save time when walking over the ground again, a most important gain towards the despatch of work, and always to be kept in sight.

The base E F itself is determined in position by the triangle E *t s*, and by the line *u t*, produced to *r*, then to tie into H G produced backwards to this point; there are two further lines run *r v*, *w x*, on which offsets are picked up; and now by means of cross lines, and others more or less parallel to the base, this portion of the work was completed; these parallels to the bases are often extremely useful in taking up fences, and are very conveniently set out by means of the box-sextant.

To the student we would suggest the tracing of this plan, leaving out the chain-work, which is represented by the dotted lines; that he should next with the plan and text before him lay down the base lines and their connecting ties in the manner above described, and keeping in view the same motives, he will then have a better opportunity of distinguishing how these lines are, as it were, *framed* together, laying at first loose and uncertain to some extent, until gradually they become all locked together by the tie-lines: when once satisfied that this is fully accomplished, he need not lay out any more, but fill in as quickly as may be; that is, set out the remaining lines to complete the offsetting, by chaining from station to station, in the easiest manner presented by the nature of the ground and the fences. In carrying out the above suggestion the student need not apply himself, to lay the chain-lines on the tracing-paper exactly or minutely as in the example; it will only be necessary for him to do this to such an extent as to satisfy himself as he proceeds that he understands the rationale of what he is about. Let him remember also that no two cases are exactly alike; that all, therefore, will require treating in a manner different more or less.

In running a line we sometimes unexpectedly find that, to carry it on, we must chain *on* a fence, or abandon it, as in Fig. 78; in such a case, it is very simple to lay out a parallel line for a length sufficient to overcome the difficulty, and return to the main line when this is done; *a b*, *c d*, *e f*, *g h*, are all equal, and set out at right angles to the line A B; *d f* is therefore parallel and equal to *c e*; a sketch and memorandum of this is of course made in the field-book.

When buildings are numerous on such work, we advise, to avoid the false economy of being sparing with chain-lines at the expense of long offsets, thereby going over the length of chain-line perhaps a dozen times; as an invariable rule, the nearer your work, the shorter your offsets, the better you can see to sketch the plans of your buildings, and the more correct the offsets will be in length and direction; it is also to be remembered that the angles and sides of buildings are much more defined on the ground than hedge and ditch.

We may now close these remarks, as under the head of "Chain Surveying," much more has been said referring to this as land surveying generally; but it should be observed that, in some parts of the plan by which we have illustrated the above remarks, a much greater width has been taken in than is generally done; this partly arises from there being a junction of two lines in the middle of the survey, and partly from the line of rail being undetermined in position when the survey was made; the average maximum of width is ten chains on each side of the centre line, where more is required special instructions should be given for the purpose.

An engineer, when making a survey, will take every opportunity of making himself acquainted with as many geological details, or subsoil information, as can possibly be acquired during the survey; he will also acquire of his own knowledge, as much information as possible as to the effect of floods on brooks and streams, the span of arches over them, and the diameters of culverts; the widths and inclination of roads; the kind and cost of building materials, and, if opportunity offers, any information as to traffic along the roads.

In making an *exploring* survey, the engineer's object is to ascertain the general direction of the route along which the best paying and most cheaply constructed line may be found; at least where neither the one nor the other of these important elements have to be sacrificed to some local interest, instances of which in this country have been but too flagrant; with a *good general map*, an exploring survey may almost be resolved for many purposes into a local examination, and noting certain lines on the ground from which a selection may afterwards be made according to circumstances, as they become developed, either as regards levels, construction of works, traffic, and local interests. But where there is no such map, it is not detail we have to explore for, but the general features of the country; in this sense, a survey of the details only would be so much waste paper, and under any circumstances would be at first, waste of time. A few main lines showing the general position of rivers, streams, roads, or tracks, woods, forests, villages, valleys, and summits of hills, with their general angle of inclinations and altitudes, with notes as to geological features; these are the points on which the engineer requires information, and it will be quite sufficient if they are shown on a map of from one to three inches to the mile; until the direction of a line proposed is determined on, a survey of details is of no value.

As regards surveys of railways and canals, much analogy will exist between both, as regards the different levels to be overcome, hills and valleys, cuttings and embankments, tunnels and viaducts, &c.; as regards materials, where in the first case we should look for a gravelly district, in the latter we should seek for clay. It is more particularly as regards water for feeding the canal, and the position of locks and chains of locks, and length of ponds, that the survey for a canal will differ from that for a railway.

Setting aside the question of traffic in a canal survey, for which, however, it must generally consist largely of minerals, and particularly coal, besides agricultural produce, to be of any value to shareholders, the engineer will have to examine for the supply of water more particularly at the summit levels; where it is to come from and how far brooks and streams are appropriated and to what purposes, as for mills, irrigation, &c.; what is to be the connexion with any neighbouring river or canal, if any; the length of long level ponds to be obtained, thereby saving the trouble and loss of time attendant upon locks; by this means a number of locks are often brought together, so that they can be attended to by one lock-keeper; the trial sections will enable him to judge of the position of long levels to be obtained, of the lowest summits to be found between the termini; whether a summit should be reached by a cutting or a tunnel, and also to decide as to other works involved by adopting the whole or part of any particular line, the features of which are shown on a trial section. If after adopting a summit level, and calculating the obtainable quantity of water, with its cost by compensation, if any, or otherwise, or by pumping if necessary, it may be found desirable to trace the level of the summit along the sides of the range of hills, which will involve a series of contour levels,* and at the same time a survey of every spring and stream crossing this level line;† care is to be taken all along the line to distinguish landsprings, or such as only run in winter. The line of contour level will have to be laid down on a plan, on which must also be shown every hill and valley above this level and draining towards it, as also the water ridges, in order that from such data the area of drainage may be calculated above the contour level; from this, if it should prove desirable, may be calculated the capacity required for reservoirs, as well as the quantities of water to be expected to flow into them; the best information attainable must be sought for with regard to the depth of rain-fall, and numerous rain-gauges must be established at different points to corroborate or correct the information obtained. Not only all information obtained, and the observations made, must be entered in a book kept for the purpose, but they must be recorded with all details connected with them in such manner that they may be distinctly and accurately referred to hereafter.

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* See "Contour Levelling."

† All the streams running into or crossing the proposed canal will require gauging, and these gauges will require to be established both above and below the level of the crossing of the canal. See "Gauging Streams."

Certain points being fixed by the levels of the trial section, it will often happen that long level reaches may be established by tracing a contour level along the ground, as, for instance, by the side of a valley until such fall occurs as renders necessary one, two, or three locks, or a fall of from 20 to 30 feet; the depth of this fall being transferred, the level thus found may again be traced forward until the same operation is repeated. As these levels are thus traced a stake may be driven where the levelling staff is held, so that a surveyor coming after may make a survey of them and lay down the lines on the plan; with the assistance afforded by the levels on the trial sections, all referring to one datum, a new and improved general direction may be given to the line, along which another section may be afterwards taken, such deviations being made as seem advisable under contingent circumstances.

In tracing these levels it will be remembered that for canals we are not tied down to curves of large radius, as in railways, and that we have here considerable latitude, it being only requisite absolutely that the boats may be conveniently worked.

Provision will also have to be made for weirs and overfalls, and generally these should be constructed where the floods occur; it will, therefore, be necessary to examine into this subject also.

Such observations being made, and levels traced and laid down on the plan, it will be time to revise the whole scheme, and see what bends may be avoided by cuttings or embankments, as by the plan above-mentioned of tracing contour levels it will be perceived that in many cases the line would be very crooked, and the bends must be overcome.

The above contingencies connected with the supply of water make the circumstances of an exploring survey for a canal considerably different from that for a railway, although the works from point to point are the same, such as cuttings and embankments, tunnels and viaducts, bridges, &c., with the exception, however, of the way itself.

In both we have to reach from one place to another, supplying directly or indirectly certain towns and villages on our way, and we have to join the two ends by the shortest and most level route compatible with economy, safety, and rapidity of transit, &c.; generally there are certain summits or lowest points to be overcome, and it will be a matter of study whether these are to be passed by tunnels or viaducts with easy gradients more or less all the way, or whether a steeper inclination shall be assumed with a partial deviation in route, so as to have merely a cutting or embankment where a tunnel or viaduct was first assumed; sometimes the route by such a summit or low point may be abandoned by means of a lateral deviation of more or less extent, as may be required; the crossing of a river or a town, or even a turnpike road, will probably considerably affect the study of a scheme. As regards railways, there are cases where a little study will enable an engineer to take the surface pretty nearly the whole way, with merely a few feet of cutting and embankment here and there; where the roads to be crossed are not too numerous or too important for level crossings, this is of course an economical system.

CHAPTER IX.

Working Plans.—Details of Townlands' Surveys.—Precautions to be observed.—Traversing.—Object and Value of Working Plans.—Concluding Observations on Land Surveying.

THE observations made in the last chapter apply principally to preliminary surveys, such as are prepared before a working or "permanent" survey is made previous to the actual construction of the works; as already observed, a certain width, being a few chains more or less, is adopted; but for the working survey, for which we are confined for the works at least within the "limits of deviation" assigned under powers obtained from an Act of Parliament or otherwise, it is a sheer waste of time and money to extend the survey beyond such limits; otherwise than this the same system is to be adopted for the working survey as for the preliminary one, with the exception principally that the offsets are shorter and more numerous, and a larger scale adopted for the plan; the same lines may often be taken as bases, but less offsetting will fall on them, as the actual survey is now restricted to the limits of deviation. Another plan to adopt with regard to a working survey is to "stump out the line" first, as soon as final alterations are determined on, and then to make a survey of the line thus stumped out, extending the width of it from 30 or 40 feet up to 100 feet on each side, according as such greater width may be required by the extent of the works, such as the slopes of cuttings and embankments or the inclination of approach-roads.

The above remarks apply equally to the working survey of townlands; here, however, a traverse is almost the only mode to be adopted, the sides of the traverse running down the sides of the street more or less, and where these are wide there being a chain line on each side; on the accuracy of such surveys often depends the purchase or non-purchase of certain property, and the greatest care must be devoted to the work; the offsets should not exceed 20 or 30 feet; in fixing the positions of buildings it is not sufficient to do so by merely taking an offset at right angles to the chain; but two and sometimes three measurements should be taken from different links on the chain to one angle on the buildings, thus, in fact, setting up two or three small triangles, the sides of which are checks on each other. The offsets off the chain lines A B and C D, Fig. 79, will express this as well as a more lengthened explanation. At the same time always measure with the tape all the sides of a building, as it will prove a further check upon the chaining and offsets, for unless this has been correctly done the side measurements of the buildings will not plot.

To every side of a building of any length always take three or four offsets, as they will mutually check each other; for the straight side of every building must pass through all the punctures made if all the offsets are correctly taken.

It will often occur, more particularly in old-fashioned places, that some buildings are so irregularly dovetailed into others that plans of the interiors must be surveyed before it is possible to make a plan of the buildings. An instance of this kind is shown on the right hand of Fig. 79, as also the lines that were measured in order to take the plan. In these cases measurements must be taken of the thicknesses of the walls, as the angles formed by the faces are the points measured to. Instances of this kind sometimes arise which are not difficult, but very troublesome. Impatience on the surveyor's part is then what he has most to fear; such matters *will* take their time to be properly done; he will feel far more annoyed and mortified if, at a future period, it proves necessary to take down a building which his plan showed would not be touched by the construction of the works; for it must be remembered that to take even three inches off a building it must be purchased, if the owner thinks proper, as in such cases he is pretty certain to do. If the error is only discovered after other buildings from that particular owner have been purchased, it may result in a troublesome business. It will thus be seen that townlands' surveying may afford many instances where it is scarcely possible to observe too much care and caution with regard to the details, as well as with regard to the whole.

It is very good practice to ascertain the exact chainage at which a side produced of any building will intersect on the chain line, and to measure the length of such produced line.

It may be observed that to do all this the sketching in the field-book will have to be drawn to a large size, and the writing and figures to be kept small, and written with a hard and sharp-pointed pencil.

Another troublesome case often arises where many of the buildings are not right-angled or square built. This is often the case where it is not sufficiently so to be detected at once by the eye; and although the surveyor should never encumber himself or his men with things not absolutely necessary for the work to be done, it will often be found a very good plan to employ a boy to carry a large carpenter's square, by means of which it may be directly seen whether the angles of a building are square or not, when it may be treated accordingly.

A court-yard, through which there is no thoroughfare, will often take up a good deal of time before a sufficient number of lines are measured to ensure the production of a correct plan, as in Fig. 80; set out a line through the passage, making an angle A C D, with the chain line A B, and measure such angles with the theodolite, for they are too short to be measured with the sextant, on account of the principles of construction of the instrument; then measure the lines C *a* D, D *b*, *b a f*, *f e*, *e c*, *c b*, *c a*, *a e*, D *f*; a sketch should be made of this in the field-book, and the lengths of the lines written in upon them as they are measured. It is not to be supposed that all the work in a townland's survey will prove as troublesome as the two instances we have given in the illustrations, but it will generally be found to occur on inferior property, and the older portions of a town.

For a survey such as we are describing, there will be required a five or six-inch theodolite, a one hundred feet chain, and a tape of equal length, a couple of five-feet rods, half a dozen of six-feet poles, plummet and line, the square we have above mentioned, and a mason's chisel and hammer to cut station marks, and perhaps a few spikes to drive in where there is no horse traffic. With these will be required an assistant and three or four men.

With regard to the traverse, we recommend its being set out so as to include the whole of the survey, if not more than half a mile in length; but whatever may be the extent of it, include as much area at a time as possible.

Supposing that you are going to traverse the whole survey at once, commence at some point where you can take as many angles or bearings as possible; and if at a point *to* which you can see *from* other stations, or some other side of the traverse, so much the better. Make the sides of the traverse as long as possible, so as to have the least number of stations, and therefore the least number of angles to take; but do not fail at the same time to take angles or bearings down every street crossed by the sides of the traverse. Where there are many of these, there are two precautions which we recommend being taken,—the one is, that the station where the theodolite is to be planted for taking these angles with the sides of the traverse after the whole of the bearing of the traverse itself shall have been taken, be sighted by the theodolite, and that such station be marked at once by a small "crow's foot" being cut with a chisel on the kerb-stone if possible, or some other mark made; the other is, that, to prevent such station falling upon a fraction of a foot, the length be measured out at once. Some trouble should be taken with some of these stations, and those on the other side of the work, so that it may be practicable to take observations from each to each.*

If the survey is too extensive to include the whole in one traverse, then commence at such a point that one of the sides of which you first take the bearing shall form a junction line with the next traverse, and if possible extend through the whole width of it.

Where any side of the traverse is of considerable length, we advise its being boned out with the theodolite at about every two or three hundred feet, by having marks cut on the pavement, or driving in spikes, if off the actual horse road. The marks cut need not be more than a couple of inches long, if a memorandum be made as to the house or door opposite to which they are made.† Although some of these precautions will obviously cause a little more time, about a couple of hours generally, in taking the angles and setting out the traverse, it will be found that afterwards time is saved in not having to bring out the theodolite again until wanted for the next enclosure, and also in chaining, as the "leader's" end is much more readily and accurately placed on the chain line when the fore object is only two or three chains in advance, instead of eight, or ten, or more. Indeed, our own practice for such work is to take or set out the length of the sides and the station lengths at first, at the same time the angles are taken; thus avoiding entirely all fractions of feet. By means of these precautions at first starting, there is a very fair chance of being on the way to produce a survey that shall not be out so much as three inches anywhere.

With regard to objects to sight upon, the poles we have mentioned above are by no means constantly required for this purpose, and it is more desirable to chain on some permanent object, such as the angle of a building, a door-post, or any object near the ground; where none such occurs, a vertical line may be drawn in chalk on a wall, which will last a surprisingly long while, but some slight mark should be cut besides for reference to hereafter.

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* See what we have said on this head under the subject of "Surveying by Traverse."

† These marks must be crosses instead of crows' feet, to distinguish them from station marks.

In chaining, instead of the leader making a mark with a pin on the pavement, or paving, which is sure to be scarcely any mark at all, we recommend his being provided with a piece of chalk, with which he is to make a small cross at each chain's length. There are many cases where a hundred-feet chain may have to lay half an hour on the ground before being laid out again; during this time passengers moving backwards and forwards, kick it out of its proper line; now by means of the two chalk marks, one at each, it is brought back immediately and accurately to the proper situation, both as regards length and direction. It is true the chainman should keep it stretched between the marks, but after laying any time on the ground it is difficult to make them do this, and the chalk marks we have just mentioned are much more simple and sure than trusting to them, for the surveyor can then easily see, from where he stands, whether the chain handles are in their proper place.

When the field-work has been completed, and the work plotted, which should be done more or less every day whilst all the objects are fresh in the surveyor's memory, it will be necessary to submit the work to a severe check, and this can almost always be done in the following manner:—Select two or three spots on the ground from which a view may be commanded of the whole, more or less; such, for instance, as the roofs or flat leads of some of the loftiest houses; find the exact points on the plan, and from these draw any convenient number of lines intersecting as many prominent features as possible, and examine with the plan on the spot how the work agrees with these points on the ground. It is generally quite possible to select some spot to which the theodolite may be raised, and on which it may be thus planted, when the plan may be submitted to any amount of scrutiny the surveyor may please, or judge necessary.

It may be necessary to observe that the above remarks on the working plan of a townland's survey refer principally to one plotted to a large scale, such as 30 or 40 feet to 1 inch, and on which therefore the general plan of the works of construction, such as a dock or station, a viaduct or cutting with its retaining walls, may be laid down with a considerable degree of precision; if the plan of the survey is correct, it will at once be seen exactly what properties are and what are not required for the "emplacement" of the works; and if the plan does not enable us to do this with accuracy, it is of no use, and if we are misled by it, then it is worse than useless. As an instance, a long viaduct or cutting with retaining walls may be taken, such works may be commenced at once in a dozen different points, as required by the conditions of the structures, or accordingly as the company obtains possession of property required in as many different places. Unless we delay until the whole property is purchased, it is not possible to set out such a work, whether on a straight line or curves from end to end at once, and as such delay is not a practicable thing, it follows that the properties must be purchased, the line set out, and the works constructed on almost the sole condition of the good faith and working accuracy of the plan; if at fault, it will follow that errors have been made in the purchase of property, and that awkward deviations have to be made in the works, and sometimes even that reconstruction may be a matter of necessity. There can, therefore, be no doubt that a correct working plan to a large scale is indispensable, though costly; that the expense of it will in the long run be an economy, and that such a plan can only be produced by the greatest care being given to it.

In a townland's survey for a general plan the same system and principles of operation will have to be carried out, but there must be greater despatch, and inasmuch as a great number of the minutiæ above noticed could not possibly be plotted even by the most careful and ingenious draughtsman, it would be useless and time thrown away to attend to them; with a scale of 40 feet to 1 inch it is very easy to plot accurately dimensions of six inches, but with a scale of 6 chains to 1 inch, it would be rather trying to the sight to plot one foot which would be the 66th part of a sixth of an inch. A little experience and intelligence will soon teach the practice to be carried out under a variety of conditions and requirements.

On the plan given to illustrate the observations made in this chapter, the *centre line* of the proposed railway has not been laid down, nor the "limits of deviation," that is, the limits within which it is proposed to deviate, if thought advisable, the centre of the line works before these are actually commenced. Further observations on this subject will be found under the head of "Levelling," and also in the pages of my work on "Railway Construction."

In concluding this portion of our subject, we may mention a system which is sometimes adopted, when the time by which the survey is to be completed is so short that nothing better can be attempted; at the best it can only be applied to surveys of a very preliminary character; nevertheless, where judgment and intelligence have been combined in doing the work, it has often passed the ordeal of "Standing Orders;" there are besides sometimes portions of a survey where it is less inapplicable than in others.

In making a survey in this manner, the bases are chained on some distant objects, if any can be selected; these bases are generally short, and the angles of deflection which they make with each other should be carefully observed with the theodolite if possible, or with the box-sextant; at the intersection which the fences make with the base line, the bearings of such fences on *both sides* of such base should be taken, and the distances from the intersection to the junction, with any lateral fences within about 4 chains' distance on each side; the bearings of such lateral fences should also be taken, and any observations required should be made as to deflections in these fences; the surveyor should have a competent assistant with him if possible, to make these observations on the *lateral fencing*, and from whose field notes he must take care to make entries into the field-book as the work proceeds along the narrow slip surveyed; in such cases as this it is very desirable that the base lines be laid as nearly as conveniently may be along the centre of the work. One considerable disadvantage with such work is, that there is no check even upon the main length of the survey, until the section is taken over the line; a double chain line therefore is very desirable instead of a single one, in which case they would be a few chains apart; these circumstances are mentioned to show the beginner the straights to which we are put sometimes in practice.

When a railway or canal survey is divided between two surveyors, they should, previously to commencing work, decide upon a common base line, which should extend for some length into both portions of the survey; this line will afford the means of correctly joining the work together.

It may not be improper here to observe, that there are two mistakes generally attending the first attempts at surveying: these are either too much or too little attention to details according to circumstances and the scale of the plan, that is to say, that the surveyor is apt to bestow as little pains on the survey for a working plan as for a preliminary one, or *vice versâ*. To make this plainer, take a plan prepared to a scale of one chain to an inch, and endeavour to reduce this to a plan having a scale of three inches to the mile, and showing every detail on the large plan, it is simply impossible; again, take the plan of a survey plotted to a small scale, enlarge it to one of eight or ten times the original, take your enlarged copy on to the ground, and you will find your details all wrong. Again, take a field-book of a survey for a large working plan, and endeavour to plot it to a small scale, you will find you cannot get all the details in; they are simply too small to be shown. The surveyor must learn to discriminate between such cases, so as not to waste his time, on the one hand, or under one state of things, and not to neglect his work on the other; a little thought and observation, after a few days' field-work and plotting, will make these remarks obvious enough.

If, during the perusal of the above pages, the reader has taken the trouble to practice the instructions here given, and has endeavoured gradually to rely upon his own intelligence and discrimination, equally as upon our advice, there is every reason to hope that he will by this time have obtained considerable insight into the business of Land Surveying, including the use of all the instruments required; but inasmuch as it is much more an art of experience than of theory, if we may be allowed the expression, we must strongly impress on his mind that it is only by practical study *on the ground* that he can attain skill combined with readiness in the performance of his work; without practice no amount of reading or teaching will ever make a surveyor.

CHAPTER X.

Ranging Curves.—General Observations.—Tangent, Secant, Sine, &c.—Practical Explanation and Application of Trigonometrical Tables.—Practice of Ranging Curves on the Ground.

THE author feels that he will be accused of pursuing a very unusual course, in placing this chapter amongst the remarks on the different systems of surveying; he can only submit as an excuse for doing this, that he considers the subject as affording the means of introducing the non-mathematical reader to the understanding and practical application of a little plain trigonometry; such as many circumstances in field engineering operations render *at times* of every-day use, when, if the reader happens not to possess such elementary knowledge, he has only to follow some rules which he only partially understands, and is consequently apt sometimes to misapply; moreover, if he happens to meet with an unusual difficulty, and he may expect to meet with such a condition at any time, he is certain to have double and treble labour to overcome it, from the want of understanding a few rules as simple as the arithmetical calculations he has to perform every day. A sound knowledge of the *rationale of the system* to be pursued in setting out or ranging curves is not only requisite in railway work, but to engineering field-work generally; and this *rationale* is based on "right-angled trigonometry," which it is as simple to understand as the easiest rules of geometry. Circumstances constantly arise where one method is preferable to another, and not unusually cases occur when the engineer has to construct some system to meet special difficulties; to effect this he must understand the principles on which all the systems are based. We are quite aware, that *many* practical men will at once turn away from the mere term "trigonometry," for they appear to entertain an intuitive dislike to everything approaching to mathematics, little aware, as they consequently must be, that they are every day performing some mathematical operations on the ground. As a practical man myself, I am not without hopes that such readers will now glance over and examine the following half-dozen pages, when I think that much of this feeling will be removed; "plane right-angled trigonometry" will then be easily understood as well as the use of the tables; this will form an introduction to oblique-angled trigonometry, or at least as much of it as we require for "surveying by observation," or for harbour and coast work, and occasionally under other circumstances.

Such is the writer's excuse for placing this subject where he has, and also for treating it in the following manner, very unmathematical and unsymmetrical, but such that any practical man will understand.

In Fig. 81, let A B be a curve, which it is required to trace on the ground; let A C be a *tangent* (so called because it *touches* the curve without cutting it), from which at the point C it is required to measure off the offsett C D, so that D shall be a point on the curve.

Observe that the length C D is the only line which, measured off at right angles to A C, from the point C, will exactly intersect the curve at D; also that C D is equal to the trigonometrical sign A E, which is called coversine, and that A C is equal to the line O I, called cosine of the angle D O B; also that the end D of the line C D, intersects the curve at the point where this curve is also intersected by the line O F, called the secant, and which forms one side of the angle D O B.

Observe also that C D, plus the line called the sine, is equal to O E, plus the line called the coversine, and that the sine, plus C D, or plus the coversine, is equal to the radius of the curve.

Observe again, that whatever fraction or portion of the radius A C may be, these proportions must always exist; see Figs. 82 and 83 also.

The angle D O B has been mentioned above, as also some of the trigonometrical lines; it will now be time to refer to the others, premising, however, that we are not going at first beyond the right angle, or an angle of 90 degrees, because this will be sufficient for our present purposes, and because by attempting more it would perhaps only confuse the reader, and interfere with the speedy application to practice of what we are about to say, which speedy application will we trust lead the reader on to pursue this subject, as he must see at the very commencement how useful, the very simplest knowledge of the merest elements of trigonometry may be made.

An angle is subtended by an arc, as D O B is subtended by the arc D B, or if a straight line be drawn from D to B, the angle D O B will be subtended by the chord D B. The circle being divided into 360 degrees, and each degree into 60 minutes, and each minute into 60 seconds, the angle may be measured by the number of such divisions and subdivisions contained in the arc.

Tangent and Secant. The tangent of an arc, or of the angle it subtends, is the straight line drawn from the beginning of the arc B, at right angles to the radius, which forms one side of the angle, and is terminated where it is intersected by the other side of the angle produced; in the three figures already referred to, B F is the tangent of the angle D O B. The leg or side of the angle produced to intersect the tangent is called the secant, thus, O F is the secant of the angle D O B, of which B T is the tangent.

Sine. The sine of an arc, or of the angle subtended by it, is a line drawn from one extremity of the arc at right angles to the radius, drawn from the other extremity of the arc, and through the centre of the circle, thus, D I is the sine of the angle D O B. Both the sine and the tangent are perpendicular to the radius, drawn from and to the same point, and they are therefore parallel.

Each of the above three signs increases as the angle increases.

Cosine. The cosine of an arc, or of the angle subtended by it, is a portion of the radius, intercepted at the centre by the secant at one end, and by the sine at the other end; it is at right angles to the sine and to the tangent. This sign decreases as the angle increases. O I is the cosine of the angle D O B.

Versine. The versine of an arc, or of the angle subtended by it, is that portion of the radius intercepted between the sine and the tangent, to both of which it is perpendicular. The versine increases as the angle increases. I B is the versine of the angle D O B.

$$\text{Secant}^2 = \text{Radius}^2 + \text{Tangent}^2,$$

or the square of the Secant = the square of the Radius + the square of the Tangent.

The square of the Radius = the square of the Sine + the square of the Cosine.

$$\text{Cosine} : \text{Sine} :: \text{Radius} : \text{Tangent}.$$
$$\text{Cosine} + \text{Versine} = \text{Radius}.$$
$$\text{Radius} - \text{Cosine} = \text{Versine};$$
$$\sqrt{(\text{Radius}^2 + \text{Tangent}^2)} = \text{Secant}.$$

Complement of an Angle. The complement of an angle is that portion of a *right angle* which *an angle* wants of a right angle; or the complement of an angle is that portion of a right angle which the angle wants of 90 degrees; thus, if the given angle be 50 degrees, then the complement of the angle is 40 degrees. D O B being the angle, A O D is the complement of the angle; and, therefore, an angle and its complement are always equal to 90 degrees.

COTANGENT AND COSECANT.—The cotangent of an arc, or of the angle subtended by it, is the tangent of the complement. It is a straight line drawn from one end of an arc at right angles to the radius, such radius being one side of *the complement*, until intercepted by the other side produced; this other side produced, until it intersects the cotangent, is called the cosecant, and is to *the complement* what the secant is to the given angle, that is the secant of the complement. The cotangent is parallel to the cosine or versine, and the cosecant is on the same line as the secant. Thus A G is the cotangent of the angle D O B, and the tangent of A O B; O G is the cosecant of the angle D O B, and the secant of the complementary angle A O D.

COVERSINE.—The coversine of an arc, or of the angle subtended by it, is to *the complement* that which the versine is to the given angle; thus A E is the coversine of the angle D O B, and it is the versine of the complementary angle A O D.

The coversine of an angle *plus* the sine of that angle = radius, for I C = A O = O B = to radius; therefore radius minus sine = coversine. Thus much as to all we shall require from a knowledge of the trigonometrical signs as regards the setting out curves.

Now, let it be supposed that in the figures 81, 82 and 83, the radius represents 1, 10, or 100 miles, chains, feet, inches, or any other measure, it will be seen at a glance that the signs we have above referred to, that is the tangents, secants, cosines, sines, &c., will be either less or greater than, or equal to, the radius, and that their proportions to the radius will increase or decrease as the angle D O B is more or less acute.

Thus in Fig. 83, where the angle D O B is greater or more obtuse than in Figs. 81 and 82, the tangent B T, the secant O T, the sine I D, the versine I B, are greater in Fig. 83, in proportion to the radius, than these signs are in Figs. 81 and 82; but as regards the cosine O I, the cotangent A G, the cosecant O G, the coversine A E, these signs will be less in proportion to the radius in Fig. 83, where the angle D O B is greater than in Figs. 81 and 82, where the angle D O B is less.

APPLICATION OF TRIGONOMETRICAL TABLES.

If, therefore, we take the radius as equal to any given lineal measure, it is clear that the signs above referred to will have numerical representatives according to the measure of the different angles subtended by the arcs. In the tables* the radius is equal to 1, or unity, and the trigonometrical signs are less or greater than unity, or equal to it, accordingly as the angles varies. With Hutton, from 0° to 45° we read from the top of the page downwards, and from 45° to 90° we read from the bottom of the page upwards; the minutes being on the left hand in the first instance, and in the second being on the right.

Example.—Let the given angle be 11 degrees and 27 minutes, then the proportions will stand thus:—

Radius = 1, sine ·1985, coversine = ·8015, cosecant = 5·03746, tangent = ·2025, cotangent = 4·93720, secant = 1·02030, &c.

Again, let the angle be 42° 22′, and the proportions will stand thus:—

Radius = 1, sine = ·67387, coversine = ·32613, cosecant = 1·48396, &c.

Or, let the angle be 88° 35′, the proportions will be as follows:—

Radius = 1, sine = ·99969, coversine = ·00030, cosecant = 1·00030, &c.

Note that in the operations connected with ranging curves, it is not necessary to take into calculation more than four figures of decimals.

To elucidate further the use of the tables, let the angle be 30° 25′, and let the radius be 40 chains; required the lengths of the tangent, secant, and cotangent.

			Tabular values.				Required lengths in Chains.
Angle 30° 25′	Tangent	=	·5871	×	40	=	23·484
	Secant	=	1·1596	×	40	=	46·384
	Cotangent	=	1·7033	×	40	=	68·132

Take now the same angle, but with a radius of 5000 feet; required the lengths of the same signs.

			Tabular values.				Required lengths in Feet.
Angle 30° 25′	Tangent	=	·5871	×	5000	=	2935·5
	Secant	=	1·1596	×	5000	=	5798·0
	Cotangent	=	1·7033	×	5000	=	8516·5

With the same angle again, and a radius of 60 chains, required the lengths of the cosecant and coversine.

			Tabular numbers.				Required lengths in Chains.
Angle 30° 25′	Cosecant	=	1·9752	×	60	=	118·512
	Coversine	=	·4937	×	60	=	29·622

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* See Hutton's or Young's Trigonometrical Tables.

Let us now suppose that we have a trigonometrical figure, of which a line we assume as cotangent is equal to 10·57, the radius being 45·00, required the angle, the cosine, and coversine. Divide the given cotangent by the radius, and we have ·2348 for cotangent to radius 1; look for this number ·2348 in the column of cotangents, and we find the corresponding angle to be 76° 47′; on the same line we find for cosine ·2286, and for covers ·0265; these multiplied by our radius of 45·00 give respectively 10·287 and 1·1925 for our cosine and coversine.

The foregoing observations and the examination of the Figures 81, 82, and 83, will readily suggest a method of setting out a curve in a manner mathematically correct, and quite as simple and practical. Let A C, in either of the figures referred to, be tangent to a curve of any given radius; and let C D at right angles to A C be the offset of which the length is required. A C is equal to the cosine O I of the angle D O B; if we divide A C = O I by the given radius we shall have the length of the cosine of the angle D O B, whatever that angle may be, but the radius will be 1, or unity. In the columns of cosines seek for that number thus found, and on the same horizontal line we shall find the minutes, the angle itself being at the top or bottom of the page, accordingly as we find our figures to read from the top or bottom of the page.

Ex: Thus let the length of A C be 100 feet, and let the radius be 400, and let it be required to find the length of the offset C D, so that when C D is measured off at right angles to A C, D shall be a point on the curve.

400)100(0·2500 = natural cosine to radius 1.

In this case, as in almost every other connected with the setting out of curves in this manner, unless the lengths of the given line and the given radius approach to equality, we shall have to seek for the number ·2500 in the column cosine from the *bottom* of the page; the required angle will therefore also be at the bottom of the page, and the minutes in the column signed or marked (′) on the *right hand.* We shall in this manner find 75° 31′ for the angle, by means of which we shall obtain other figures which will not only give the offset required, but the means of checking our work.

Observe that in either of the Figures 81, 82, and 83, the offset C D is equal to the coversine A E of the angle D O B; then if on the same horizontal line where we have found ·2500 we look in the column signed coversine, we shall find ·03177 as coversine to the angle 75° 31′ for a radius equal to unity; and if we multiply this by our given radius of 400, then we shall obtain the length of C D required.

$$\cdot 0318 \times 400 = 12\cdot 72$$

This measured off at right angles to A C from the point C will give the point D on the curve, *but without any reference to the length of the arc* A D.

Similarly let A C = 20 chains and radius 70 chains; required the offset C D:

$$20 \div 70 = 0\cdot 2857 = \text{cosine of angle } 73^\circ\ 24', \text{ near enough,}$$

of which the coversine = ·0417; and ·0417 × 70 radius = 2·919 = offset C D.

Before, however, proceeding any further let us return to the subject of *checking our work.*

Observe that in the Figures 81, 82, and 83, A G is the cotangent, and O G the cosecant of the angle D O B; and referring to our first example in the tabular numbers the cotangent = ·2583, and the cosecant = 1·0328.

Then,

$$\cdot 2583 \times 400 = 103\cdot 32 = \text{A G.}$$

And with regard to the cosecant O G,

$$1\cdot 0328 \times 400 = 413\cdot 12 = \text{O G.}$$

But G O − radius = G D;

Then,

$$413\cdot 12 - 400 = 13\cdot 12 = \text{G D,}$$

which will be the exact length measured from G to D, if the point D has been correctly set out.

Similarly in our second example, A C will be found equal to 20·867, and G D = 3·044.

As before observed, the above examples have been gone through without any reference to the *length of the arc* A D, and we have taken the coversine, the cotangent, and the cosecant of the angle D O B, because A C = O I = cosine ÷ radius, enabled us to get at the *tabular* cosine; this once obtained we find, as above shown, the coversine A E = C D, and the cotangent = A G, and the cosecant = O G, and therefore by subtraction G D also. But A E, the coversine of the angle D O B, is the versine of the complementary angle A O C; and A G, the cotangent of the angle D O B, is the tangent of A O C; O G, the cosecant of the angle D O B, is the secant of A O C; but 90° − 75° 31′ = 14° 29′

= the angle A O D, of which, therefore, A E, A G, and O G are respectively the versine, the tangent, and the secant of the angle A O B.

By referring to the angle 14° 29″ in the tables this will be found to be the case; for,

Angle.		Tabular numbers.	
14° 29″	Versine . . . =	·0318 × 400	. . = 12·72
	Tangent . . . =	·2583 × 400	. . = 103·32.
	Secant . . . . =	1·0328 × 400	. . = 413·12

Also let it be observed that we have now obtained the angle at the centre = A O D, and this the reader is requested to keep in sight.

Observe also, that by subtracting the angle A O D from 90°, we obtain the angle A G O, for the three angles of a triangle = 180°, or two right angles, and the angle at A is equal to one of them.

Again, with regard to the angles here concerned, let a straight line be drawn from A to D, required the angle C A D, which may be called the tangential angle; the three angles of a triangle being equal to two right angles, subtract the angle at the centre from 180°, and divide the remainder by 2, which will give the angles D A O or A D O, and the angle O A C being a right angle 90° minus the angle D A O = C A D, the tangential angle.

It is not to be supposed that the knowledge of the values of all these lines and angles is always indispensable in setting out curves, but let it be remembered that an attempt is here made to explain thoroughly the *rationale* of the ranging of curves; in some cases the point D may be quite inaccessible except in a boat, or it may be impossible to measure from C D. Now, having the angles at A and at G, D may be set out by the use of two box-sextants used simultaneously at A and G; and as a check upon this, always requisite in these cases, note that a line at right angles to A C, from the point C, set off by instrument only, and therefore without linear measurement, should exactly intersect the point D, already set out from A and G. This may be done very readily and quickly by the box-sextant.

The reader's attention has been already particularly directed to the angle at the centre, A O D. Refer now to Fig. 84.

By the methods given above, we find the angle A G O, for

$$90° - A\,O\,G = A\,G\,O,\ O\,A\,G \text{ being a right angle.}$$

From G as a centre, with distance G A, set off G E; then the angle G O E = A O G, because the arc D E = A D, for equal arcs subtend equal angles.

One line falling upon another makes the adjacent angles equal to two right angles; the A G O plus O G T = 180°, and also A G O, O G E, E G T = 180°.

But A O G + A G O = 90° + E O G + E G O = 180°;

and, subtracting in both cases A G E = A G O + E G O from 180°

leaves E G T = A O E,

which we may call the double angle at the centre, since it is compounded of A O G + E O G.

Therefore in practice, having the direction of A T given, and the length of A G found by means of the tables, if we set out by instrument the angle T G E equal to twice A O D, and then make G E = A G, E will be another point on the curve. Evidently also, if we have the *direction* given of the two tangents A G and E G, and we wish to ascertain the *distances* G A and G E from G, at which the curve is exactly tangent, we have only to measure the angle T G E, divide it by 2, look for the quotient in the angles in the tables, and on the corresponding line in the column signed tangent we shall obtain the *tabular* length or number corresponding to the angles at the centre, A O G, E O G, each equal to half T G E, and which tabular number multiplied by the radius, will give the distances G A and G E.

In the last page we have supposed the angle A O D = 14° 29′; then the angle T G E will be equal to 28° 58′, and if we have the same radius of 400 feet (tabular tangent being ·2583) we have only to make G E = 103·32, and E will be a point on the curve.

Length of Arc. Let us now take into consideration a given length of arc upon a curve of a given radius.

Ex. Radius 60 chains, and the length of a portion of the curve = 10 chains.

A quadrant embraces 90 degrees, which multiplied by 60 minutes, gives 5400 minutes; the constant number 3·1416, multiplied by the diameter of a circle = the circumference, and in this instance therefore, 3·1416 × 120 = 376·992; but as we only require the length of the arc for the quadrant, or a fourth part of the circumference, we may multiply 3·1416 by half the radius, and we shall have a length of arc = 94·248, (of course in chains) subtending the 5400 minutes in the quadrant.

94·248 : 5400′ : : 10 : 572′·95 = 9° 32′·95 = A O D, the angle at the centre, subtended by the arc of 10 chains with a radius of 60 chains.

Or, $9{\cdot}4248\,)\,5400\,(\,572'{\cdot}95 = 9^\circ\ 32'{\cdot}95 = \text{A O D}$,
the angle at the centre.

That is, divide the length of the arc for the quadrant by 10, and by the quotient divide 5400, the number of minutes in the quadrant, which will give the number of minutes in the angle at the centre A O D, and this divided by 60, gives the angle in degrees and minutes.

As another example, let the length of the arc still be 10 chains, but the radius 25 chains.

$3{\cdot}1416 \times 12{\cdot}5 = 39{\cdot}27 =$ length of arc in quadrant.

And,

$3{\cdot}927\,)\,5400\,(\,22^\circ\ 55' =$ angle A O D at centre.

Let us return now to the radius of 60 chains. The first thing we shall require will be the length of a tangent, (not a trigonometrical tangent) but a line so called, because it touches the circle without cutting it, as before observed; this line A C, see Fig. 85, we require in order to set off from C, and at right angles to A C, the length of the offset C D, which will give a point on the curve.

In our first demonstrations, A O D has been called the complement of the angle D O B; we may reverse this, and call A O D *the angle*, and D O B the complement.

It has been shown that A C = cosine of D O B.

$90^\circ - 9^\circ\ 32'{\cdot}95 = 80^\circ\ 27'{\cdot}05$;
$\text{A O D} = 9^\circ\ 32'{\cdot}95$, and $\text{D O B} = 80^\circ\ 27'{\cdot}05$
Nat. Cos. of $80^\circ\ 27' = {\cdot}1659$;
${\cdot}1659 \times 60 = 9{\cdot}954 = \text{A C}$;
Nat. Vers. $9^\circ\ 32'{\cdot}95 = {\cdot}01386$;
${\cdot}01386 \times 60 = 0{\cdot}8316 = \text{C D}$;
Nat. Tang. $9^\circ\ 32'{\cdot}95 = {\cdot}1682$;
${\cdot}1682 \times 60 = 10{\cdot}092 = \text{A G}$;
Nat. Secant $9^\circ\ 32'{\cdot}95 = 1{\cdot}01405$;
$1{\cdot}01405 \times 60 = 60{\cdot}843 = \text{O G}$;
$60{\cdot}843 - 60$ (Radius) $= {\cdot}843 = \text{G D}$.

To find A *c*, and *c d*;

Half the angle $\text{A O D} = 4^\circ\ 46'\ 30''$
$90^\circ - 4^\circ\ 46'\ 30'' = 85^\circ\ 13'\ 30'' =$ Complement of Angle;

$A\,c$ = Cosine of 85° 13′ 30″ × 60
$c\,d$ = Versine of 4° 46′ 30″ × 60
We shall by this means find $A\,c$ = 4·995
and $c\,d$ = 0·2082, or very nearly 21 links.

With regard to the lengths of the intermediate offsets, observe that, as an approximation, the lengths of these will be at the consecutive chains as the squares of the times; that is,

if at 1 chain = 1
then at 2 „ = 2 × 2 × 1 = 4
at 3 „ = 3 × 3 × 1 = 9
at 4 „ = 4 × 4 × 1 = 16, &c., &c.
and at 10 „ = 10 × 10 × 1 = 100.

In the case now in hand, it is at 10 chains = 0·8316;

then at 1 chain it will be 100th part of 0·8316 = 0·008316
at 2 chains it will be 4 times this = 0·0332
at 3 „ „ 9 „ = 0·0748
&c. &c. &c. &c.

The ten first chains are now set out.

To carry on the course of the curve, make the angle $T\,G\,T^2$ = 9° 32′·95 × 2 = 19° 5′·90, and repeat along G E and E F the same operations we have been carrying out along A G. Having reached F, make the angle $T^2\,F\,L = T\,G\,T^2$, that is = 19° 5′·90.

On reaching K, after passing H, the length of the arc measures 4·40. To reach the tangent K L, we must multiply our offset at 1 chain = 0·008316 by (4·4 × 4·4) = 19·36, which will give for offset 0·1607 . . . which if the curve has been correctly set out, will be the exact measurement from the tangent to the point K, supposed previously fixed.

In practice the fixing of A and K is the first thing to be done, in order that we may have the exact points at which the curve joins the tangents A T, K T. To ascertain this, range out as correctly as possible A T and K T from the plan, measure the angle K T X, one half of which, as already shown, will be equal to *one* of the angles at the centre, and which has in our former demonstrations been called the angle A O D.

From the tables find the natural tangent of *half* the angle K T X, and multiply it by the radius, which will give the length of T A and K T, A and K being the points at which the curve is to commence and terminate.

At the same time that the natural tangent is taken from the tables, take out also the natural secant of the angle at the centre, multiply it by the radius, and then subtract the radius; the remainder will be equal to T S, which in the Figs. 81, 82, 83, and 84, we have hitherto called G D.

Ex. Angle K T X = 42° 24′.

$$\frac{42^\circ\,24'}{2} = 21^\circ\,12'' = \text{one of the two angles at the centre.}$$

Nat. Tangent 21° 12′ = $\cdot 3879 \times 60 = 23\cdot 274$ = A T or T K.
Nat. Secant 21° 12′ = $1\cdot 07258 \times 60 = 64\cdot 3548 - 60 = 4\cdot 3548$ = T S.
But $180^\circ - 42^\circ\,24' = 137^\circ\,36' \div 2 = 68^\circ\,48'$

If then we make either of the angles A T S, K T S = 68° 48′, and make T S = 4·3548, we shall have the point S on the curve.

To find the Length of the Curve.—Refer to the proper table for the lengths of arcs, take out the tabular numbers referring to the degrees and minutes subtended by the angles at the centre, and multiply them by the given radius, which will give the length of curve.

Ex: Angle 42° 24′;

$$\begin{array}{lr}
\text{Tabular number for } 42^\circ = & \cdot 73303 \\
\quad ,, \qquad ,, \qquad 24' = & \cdot 00698 \\
\hline
 & \cdot 74001 \\
 & 60 \\
\hline
\text{Length of curve} = & 44\cdot 40060.
\end{array}$$

Another method to ascertain the length of the curve will be found further on.

In Setting out by Secants.—If we choose to set out, as, for instance, in the case of a long curve, more points than the central one S by means of secants before proceeding with any other method, we may take *half the tabular tangent* already found, and ascertain by this in the tables the corresponding angle, and consequently the tabular secant.

Ex: In Fig. 86 the tabular tangent for A T = ·3879 as above;

$$\frac{\cdot 3879}{2} = \cdot 19395 = \text{nat. tangent } 10^\circ\,58'\,30'',$$

nat. secant 10° 58′ 30″ = $1\cdot 01863 \times 60 = 61\cdot 1178 - 60 = 1\cdot 1178$ = G D; and,

$$90^\circ - 10^\circ\,58'\,30'' = 79^\circ\,1'\,30'' = \text{A G D}.$$

The method above explained we will call setting out by tangents and secants, in order to distinguish it from the method to be given next, and which is almost entirely performed by instrumental observation, and consists in setting off with a theodolite the angles T A *a*, T A *b*, T A *c*, &c., Fig. 87, at the same time that the chainage A *a*, *a b*, *a c*, &c., is carried forward, A *a*, A *b*, A *c*, &c., subtending so many angles at the centre.

In setting out by this method we have first to ascertain the points A and *k*, at which the curve commences and terminates; this is obtained by measuring the angle X T L, Fig. 85, explained above. We thus ascertain the lengths of the tangents T A and T *k*, and these being measured off from T give the points A and *k*.

Then divide the constant 1718·9 by the radius of the curve to obtain the angle T A *a*, and call this angle A.

$$\begin{aligned} \text{Then } TAb &= 2A \\ TAc &= 3A \\ TAd &= 4A, \text{ \&c., \&c.} \end{aligned}$$

Ex: Let the angle X T *k*, Fig. 85, be equal to 38° 30′, equal to twice the angle at the centre, which we have called A O D in the former figures; and let the radius = 60 chains.

$$\begin{aligned} &\text{Nat. tangent } 19^\circ\, 15 = \cdot 34921 \\ &\cdot 34921 \times 60 = 20\cdot 953 = TA \text{ or } Tk. \end{aligned}$$

This distance measured off on the tangents gives the points A and *k* for commencement and termination of the curve.

$$\begin{aligned} 1718\cdot 9 \div 60 &= 28'\cdot 65 = TAa = \text{tangential angle, Fig. 87.} \\ 28'\cdot 65 \times 2 &= 57'\cdot 30 = TAb \\ 28'\cdot 65 \times 3 &= 1^\circ\, 25'\cdot 95 = TAc, \text{ \&c., \&c.} \end{aligned}$$

Length of Curve.—Observe that half the angle X T *k* (Fig. 85) = the angle A O D, Figures 81, 82, and 83, is also equal to the *angle of deflection* G A D in those Figures, and T A *k*, Figures 85 and 87.

Divide half the angle X T *k* or the angle of deflection *in minutes* by the tangential angle, and the product will be the length of the curve.

Ex: $38^\circ\, 30' \div 2 = 1155'$,

and $28'\cdot 65)1155(40\cdot 314$ = length of curve in chains.

Before removing the instrument from T set out the angle A T S, which is equal 90° − 19° 15′.

$$\begin{aligned} &\text{Nat. secant } 19^\circ\, 15 = 1\cdot 05922\,; \\ &1\cdot 05922 \times 60 = 63\cdot 553 - 60 = 3\cdot 553 = TS. \end{aligned}$$

We thus obtain the point S at the centre of the curve, by the means of which we have check on reaching that point.

To Range the Curve by the Tangential Angle.—Plant the theodolite at A, with the zero on T, set off the tangential angle T A $a = 28'\cdot65$, and measure off 1 chain from A to *a;* the intersection of the visual line with the end of the chain fixes the point *a;* for the second chain set the vernier of the instrument at twice T A $a = 57'\cdot30$, and measure another chain from *a* to *b*, when the intersection will fix the point *b*, and so on.

Having thus set out a certain number of chains, say to *d*, move the instrument with the vernier fixed at the angle T A *d* to the point *d*. There fix the instrument with the vernier on A, and then move round to zero, which will place the visual line on the tangent $d\,T^1$; reverse the telescope, which will give the tangent produced to T^3, and set out the angle $T^3 d\,e = 28'\cdot65$, $T^3 d f = 57'\cdot30$, &c., as before.

Repeat this operation till we get to the end of the curve.

Generally 10 chains at each plant will be sufficient;* when after reaching *i*, *i k* is a fraction of a chain, then the angle *i d k* will be a corresponding fraction of the tangential angle; thus if $i\,k = 0\cdot40$, the $i\,d\,k = 11'\cdot46$.

Given Two Points on the Ground to Pass a Curve through them when each is visible from the other.—Measure the length A *k*, Figures 85, 86, 87, and the angle T A *k*, and divide the angle in minutes by the length, which will give the tangential angle. Possibly the difference between the length of the chord and that of the arc may be sufficient to prevent the last stump set out from reaching *k* properly; but we shall then have the correct length of the arc, by which again dividing the angle of deflection we shall obtain the correct tangential angle.

To find the Radius of the Curve thus set out divide the constant 1718·9 by the tangential angle, and the quotient will give the radius.

Therefore if from A we can obtain sight of *k* and the distance A *k*, we ascertain the angle of deflection, the tangential angle, the angle at the centre subtended by half the arc, and also the radius. On reaching the point *k* we have merely to fix the direction of the straight tangent at that point by the method already explained.

Given Two Points on the Ground not visible from each

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* Unless the verniers should read a fraction of a minute.

other.—In this case we must set out the tangents A T and T *k*, Fig. 88, measure the angle X T *k*, and proceed as before explained to find the length of A T and T *k*, and divide the constant 1718·9 by the radius to find the tangential angle by which to set out each chain's length.

To find Radius when this is not given, but the two points A *and* k *on the ground still fixed.*—Here again measure the angle X T *k*, Fig. 88, half of which is equal to the angle A O T = the angle of deflection T A *k*; from the tables of natural lines and tangents take out the natural tangent of half the angle X T *k*, and measure the distance of tangent T A or T *k*; then, as nat. tangent of half the angle X T *k* is to 1, so is tangent T A to the radius required.

Ex: Let the angle X T *k* = 38° 30′, half of which = 19° 15′,
and let T A or T *k* = 20·953·
nat. tangent 19° 15 = ·34921;

then,
·34921 : 1 : : 20·953 : 60 = radius required.

That is to say, divide the measured length T A or T *k* by the tabular tangent of half the angle X T *k*, and the quotient will be equal to the radius sought;

and,

$$\frac{1718{\cdot}9}{\text{radius}} = \text{tangential angle for setting out,}$$

and,

$$\frac{\text{half the angle X T } k}{\text{tangential angle}} = \text{length of curve as before.}$$

Setting out with a 100 *Feet Chain.*—It is very usual in England to range a centre line with the "Gunter's chain" of 66 feet, but it is not, however, always the case. In the colonies and elsewhere it is very common to use a chain 100 feet long, and it is by no means a bad plan, as it expedites the work, stumps being then driven at a distance of 100 feet apart instead of 66. In using the 100 feet chain we may use any of the methods above described, but the calculations will slightly differ.

For instance, in setting out with the instrument and by tangential angles, instead of the *constant* 1718·9, we must make use of the *constant* 171872·3, the radius being given in or reduced to *feet*, instead of being given in chains. By dividing this last constant by the radius in feet, we shall obtain the *tangential angle for the chords of* 100 *feet each* by which we set out.

Ex: Let the exterior angle X T *k*, Fig. 5, = 38° 30′, half of which = one of the angles at the centre, or = the angle of deflection, and let the radius = 3960 feet.

Nat. tangent 19° 15′ = ·3492;
·3492 × 3960 = 1382·8 = tangent T A or T *k*.
3960)171872·3(43′·4 = tangential angle for 1 chord = 100 feet.
19° 15′ = 1155′,
and,
43·4)1155(26·61 = length of curve, in chains of 100 feet each.
Nat. secant 19° 15′ = 1·0592 × 3960 = 4194·43
4194·43 − 3960 = 234·43

It is scarcely needful to add that the 100 feet chain may be used equally well in setting out by the method previously explained, that is, by tangents and secants, &c.

Of the two methods it may be observed, that under ordinary circumstances the work may be equally well done by either method, and with equal despatch. In hilly, broken country the second method has perhaps the advantage slightly, but for curves of short radii the first method is the most accurate; indeed, with a fair average amount of care, it is mathematically correct; in either case some calculations on the ground are necessary, more so by the first method than by the second, and to shorten these tablets will be found at the end of the volume for arcs of 10 Gunter's chains, corresponding to all the radii usually laid down; by the help of these the reader may get through a very fair amount of work in the course of a day.

It will not be irrelevant to add that although in England we almost always stump out at every chain's length, it is different in some of the wild lands in which the engineer pursues his avocations. Here very often five and ten chains apart is sufficient at first, particularly in feeling his way through a rough country with perhaps a very indifferent plan.

Reverse Curves.—In setting out reverse curves the case is so very simple usually that nothing need be added after the foregoing explanations. Reference to Fig. 87 will alone point out that to set out the reverse curve, K *g*, we have only to set out from *right to left*, instead of *left to right*, as in setting out from A to K, varying the offsets or the angles, as we may be setting out by the one or the other of the methods above explained, according to the radius of the curve. *But*, let it never be omitted in setting out *reverse curves*, to leave about two chains of straight line between the curves.

Setting out a Curve where the Vision is Obstructed.—This is by no means an unusual case; we may have some 20 or 30 chains of curve to range through a wood, where we can neither produce our tangents, nor measure the angle of deflection, T A K, Fig. 89. Suppose the preceding curve ranged as far as C, and the straight tangent as far as A.

Take the radius from the plans, and by either of the methods explained above range as far as K, and if the curve be of any length, and there is reason to doubt the accuracy of the plan, then for expedition we may range out five chains at a time. On reaching K, let the error be equal to K K^1, or KK^2.

In ranging from A to K, we have set out a curve subtending a certain angle at the centre, half of which is equal to the angle of deflection, T A K.

To find the Angle at the Centre, and the Angle of Deflection, from the Radius and the Length of Curve.—Divide the length of the curve by the radius. From the Table giving the length of circular arcs, take out the number nearest to the quotient we have obtained, and on the same horizontal line we shall have the *degrees*; to find the minutes, subtract the tabular number just taken out from the quotient, and look for the remainder amongst the length of arcs corresponding to minutes, when we shall obtain the minutes, which, with the degrees, will give the angle sought, half of which will give the angle of deflection, T A K, which we have just set out.

But the curve sought to be set out would have given the angle T A K^1, or T A K^2, accordingly as the error may be; and all three of these angles are subtended by the arcs, K T, K^1 T, K^2 T.

Of these arcs we must find the radius. If the curve A K be tolerably flat, and not a very considerable portion of the circumference, we may take the curve A K as = radius; if otherwise we must find the actual length of the *chord* A K.

To obtain this, observe that as the sine of O A K is to radius, so is the sine of A O K to A K required.

If we can find the angle K^1 A K, or the angle K^2 A K, we shall only have to add the one to, or subtract the other from, the angle T A K, to obtain the angle of deflection required.

Now the length of arc, K K^1, or K K^2, divided by radius, will give a number which we have only to treat as we have a similar one just above, to obtain the angle required, and which, added to, or subtracted from the angle of deflection, T A K, to obtain the correct angle of deflection, T A K^1, or T A K^2.

If we are going to set out by instrument, then treat this new angle of deflection as explained above, the first step being to divide the angle of deflection by the radius to obtain the tangential angle.

If we are going to set out by tangents, secants, &c., with the tables of offsets, we must first find the radius. To do this, divide the angle of deflection by the length of the curve which gives the tangential angle, then divide 1718·9 by the tangential angle, which will give the radius of the curve to be set out.

But if we are using the 100 feet chain, then the calculations will be similar, but *the constant will be as already shown*, 171872·3, or 1718·9 divided by the radius in feet, will give the angle for 1 foot.

Intersection of Tangents Inaccessible.—In Fig. 90, on the tangents A T, K T, let the distances *b* T and K T be inaccessible by reason of their extending into masses of buildings, or into an impenetrable wood.

To obtain the angle X T *c*, measure the angle A *b c*, and the angle K *c b*, and subtract each of them from 180°; the two remainders will together be equal to the angle X T *c* required, the tangent of half of which gives the distance, T A, or T K, but since *b* T and T *c* are both inaccessible, we must ascertain their lengths before we can measure off *b a*, or *c k*. Carefully measure *b c*, then

As sine *b* T *c* : *b c* : : Sine T *b c* : T *c*;
Also,
As sine, *b* T *c* : *b c* : : Sine, T *c b* : T *b*.

Observe that the sine of the angle, *b* T *c*, is equal to the sine of X T *c*, the angle, *b* T *c*, being an obtuse angle, or greater than 90°.*

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* At page 128, and at Figures 81, 82, and 83 we have entered into some explanations of the trigonometrical signs. Referring to the same figures, we have here to add, that as the *complement* of an arc or angle is that which such arc or angle wants of 90°, so is the *supplement* of an arc or angle that which such arc or angle wants of 180°; thus D′ B′ is the *supplement* of the arc B D′, and the angle D′ O B′ is *supplement* of the angle B O D′; and D′ I′ the sine of the

Given in words the rule will be—*As the sine of the angle* bTc *is to the side* bc, *so is the sine of the angle* Tcb *to the side* Tb.

Having found the lengths of Tb and Tc, it only remains to subtract each of them from the length of the tangents, TA and TK to obtain the distance bA and cK, taking care, however, to subtract Tb from TA, and Tc from TK; as although TA and TK are exactly equal, Tb and Tc may not be equal to each other.

Example.—In Fig. 90, let AK be a curve of 40 chains radius; let the tangents aAb, Lkc be set out from the plan as usual.

Measure the angle Abc, and the angle Lcb, and the length bc;

let the angle $Abc = 158^\circ\ 20'$,
and „ $Kcb = 153^\circ\ 12'$.
and $bc = 14{\cdot}10$.

Then $180^\circ - 158^\circ\ 20' = 21^\circ\ 40' =$ the angle Tbc,
and $180^\circ - 153^\circ\ 12' = 26^\circ\ 48' =$ „ Tcb.
$Tbc + Tcb = 21^\circ\ 40' + 26^\circ\ 48 = 48^\circ\ 28' =$ the exterior angle XTc;

$$\frac{48^\circ\ 28'}{2} = 24^\circ\ 14', \text{ and tangent } 24^\circ\ 14' = {\cdot}45012.$$

Then $\cdot 45012 \times 40 = 18{\cdot}005 = TA$ or TK.
$180^\circ - XTc = bTc$, or $180^\circ - 48^\circ\ 28' = 131^\circ\ 32' = bTc$.

$$\text{Sine } 131^\circ\ 32' = \text{sine } 48^\circ\ 28' = {\cdot}74857,$$

and,

$$\text{Sine } 21^\circ\ 40 = {\cdot}3692, \text{ and sine } 26^\circ\ 48 = {\cdot}45087,$$

then,

$$74857 : 1410 :: 3692 : Tc, \text{ or } 7{\cdot}088;$$

and,

$$Tk - Tc = ck, \text{ or } 18{\cdot}005 - 7{\cdot}088 = 10{\cdot}917;$$

then measure off 10·917 from c to k;

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supplement D′OB′ is also the sine of the angle BOD′, and the cosine I′O *supplementary* angle D′OB′ is also the cosine of the angle BOD′. Therefore in looking in the tables of Sines and Tangents, &c., for the sines and cosines of angles greater than 90°, subtract such angle from 180°, and look for the sine and cosine of the remaining angle. Thus the sine or cosine of 120° is merely the sine or cosine of the *supplementary* angle equal to 60°. The correct understanding of this, and of what has been said already upon the subject, is all that will be required for our purposes at present.

and

$$\cdot 74857 : 1410 :: \cdot 45087 : 8\cdot 494 = \mathrm{T}\,b,$$

and

$$18\cdot 005 - 8\cdot 494 = b\,\mathrm{A} = 9\cdot 511$$

then measure off 9·511 from *b* to A.

Fig. 90 is drawn to a scale of 8 chains to 1 inch.

Another case of inaccessible distance or obstruction is shown at Fig. 91, *where after ranging from a* to A *the curve runs on to* D *and* L, *after passing for some distance through water.* —This is by no means an unusual case, where a line runs along the bank of an estuary or along the sea-shore. From the plan measure carefully the distance A D, and make A D some round number, as in the figure where it is made equal to 21 chains, the *radius being* 30 *chains.*

Divide the length of the arc by the radius, and by means of the quotient, from the table of the lengths of arcs, find the angle subtended by the arc A D.

21) 30 (·7000.

Length of arc ·69813 corresponds to 40°
·0020 „ „ 0° 7′ sufficiently near.
·70013

Then the angle subtended by the arc = 40° 7′;

$$\frac{40^\circ\ 7'}{2} = 20^\circ\ 3'\ 30'' = \text{angle of deflection at D};$$

$$90^\circ - 20^\circ\ 3'\ 30'' = 69^\circ\ 56'\ 30'' = \text{D A O or O A D}.$$

Now take out the tabular *versine* of the angle subtended by the arc A D = 40° 7′, and also the sine of the same angle.

$$\text{Nat. Versine } 40^\circ\ 7' = \cdot 23526 \times 30 = 7\cdot 058 = \text{A V};$$
$$\text{Nat. Sine } 40^\circ\ 7' = \cdot 64434 \times 30 = 19\cdot 330 = \text{V D}.$$

Supposing that as yet we have been setting out by tangents, &c. and offsets, then the tangent A *t* is already fixed on the ground; with the box or pocket sextant set out A V at right angles to A *t*, and make A V = 7·058; and at V set out V D at right angles to A V, and make V D = 19·330.

Now at A make the angle D A O = 69° 56′ 30″, when the visual line will intersect the point D measured off from V.

To carry on the course of the curve from D to L. The three angles of a triangle being equal to 180°, and O V D being one of them, observe that 90° − 40° 7′ = 49° 53′ = the angle V D O; by means of this angle V D O, and then at D, set off T^1 D at right angles to D O; the curve may then be continued on to L by tangents, &c. and offsets as before.

If setting out by theodolite, then at A fix A *t*, by the method already shown, reverse the telescope and make the angle T A D = 20° 3′ 30″, and also make A V at right angles to A *t*, and from V, set out V D at right angles to A V; if D is sighted off from V by an assistant at the same time that the angle T A D is being set off, the visual intersection will be at D, and V D will measure as above 19·33. To carry on the curve from D, plant the theodolite at that point; fix the vernier at 21 times the tangential angle for 1 chain, there being 21 chains from A to D, and move round to zero; then reverse the telescope which fixes D T^1.

A very analogous case of inaccessible distance, but requiring different treatment, occurs in Fig. 92, when the curve A K crosses a river at *b* and *c*. The point *b* we may set out by any of the methods above explained, but not so the point *c*, for we cannot measure from *b* to *c*; neither can we set out the *versine* A V, nor the *sine* V C.

By the means already explained, after measuring A *c* on the plan, we obtain the angle at the centre, half of which is equal to the angle of deflection. Having the angle at the centre, we obtain the length of the tangent, which set off as at A T. Take out the secant and subtract the radius, which gives T *d*. To obtain the angle *d* T A, subtract the angle at the centre from 180°, and divide the remainder by 2, which gives the angle O *d* A, O being the centre and left out in the figure to save space; 90° − the angle O *d* A = the angle T *d* A; and 180° − T D A + *d* A T = A *d* T, which last angle set out at T. Now at *d* make the angle A *d* V = to the angle of deflection, and at A set out the angle of deflection T A *c*, and the intersection at *c* will give the point on the curve which was required.

Having the angle at the centre, we may set out the sine of the angle, as at A C, and the versine at C, at right angles to A C, will give the point *d*.

A perfect understanding of the above very simple, though perhaps very tedious remarks, will enable us to find out another way to extricate ourselves from some of the difficulties which often arise, more particularly in uncultivated districts, where we may be at times, to a certain extent, feeling our way, and where time is precious.

In Fig. 93, let the curves and the tangents be altogether in the midst of a large wood, through which we have reached our way as far as A; required to set out the curve A L, with a radius of 20 chains; the direction of the tangent A T, however short it may be, is fixed by our previous work. With the theodolite at A, and the telescope on A T, pick out some line A D, on which there is a clearing, or where it is most easy to make one. Measure the angle T A D, and subtract it from 90°, which leaves the angle D A O = A D O^1; twice either of these subtracted from 180°, gives the angle at the centre subtended by the chord or arc A D.

Thus, let T A D = 12° 15′; 90° − 12° 15′ = 77° 45′ = D A O or A D O^1;

180° − (77° 45′ × 2) = 24° 30′ = the angle at the centre.

Observe that *twice the sine of half the angle at the centre is equal to the chord of that angle;* the sine of 12° 15′ = ·21218;

·21218 × 2 × 20 (radius) = 8·487.

Then 8·487, set out in the direction A D, will be a point on the curve.

To find the chainage of this on the arc itself, take out from the Table, giving the lengths of circular arcs to radius 1, the tabular numbers to 24° 30′, and multiply them added together by the radius, which will give the length of the arc, thus,

Tab. No. 24° = ·4188790
„ 0—30′ = 87266
·4276056 × 20 = 8·552 = length of arc,

which added to the chainage at A, will give that at D.

Removing the instrument, with the verniers clamped to the point D, bringing the instrument to bear bodily on A, and there clamp it; then setting the verniers to 360° and 180°, will give the direction of the tangent $T^2 T^3$, when we may repeat the operation for a longer arc, as D L, or a shorter one, as D *d*, as circumstances may make convenient.

CHAPTER XI.

The Variation of the Magnetic Needle.—Rules to find the Variation.

To find the variation* of the needle. Having found a convenient spot, set up a prismatic compass, when the sun's lower edge or limb is a semidiameter above the horizon,† take the bearing of its centre from the north or south, whichever is nearer; the bearing subtracted from 90° will be the sun's *magnetic* amplitude,‡ from the east or west by the needle. Now to find the sun's *true* amplitude for that day.

As the cosine of the latitude§ is to radius, so is the sine of the sun's declination‖ at setting or rising to the sine of his amplitude from the west or east, which will be north or south, as the sun's declination is north or south, and the distance in degrees and minutes between the true east or west, and the magnetic, is the variation of the needle. To prevent mistakes, sketch the horizontal circle to represent the visible horizon, and on it the several data or angles, when it will easily be seen how the variation is found, whether by addition or subtraction, and on which side of the north it lies.

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* That is the variation of the magnetic north, as shown by the needle, from the true north; this variation is different at different parts of the world; and there is also a diurnal variation.

† At this time, although apparently elevated, on account of the refraction of the atmosphere, as explained in the article "Sextant," the centre is in fact about on the horizon.

‡ The *magnetic* amplitude is an arc of the horizon, contained between the sun or star at its rising or setting, and the magnetical east or west point of the horizon, indicated by the magnetical compass, or the azimuth compass;—or it is the difference of the rising or setting of the sun, from the east or west points of the compass. The *true* amplitude is an arc of the horizon, intercepted between the *true* east or west points, and the centre of the sun or a star at its rising or setting; the amplitude is therefore of two kinds, eastern or western; these are also northern or southern, accordingly as they fall in the north or south quarters of the horizon. The complement of the amplitude, or what it wants of 90°, or the angular distance of the point of rising or setting from the north or south point, is called the AZIMUTH. Azimuths, called also vertical circles, are great circles of the sphere intersecting each other in the zenith and nadir, and cutting the horizon at right angles.

§ The latitude of a place on the terrestrial globe is its angular distance from the equator. It is measured on the meridian, being that part of it which is intercepted between the zenith of the place and the equator. If the place is situated north of the equinoctial line, it is said to have north latitude; if on the other side it is said to have south latitude.

‖ The necessary tables for declination and corrections will be found in the *Nautical Almanac.*

Suppose the variation sought at sunset; draw by hand the circle N W S E, Fig. 94, to represent the horizon; the centre C will be the observer's station; through this draw N S and W E; then on the north or south of W, accordingly as the sun sets north or south of the true west, draw C ⊙, representing the position of the sun at its setting; and another line C *w*, for the magnetic west, either north or south of ⊙, as observed to be; by means of these it will be easily seen whether the magnetic amplitude and true amplitude are to be added or subtracted, to give the variation. In our diagram *w* ⊙, is the magnetic amplitude, and W ⊙ the true amplitude; the angle W ⊙ must therefore be subtracted from *w* ⊙, to give W *w*, the distance of one from the other; and *n*, the magnetic north, 90 degrees from *w*, must be westward of N, the true north. Observe also, that in the above rule the fourth proportional found is the *sine* of the amplitude, and therefore opposite to this sine in the tables we shall find the angle or amplitude; and this will be of the same name as the declination; that is north, when the declination is north, and south, if the declination is south.

A second method, more simple, though it takes more time, is to observe the azimuth of the sun at equal altitudes on different sides of the meridian, which will be at equal hours before and after noon; the mean reading will give the true meridian.

An approximation may be determined by setting up a rod vertically on a level piece of ground, having first, with the length of the rod as radius, and from the exact spot where it is to be set up as centre, described an arc of a circle, and noting where this arc is intersected by the shadow of the rod a few hours before noon, and then again at the same time in the afternoon; a line through the middle point on the arc between the intersections, and the foot of the rod will be the meridian.

Whichever of these two ways is adopted, the difference between the meridian thus found, and the magnetic, will be the variation, which should be shown on all maps of whatever description.

When the meridian has been thus determined, it should be permanently staked out, unless fixed objects can be found to answer the same purpose, and this more particularly on extensive surveys.

CHAPTER XII.

SURVEYING BY OBSERVATION.

A few observations on the great "practical" value and extreme simplicity of the elements of plane trigonometry.—Solution of right angled and oblique angled triangles, with examples showing application to numerous practical purposes, with the use of the tables.—General observations.—Maxims.—Base of operations.—Measurement and position of Base.—Primary and Secondary Bases.—Practical examples.

THE title of "Surveying by Observation" has been adopted for this chapter, in preference to that of "Trigonometrical Surveying," as being more suitable to such engineering field-work surveys, as require some few stations to be fixed by observation and calculation. The expression of trigonometrical surveying appears more applicable to operations connected with geodæsy, for the formation of a territorial or county map, extensive military plans, and long lines of coast, where details become secondary matters to those more important of position and distance as regards towns, villages, hamlets, hills, mountains and valleys, headlands and bays, and the general directions of roads and rivers. Besides, also, this latter expression would convey an idea of "matters mathematical," which to many readers would be sufficient to make them throw down our poor book, in which we have striven with no little pains to divest every page of all appearance of scientific attempts, and to give such modest information as we have to offer in such manner that any one possessing an average knowledge of arithmetic may understand and practise it. Such information, and indeed much more, is now very generally possessed by the best surveyors connected with engineering works, and it is more or less indispensable for conducting the surveys of river, estuaries, harbours, marshy districts, lands to be recovered, extensive gathering grounds for waterworks; it is often also very applicable where a long and tedious process of many miles of chain triangulation is, we must say, *submitted* *to*, in preference to taking, from a well-measured base, perhaps a dozen angles, and calculating a few sides of triangles.

But it will be observed by many persons "it is not practical;" those who have put it in practice know better; "it is difficult to learn;" in one week the whole may be mastered. "It is troublesome to remember;" no more so than remembering how to measure the brickwork in a bridge, or the excavation of a cutting, when once it has been practically applied, and this remark may be made of every operation whatever connected with engineering field-work. Perhaps, however, no expression in the language has been more thoroughly abused than that of "practical man," which may, or should be very often taken as synonymous with "quack."

We very well remember an illustration of this which occurred to our own knowledge some years since; an "eminently practical man" of the name of Mr. A——, who was in charge of some railway works, had to set out a tunnel on a reverse curve; after keeping the contractor with his miners and bricklayers waiting for about three weeks or a month, he abandoned the task, and sent for his friend Mr. B——, the engineer in charge of the next length, to extricate him from his difficulties, and the tunnel was set out; Mr. B——'s assistant, who was another practical man, afterwards related that the men on the works inquired where that "stranger's peculiar instrument" was, when he replied, "in his head, I suppose—why he has taught me so plainly that I think I could do it myself,"—and so he could. But we have another very far more remarkable instance at this present time of an educated man becoming practical, in the person of one of our professional men, and whom we would name but that "comparisons are odious;" certainly a few years ago he *was* not a practical man, but he brought much knowledge of a high order to bear upon the subject he was engaged on, and, besides very soon mastering the practice, he is now one of our most eminent iron bridge engineers.

We should certainly very much regret if any reader should suppose from this that we deprecate that knowledge which is practical, and which at starting in life very great sacrifices must be made to obtain; but we have been plain in the above remarks because in our experience we have often seen a good method abandoned for a bad one in mere deference to the cabalistic "practical;" the other extreme error or shoal is equally to be avoided, which is to imagine that by mathematical formulæ *alone* practical results can be carried out; here again it may be truly observed that extremes meet, and that they must always be avoided. Our profession, however, is based upon mathematical knowledge, however obtained, and it is ridiculous to suppose that the elementary truths of the science are not always useful and applicable; every year the study of these truths is becoming more common, the really practical man, who is induced to apply himself to such study, eagerly seeks for more information on the subject, and if in the course of early professional practice, the reader is led to imagine that these elementary truths are less valuable than he at first supposed them to be, we would advise him to delay judgment, and to believe, on the contrary, that he himself has not yet acquired sufficient practical knowledge to know how, when, and where to apply them.

In the chapter on "Ranging Curves" we have endeavoured to explain in the most simple manner the nature of the trigonometrical signs as regards an angle not exceeding 90°, and we hope it has been made quite plain and intelligible that the cosine, cotangent, and cosecant of an arc or angle are so called, because whatever angular or arcual measure it may be deficient of 90°, is called the complement of the angle, for the cosine, cotangent, and cosecant of *the angle* are the sine, tangent, and secant of the complement.

Exactly as the term *complement* is applied to that which an arc or angle wants of 90°, or to the difference between any arc or angle and one of 90°, so is the term *supplement* applied to the difference between any arc or angle and 180°; thus in Figures 81, 82, and 83, as D O A is the *complement* of the angle D O B, or as the arc D A is the *complement* of the arc B D, so is the angle D O B′ the *supplement* of the angle B O D, and consequently so is the arc D B′, the supplement of the arc B D. It has already been shown that a right line falling from the one extremity of an arc perpendicularly to the radius which passes through the other extremity of the arc, is the sine of such arc, or the cosine of its complement.

The sines increase in length from 0° up to 90°, so that the sine of a right angle is equal to radius, and a glance at the figures will show that the sine of an obtuse angle or of the arc subtending it is the same as the sine of its supplement; for in the two arcs B D and B′ D′, let B D = B′ D′; then B D + B D′, or B D + B′ D = 180°; therefore B′ D = B D′ is the supplement of B D, or B O D′ = B′ O D is the supplement of B O D; and sine D I = sine D′ I′. Therefore the sine of an obtuse angle is the same as the sine of the *supplement*, and consequently in looking in the tables for the sine of an angle above 90°, we must find the sine of the *supplement*. It also follows that because the sine increases from 0° to 90°, so it decreases from 90° to 180°.

The cosine of an arc or of an angle is also equal to the sine of its *supplement* in *length*, but opposite in *sign;* thus I O = O I′, but lays in an opposite direction; or as the angle increases from 0° to 90° the cosine decreases in length, but from 90° to 180° it increases in length.

From the above definitions it follows that the tangent of an obtuse angle is the same or equal to the tangent of the supplement; thus the tangent of B′ O D is the same as the tangent of B O D = B F. As the arc or angle extends from 0° to 90° the tangent is positive, increasing from 0 to infinity, but from 90° to 180° the tangent is negative, decreasing from infinity to 0. To find in the tables the tangent of an obtuse angle we must look for the tangent of the supplement.

As regards the derivation of the numerous trigonometrical formulæ, which are all based upon geometrical properties, we shall not proceed beyond the following demonstrations, but refer such investigations to mathematical works on trigonometry, and proceed as speedily as possible to the practical application of the elements of the science to engineering field-work.

Given the sine of any arc or angle, to find the cosine, the tangent, the cotangent, and the secant.—In the Figures 81, 82, and 83, given the sine D′ I, to find the cosine O I. In the right-angled triangle D I O the square of O I = the square of O D − the square of D I; therefore I O, the cosine = $\sqrt{OD^2 - ID^2}$. Or from the square of the radius subtract the square of the sine; the square root of the remainder gives the *cosine* required.

To find the tangent; in the similar triangles D O I, F O B, O I is to I D as O B to B F; then multiplying the sine by the radius, and dividing by the cosine gives the *tangent* required. And multiplying the cosine by the radius, and dividing by the sine gives the cotangent required. And $\sqrt{OB^2 + BF^2} = OF$; or extracting the square root of the sum of the squares of the radius and the tangent of any angle gives the secant of such angle.

And the square root of the sum of the squares of the radius and the cotangent gives the cosecant.

From the above it may be deduced that the cotangents of angles are inversely as the tangents of such angles, or that as the tangents increase in length, the cotangents decrease, or the reverse, as may be seen in Figures 81, 82, and 83.

Given the sine and the cosine of an angle, to find the sine and cosine of half the angle.—Let D O B, Fig. 84, be the angle of which D I is the sine, and I O the cosine;

required the sine and the cosine of half the angle D O B = B O L. Draw the chord D B, and bisect it on H, and draw O L through H vertical to D B. The definition of a sine being a line drawn from one extremity of arc perpendicular to the radius passing through the other extremity, it is evident that D H and B H are respectively the sines of the arcs D L, B L, or of the angles D O L, B O L; and O H is the cosine of each of these angles also by definition. Draw the chord B′ D, which will be equal to twice O H.

Now in the right-angled triangles B D I, and B D B′,

$$IB : BD :: BD : BB',$$

therefore,

$$BD^2 = BB' \times IB,$$

and,

$$DB = \sqrt{BB' \times IB};$$

therefore,

$$DH \text{ or } BH = \text{sine of } DOL \text{ or } BOL = \tfrac{1}{2}\sqrt{BB' \times IB}.$$

Also because,

$$BB' : B'D :: B'D : BI',$$

we shall have

$$B'D^2 = BB' \times B'I,$$

and

$$B'D = \sqrt{BB' \times B'I},$$

and

$$OH = \frac{B'D}{2} = \text{cosine of half the angle } DOB.$$

Therefore the sine of half any angle is a mean proportional to half the radius, and the difference between the radius and the cosine of the whole arc; and the cosine of half an arc is also a mean proportional to half the radius, and the sum of the radius and the cosine of the whole arc. Of course by the tables the above is done at once by inspection.

Given the sine and the cosine of an arc; to find the sine and the cosine of twice this arc.—In Fig. 95 let the angle L O B be called A, of which the sine is B H, and the cosine O H. It will be seen that D I and I O are the sine and the cosine of twice the angle A. The right-angled triangles, B H O and B D I, with the angle B common to each, will be similar triangles, and

$$OB : OH :: DB : DI.$$

Therefore,

Radius : Cosine of A :: 2 sine of A, or D B : sine of 2 A, or D I.

By the same triangles we have

$$BO : BH :: BD : BI.$$

Or,

Radius : sine of A :: 2 sine of A : Radius − cosine of 2 A.

Then,

$$RR - R \times \text{cosine of } 2A = 2 \text{ sine of } A \times \text{sine of } A.$$

And therefore,

$$\text{Cos. of } 2A = \frac{RR - 2 \text{ sine } A \times \text{sine of } A.}{R}$$

The tables given in this volume being natural tables for *all* the trigonometrical signs, the reader will not require for practical purposes the use of the above formulæ, which we have only introduced to give an idea of trigonometrical investigations.

CONSTRUCTION AND USE OF TRIGONOMETRICAL TABLES.—As already observed, the numerical expressions, or numbers given in these tables in the columns of sines, cosines, &c., are strictly abstract numbers referring to a radius equal to unity. If in the application of these tables it so happened that our radius were *one* mile, then the numerical expressions for the tangent, sine, secant, &c. of any angle, a side of which = one mile, would stand as in the tables.

Thus required the tangent 10°, *the radius being* 1 *mile.*—In the page, headed 10 degrees,* and in the column of tangents, on the same horizontal line as 0, will be found the number 0·17633; this number then will give the length of the tangent of an angle of 10°, one side of which = R = 1 mile. But let the radius = one mile in chains, or equal to 80 chains, then we shall have to multiply the above trigonometrical expression by the *numerical* value of the radius = 80, and 0·17633 × 80 = 14·1064 chains, and

Mile
1 : 0·17633 of a mile :: 80 chains : 14·1064 chains,

And,

Mile
1 = 80 chains,

And,

0·17633 of a mile = 14·1064 chains.

---

* Supposing Hutton's Tables in use.

Given the angle $A = 15° 20'$, *radius* $= 1000$ *feet, required the length of the sine.*—In the page headed 15 degrees, look for the column of sines; the column on the left headed thus ('), gives the minutes; and in the horizontal column opposite to 20′ will be found the number 0·26443.*

Then $0·26443 \times 1000 = 264·43$ = length of sine required.

Observe that in Hutton's tables the natural sines, tangents, secants, and versines, &c., from 0° to 45° are read down the page, the minutes being in the left hand column; but this is reversed from 45° up to 90°, in which case we read from the bottom upwards, the column of minutes being on the right hand. Reference to the tables will illustrate this explanation, which will appear verbose, and excite a smile in one at all acquainted with the use of the tables; but we must beg to remind him that these tables are also for the use of the beginner, or the practical man who has never been accustomed to their use.

Let us now proceed with further examples.

Angle $A = 65° 35'$, *radius* $= 50$ *chains, required the length of the sine and of the cosine*, and the *tangent.*

The angle being above 45°, we shall have to read from the bottom, and in that signed 65 degrees, refer to column signed thus ′ on the right hand up to 35′; and in the same horizontal line, and in the column signed at bottom *sine* will be found the number 91056.

Then,

$0·91056 \times 50·00 = 45·528$ equal to the sine required.

For the cosine we shall have to read upwards on the left hand page and in column of minutes on the right hand up to 35′, and in the column signed cosine at bottom, on the same horizontal line, as 35′, we find the number 49116.

Then,

$0·49116 \times 50·00 = 24·558$ chs. = cosine required.

For the tangent we shall have to look on the right hand page, and we shall find the tabular number 2·20278;

Then,

$$2·20278 \times 50·00 = 110·139.$$

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* Note that all through the trignometrical tables all the expressions are decimals except when otherwise shown, as in tangent of 50° 20′, where the tabular number is 1·20593, but tangent of 10° = 0·17633.

Angle 120° 10′, *R* = 2000 *yards, required the length of the sine.*

Here 180° − 120° 10′ = 59° − 50′ = Supplement of 120° 10′.

Sine 59° 50′ = ·86456 and 0·86456 × 2000 = 1729·12 yards = sine required.

THE RESOLUTION OF PLANE TRIANGLES.—When explaining the above means of finding the sine and cosine of half an arc or angle by means of the sine and cosine of the whole arc or angle, and also from the sine and cosine of an arc or angle to find the sine and cosine of twice such arc or angle, it must have been made evident that the sines of angles may be considered the measures of angles; and since the sides of triangles subtend the opposite angles, that in all plane triangles the sides are to each other as the sines of the angles.

In any circle, C, Fig. 96, inscribe a triangle A B D, and draw the radii C A, C B, C D; bisect these three lines at H, L, and K, and through these bisections draw the perpendiculars C T, C F, and C G, from the centre C. By the 20th, Third Book of Euclid, the angle B C D at the centre is equal to twice the angle B A D at the circumference; but B C L = ½ B C D; then B C L = B A D; by definition B L is the sine of the angle B C L; then sine B C L = sine B A D. The same may be observed of the other angles, A C D at the centre, and A B D and of A C B and A D B. But by construction B L = ½ B D, and A K or D K = ½ A D, and A H = ½ A B; then ½ A B = sine A D B; ½ B D = sine B A D; and ½ A D = sine A B D.

Therefore,

$$\tfrac{1}{2} AB : \tfrac{1}{2} BD :: \text{sine } D : \text{sine } A;$$

or,

$$AB : BD :: \text{sine } D : \text{sine } A;$$

and therefore, also,

$$BA : AD :: \text{sine } D : \text{sine } B.$$

The above proposition, that *the sides of a plane triangle are to each other as the sines of the opposite angles* is a fundamental one in trigonometry, and on it are based all the former observations that have been made.

Let the angle at D be considered a right angle; then the sine of D = radius, and A + B = 90°, since the three angles of a triangle = 180°. The above proportions are now changed into the following—

$$BD : DA :: \text{cosine } B : \text{sine } B.$$

This will be made plainer by reference to Figs. 81, 82, and 83, where in the triangles F O B,

$$OB : FB :: OI : ID,$$

and in the triangles A O G,

$$OA : AG :: OE : ED.$$

RIGHT-ANGLED TRIANGLES. In the right-angled triangle A B C, Fig. 97, from B as a centre, with radius B C, describe the arc C F; then C A will be the sine, and B A the cosine, and A F the versine of the angle B.

Taking C as the centre, with radius C B, describe the arc B G; then B A will be the sine, C A the cosine, and A G the versine of the angle C.

From B as a centre, with radius B A, describe the arc A D; then A C will be the tangent, and B C the secant of the angle B.

From C as a centre, with radius C A, describe the arc A E; then A B will be the tangent, and C B the secant of the angle C.

This, it will be observed, may be done without altering the values of the three sides of the triangle, but by altering the position of the centre, and assuming one line instead of another as radius.

Ex. Let the radius B C = 1120 *feet, and* B = 26° 34′; *required the lengths of* B A *and of* A C.

The line C A is drawn from the extremity C of the arc, perpendicular to the line B F, which is drawn from the centre B to the other extremity F of the arc; the line C A is the sine of the angle B, and B A intercepted between it and the centre is the cosine.

The tabular cosine of 26° 34′ = 89441, and the sine = 44724; then

$$0{\cdot}89441 \times 1120 = 1001{\cdot}739 = BA;$$

and

$$0{\cdot}44724 \times 1120 = 500{\cdot}9 = AC.$$

The three lengths being given, the reader may now practise, by assuming B A the radius, A C the tangent, and A C the secant, B being still the centre. Or by assuming C as a centre, and C B as radius, B A will be the sine, and A C the cosine, of the angle C = 63° 26′, for 90° − 26° 34′ = 63° 26′. Or again, with centre C, make C A radius, when A B will be the tangent, and C B the secant of the angle C.

And we may reduce the resolution of plane right-angled triangles to the following analogies.

Since the sides are to each other as the sines of the opposite angles, then

Sine A : sine B :: B C : B A.

But A is a right angle, the sine of which = radius, by some termed the *Total Sine.* The tabular sine of B or 26° 34′ = 44724, then we shall have, as the sine of 90° is to sine B, so is B C to B A; or,

1 : ·44724 :: B C : B A, or 1 : 44724 :: 1120 : 500·9 = A C

And

Sine 90° : sine C :: B C : C A,

and

C = 63° 26′, the sine of which = 89441,

then

1 : ·89441 :: 1120 : 1001·7392 = B A.

Therefore, the sine 90° : *hypothenuse* :: *sine of given angle* : *opposite side.*

Or, sine 90° : *hypothenuse* :: *cosine of given angle* : *the adjacent side*, which the reader may very readily work out.

Inversely, *the hypothenuse* : *sine* 90° : : *given side* : *sine of opposite angle*, or,

B C : sine 90° :: B A : sine C,

or,

1120 : 1 :: 1001·739 : 0·89441,

and, by looking in the column of sines for this number, the angle will be found, due attention being paid as to the angle being above or under 45°, both as regards the top or the bottom of the page, and also the right or left hand column of *minutes.*

Another analogy is, that *with one side given, and the adjacent angle, or the opposite angle, which is the complement of it, the radius is to the tangent or cotangent of the given angle as the given side is to the other side.*

Given C = 63° 26′, and C A = 500·9 ; *required* B A.

Tangent 63° 26′ = 1·99986,

then

1 : 1·99986 :: 500·9 : 1001·739 = B A required.

With regard to the line B C, treat it as the secant of the angle B, or the angle C, B A being the radius in the one case, and C A in the other.

At present we have only to add, that if at any time, on the ground or on a map, we assume any line for either a cosine, or sine, or tangent, we shall obtain the tabular value of such sign by dividing the line assumed as the radius, whatever that may be. Thus, assuming the line A C above for the sine of B, we have only to divide its length = 1001·739 by the proper radius 1120, and we shall find the tabular number, which in the column of sines will give us the value of the angle B.

And now, lest the reader should accuse us of departing from our promise, we will leave for the present these mathematical investigations, few and simple as they are, and turn to the subject of the application of the trigonometry of right-angled triangles to practical purposes. The resolution of oblique-angled triangles will be found treated of a few pages further on.

Let A B, Fig. 98, be an inaccessible distance across a river; set out A C perpendicular to A B, and measure the angle A C B; call C A radius, and A B tangent to angle C. From the tables take out the tangent of C, and multiply it by A C; the product will be equal to the distance A B. On reaching B, measure the angle A B C, which is equal to the complement of C.

Example.—Let the distance A C = 2·00 chains, and the angle C = 75° 30′.

Table Tangent, 75° 30′ = 3·86871; then 3·86871 × 2·00 = 7·737 chains.

90° − 75° 30′ = 14° 30′, which should be the measure of the angle B, if C has been accurately taken.

From its extreme simplicity this problem may be made of the greatest practical use in almost every description of survey, as the next two examples will tend to show.

Let A B, Fig. 98, be a river of which it is desired to obtain a section; as before, set out A C perpendicular to A B; let a careful man, or an assistant, with a levelling staff, proceed in a boat to stations 1, 2, 3, 4, B, or to as many more as may be required according to the contour of the bed of the river, taking care to keep exactly on the line A B, and guiding himself in so doing by ranging poles carefully set up at A and D; with a sextant or theodolite at C measure the angles A C 1, A C 2, A C 3, &c., on the levelling staff being properly planted at 1, 2, 3, &c., of which a signal is made from the boat. At the same time the levelling staff is read off by means of a level planted near C, so as to obtain the levels by means of which the section of the bed of the river is to be plotted. With regard to the distances A 1, A 2, A 3, &c., it will be observed that they are tangents to the angle C, assuming A C as radius.

Example.—Let the following be the angles at C, A C being equal to 5·00 chains.

			Chains.
A C 1 = 25° 30′.	Tangent 25° 30′ = 0·47697	=	2·38 = A 1.
A C 2 = 45° 10′.	„ 45° 10′ = 1·00583	=	5·03 = A 2.
A C 3 = 50° 20′.	„ 50° 20′ = 1·20593	=	6·03 = A 3.

Again we may have recourse to this system of measuring inaccessible distances in such a case as in Fig. 99, where the base A B passes through a country intersected by two or three rivers, numerous water-courses, marshes, high vegetation—such as hop grounds, vineyards, where the difficulties in obtaining merely a fair admeasurement of a line as A B would be very considerable. In such a case, we may again set out the secondary base A C, perpendicular to A B, and plant the theodolite at C. A few ranging rods are then sent forward along the line A B, with an assistant, capable of fixing stations in useful places, as at 1, 2, 3, 4, &c., making such memoranda, or leaving such marks, that each may be identified to a certainty shortly afterwards. The angles A C 1, A C 2, A C 3, &c., are then carefully taken, and by means of the table of natural tangents, the lengths A 1, and A 2, A 3, &c., may be immediately calculated. Leave a pole at C, and plant the theodolite at A. On his return from B, let the assistant, still guided by his own judgment in selecting fresh stations, and with the help of a good chainman with him, plant two or three rods along some of the *visual line* B C, as at 8, 9, 10. From A measure the angles 8 A C, 9 A C, 10 A C, as by means of these also we obtain so many points accurately fixed, for the angle at A being equal to 90°, each of the angles 8 A C, 9 A C, 10 A C, being subtracted from 90°, will leave the angles B A 8, B A 9, B A 10, and the angle at B is the complement of the angle C; therefore subtracting C from 90°, leaves the angle B, and

This angle B + B A 8, subtracted from 180° = B 8 A.
„ B + B A 9, „ „ = B 9 A.
„ B + B A 10, „ „ = B 10 A.

We have thus all the means of carefully laying down these three points also, or as many more as it may be desirable to assume; by means of the chain alone, or at any rate the chain and sextant, the remainder of the survey may be easily completed. A C should not be less than a third of A B, in order to avoid the angles being too acute towards B; and if A B is of any considerable length, it will be advisable to have along the total distance two or three secondary bases, as A C. It may, perhaps, be as well just to remind the reader that the point C must be in a position to command sight of the ground forward, and this will, therefore, be a leading point in fixing the site of A C.

This system of getting over inaccessible distances may also be very useful on less important occasions, though of very frequent occurrence,—such as getting through gardens, or ripening wheat fields, the owners of which have generally a strange, but positive dislike to their being chained through. The mode just laid down affords a very easy way of avoiding many such difficulties, and may be practised with much more accurate results than chaining, under such circumstances, can possibly give, and more rapidly.

OBLIQUE-ANGLED TRIANGLES.—In every triangle there are six parts, namely, the three sides and the three angles; of these six parts, any three being given together, except the three angles, the remaining parts may be calculated.

Thus we may have—

1. *Two sides given and one of the opposite angles, to find the third side and the two remaining angles.*

2. *One angle and the two adjacent sides, to find the other two angles and the remaining third side.*

3. *The three sides, to find the three angles.*

The first problem of "*Two sides given, and one of the opposite angles,*" is called the *ambiguous case*, though in practice field operations remove any ambiguity; but otherwise we have nothing else to guide us beyond the two geometrical truths, that in any triangle there cannot be two obtuse angles, or angles greater than 90°, since the three together cannot but make twice 90, and that the greater angle must have the greater side opposite to it. In Fig. 100 this case of ambiguity is explained, where the two triangles, A B C, A B′C, have two sides A C, B C, and the angle A in one, the same as the two sides A C, B′C, and the angle A in the other, provided B C = B′C.

In any triangle A B C, A = 47° 42′; C = 20, and A B opposite to C = 100, to find B C, A C, and B.

$$\text{Sine C, } 20^\circ : \text{sine A, } 47^\circ\,42' :: 100 : 216{\cdot}25 = \text{B C},$$

$$180^\circ - 47^\circ\,42' + 20 = 112^\circ\,18' = \text{B}.$$

$$\text{Sine C} : \text{sine B} :: \text{A B} : \text{A C}.$$

$$\text{But sine } 112^\circ\,18' = \text{sine } 67^\circ\,42'.$$

Then

$$34202 : 92521 :: 100 : 270{\cdot}5 = \text{A C}.^*$$

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* It will be observed that in either right-angled or oblique-angled triangles, we here work by the natural instead of logarithmic tables, although calcula-

Given A B, Fig. 101 = 1000, *angle A* = 32° 23′, *and angle B* = 60° 21′; *required the lengths of A C and B C;* also the lengths of A D and D B intercepted by the perpendicular C D, and also the length of C D.

$$180^\circ-(32^\circ\,23'+60^\circ\,21')=87^\circ\,16'=C.$$

$$\text{Sine } C : \text{sine } B :: AB : BC;$$

And

$$\text{Sine } C : \text{sine } B :: AB : BC.$$

$$\text{Sine } C\ 87^\circ\,16'=98840$$

$$\text{Sine } A\ 32^\circ\,23'=53558$$

$$\text{Sine } B\ 60^\circ\,21'=86906.$$

Then

$$98840 : 53558 :: 1000 : 536{\cdot}2=BC;$$

And

$$98840 : 86906 :: 1000 : 870=AC.$$

The angles at D being right-angles, 90° − 60° 21′ = 29° 39′ = D C B;

And

$$90^\circ-32^\circ\,23'=57^\circ\,37'=ACD.$$

$$CD=BC\times\text{sine } B=465{\cdot}9;$$

And

$$DB=BC\times\cos. B=264{\cdot}88,$$

Or

$$DB=CB\times\text{sine } DCB,$$

Or

$$DB=CD\times\text{tangent } DCB.$$

And

$$AD=AC\times\text{cosine } A=754{\cdot}3,$$

Or

$$AD=AC\times\text{sine } ACD,$$

Or

$$AD=CD\times\text{tangent } ACD.$$

As a general rule, when making any of these calculations, always make a sketch of the figure in the field-book, which will

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tions by the latter are undoubtedly shorter: our motive for doing this is that practical men, if not adverse to the use of logarithms, have only occasional opportunities of applying them, and the want of habit engenders embarrassment. With the natural tables only ordinary arithmetic is employed, and although the necessary calculations are somewhat longer, in quantities where six or seven figures enter, it is the reverse where there are only three or four, and we may say *never* more than five, as in engineering field-work.

be found of considerable service in deciding on the signs to be made use of in calculating the sides.

Given the two sides A B, A C, and the included angle A in the triangle A B C, Fig. 102, *to find the remaining angles and the remaining side B C.*

From C draw C D perpendicular to A B, then the angles at D will be two right angles; therefore $90^\circ - A = ACD$.

Observe now that C D is the sine, and A D the cosine of A, A C being taken for radius; then $AC + \text{cosine } A = AD$, and $BA - AD = DB$.

Now take C as the centre, and C D as radius; then A D will be the tangent of the angle A C D, and B D will be the tangent of the angle B C D.

And

$$AD : BD :: \text{tangent } ACD : \text{tangent } BCD.$$

And

$$BCD + ACD = C,$$

and

$$180 - (C + A) = B.$$

We have now the three angles, and the two sides A B, and A C of the triangle A B C, and we may obtain the length of B C by the rule given in the former proposition, and we shall have the following analogy—

$$\text{Sine } B : \text{sine } A :: AC : CB.$$

Example.—*Let* $A = 39^\circ$, *and* $AB = 16{\cdot}00$, *and* $AC = 8{\cdot}00$ *chains; required the angle* C *and the length of* B C.

The angle A being 39°, and $D = 90^\circ$, then $DCA = 51^\circ$.

$$AC \times \text{cosine } 39^\circ = AD,$$

or

$$8{\cdot}00 \times {\cdot}777 = 6{\cdot}21 = AD.$$

$$BA - DA = BD; \text{ or } 16{\cdot}00 - 6{\cdot}21 = 9{\cdot}79.$$

$$AD : DB :: \text{tangent } ACD : \text{tangent } BCD;$$

or

$$6{\cdot}21 : 9{\cdot}79 :: \text{tangent } 51^\circ = 1{\cdot}235 : 1{\cdot}947 = \text{tangent } 62^\circ\ 49'.$$

Therefore, the angle $BCD = 62^\circ\ 49'$, and $DCA = 51^\circ$; then

$$C = 62^\circ\ 49' + 51^\circ = 113^\circ\ 49',$$

and

$$180^\circ\ 113^\circ - 49' + 39^\circ = 27^\circ\ 11' = B.$$

$$\text{Sine B : sine A :: A C : B C},$$

or

$$\text{Sine } 27^\circ\ 11' : \text{sine } 39^\circ :: 8{\cdot}00 : 11{\cdot}02 = \text{B C},$$

or

$$4568 : 6293 :: 8{\cdot}00 : 11{\cdot}02 = \text{B C}.$$

The above solution of that trigonometrical proposition, "Where two sides are given, and the included angle," is not the most mathematical, but it has been given to show the advantage of acquiring a certain readiness in applying the different signs, and of assuming one line and then another as radius. The same proposition will now be solved in a different manner.

If the sum of any two unequal quantities be given, and their difference, half the sum, plus half the difference, will be equal to the greater quantity; and half the sum, minus half the difference, will be equal to the lesser quantity.

Example.—Let the sum of 2 quantities = 14 and their difference = 2.

$$\frac{14}{2} = 7, \text{ and } \frac{2}{2} = 1;$$

and

$$7 + 1 = 8 = \text{the greater quantity};$$

and

$$7 - 1 = 6 = \text{the lesser quantity}.$$

Again, let the sum of 2 quantities = 28, and their difference = 2.

$$\frac{28}{2} = 14, \text{ and } \frac{12}{2} = 6;$$

then

$$14 + 6 = 20 = \text{the greater quantity};$$

and

$$14 - 6 = 8 = \text{the lesser quantity}.$$

The two following propositions are taken from "Young's Plane Trigonometry."

Let A B C, Fig. 103, be any plane triangle; the sides A C, B C, and the included angle C, are supposed to be given; to find the angles A and B.

From the longer of the two given sides, C A, cut off a part, C D, equal to the shorter C B; draw B D, and perpendicular to it draw C E, which produce to F, and draw E G parallel to A B.

Then, because the right-angled triangles C E B, C E D have the angles at B and D equal, and the side C E common, C E bisects the angle at C, and also the line B D. And because D E = E B, therefore D G = G A; therefore C G = $\frac{1}{2}$ (B C + C A); that is,

C G = half the sum of the given sides;

and since A D is the difference of those sides, therefore

A G = half the difference of the given sides.

Again, since the three angles of every triangle make two right angles, the sum of the angles C B D, C D B must be equal to the sum of the angles A' B of the proposed triangle, so that C D B is half that sum; therefore

C B D = half the sum of the unknown angles;

also

A B D = half their difference,

because the half sum added to the half difference must make up the greater of the two.

These preliminaries being settled, we have, by right-angled triangles

C E = B E × tangent C B D;
E F = B E × tangent A B D,

therefore

C E : E F : : tangent C B D : tangent A B D.

But

C E : E F : : C G : G A,

therefore

2 C G : 2 G A : : tangent C B D : tangent A B D,

and this proportion expressed in words is the following rule.

As the sum of the two given sides is to their difference, so is the tangent of half the sum of the opposite angles to the tangent of half the difference.

When the given parts are the three sides of a plane triangle.

Figs. 104 and 105. When B is acute,

$$AC^2 = BC^2 + BA^2 - 2\,BA \times BD.$$

When B is obtuse,

$$AC^2 = BC^2 + BA^2 + 2\,BA \times BD.$$

But B D = B C × cosine C B D; therefore by substitution, putting a, b, c, for the three sides of the triangle, we have, when B is acute,

$$b^2 = a^2 + c^2 - 2\,a\,c \times \text{cosine B};$$

When B is obtuse,

$$b^2 = a^2 + c^2 + 2\,a\,c \times \text{cosine C B D}.$$

But C B D being the supplement of B, in the latter case, its cosine is—cosine B; hence, whatever be the character of the angle B,

$$b^2 = a^2 + C^2 - 2\,a\,c \times \text{cosine B}.$$

Here one side of a triangle is expressed in terms of the other two and the included angle, and when the two sides are small numbers this formula may be conveniently used, the natural cosine of the included angle being employed. As b is *any* side of the triangle, we of course have similar expressions for a^2 and b^2. Hence, for the cosines of the three angles in terms of the sides, we have

$$\text{Cosine A} = \frac{b^2 + c^2 - a^2}{2\,b\,c}$$

$$\text{Cosine B} = \frac{a^2 + c^2 - b^2}{2\,a\,c}$$

$$\text{Cosine C} = \frac{a^2 + b^2 - c^2}{2\,a\,b}$$

To resume then,

I. *In any triangle, having a side given and two angles, one of which is opposite to the given side.*—As the sine of the angle opposite to the given side is to the sine of the other angle, so is the given side to the side opposite the other angle.

In the triangle A B C, given the side A B, the angle A, and the angle C opposite the side A B; required the side opposite the angle A.

Sine C : sine A : : A B : B C.

A B = 600; A = 23° 45′; C = 17° 15′.

or 29654 : 40274 : : 600 : 814·8 = B C.

180° - (17° 15′ + 23° 45′) = 139° = B

sine 139° = sine 41°,

then sine C : sine B : : A B : A C,

or 29654 : 65606 : : 600 : 1327·4 .

II. *When the three given parts are two sides and the included angle.*—As the sum of the two given sides is to their difference, so is the tangent of half the sum of the opposite angles to the tangent of half their difference.

In the triangle A B C, A B = 345, and B C = 174, and their included angle B = 37° 20′; required the other parts.

$$\begin{array}{rl} AB = 345 & AB - BC = 171\,; \\ BC = 174 & A + B + C = 180^\circ \\ \hline & \quad B - 37^\circ\ 20' \\ AB + BC = 519 & \\ & A + C = 142^\circ - 40' \end{array}$$

519 : 171 : : tangent 71° 40′ = 2·96004 : ·97527 = tangent 44° 17′;

then 71° 20′ being half the sum, and 44° 17′ half the difference of the opposite angles, it follows from what has been said before, that 71° 20′ + 44° 17′ = 115° 37′ = the greater angle,

and, 71° 20′ — 44° 17′ = 27° 3′ = the lesser angle.

The third side may now be found by the former Rule I.

Sine 115° 37′ = sine 64° 23′;

then, sine 64° 23′ : sine 37° 20′ : : 345 : 232 = C A.

III. *When the three sides are given, to find the angles.*—Calling the three sides by the letters a, b, and c, accordingly as they are opposite the angles A, B, and C, we shall find the angles by calculating the cosines, thus:

$$\text{Cos. A} = \frac{b^2 + c^2 + a^2}{2\,b\,c}$$

$$\text{Cos. B} = \frac{a^2 + c^2 - b^2}{2\,a\,c}$$

$$\text{Cos. C} = \frac{a^2 + b^2 - c^2}{2\,a\,b}$$

These formulæ the reader may work out by applying them to any of the foregoing triangles. It will, however, be as well to observe, as regards the last proposition of "*Three sides given to find the angles,*" that whichever angle is sought, the sum of the two sides squared ($b^2 + c^2$), is made up of the squares of *the two sides including that angle;* and that the square of the side, (a^2), subtracted from the sum, is the square of the third side *opposite* the angle sought; and that the divisor, ($2\,b\,c$) is made up of the two sides *including* or containing the angle sought, the one multiplied by the other and taken twice.

The working out of these few rules is far more simple than many of the operations constantly performed, either before or after the construction of works; there is scarcely more than half-a-dozen to remember, and they can therefore be easily remembered; moreover, an abstract of them may be easily enclosed in the field-book or note-book which the engineer or surveyor has always to carry with him, with the Portable Trigonometry.

In the preceding pages of this chapter, and in that on "Setting out Curves," the reader will find all that he requires for engineering field-work generally, and if it be once impressed on the memory in a rational manner, he will never be at a loss in any matter of setting out skew lines or curves, wherever or however they may be, or in any survey, even of the highest order, presupposing of course that he has attended to former advice, and put it in practice.

A few observations must now be made as to the application of triangulation to different cases, for there is considerable difference as to the *manner* of applying the system to large areas of land, as for instance, hilly or flat, wooded or clear; again, to the different natural conditions of a navigable river, or an estuary; and again to the survey of a harbour, or lands to be reclaimed; always supposing of course that ordinary triangulation and traversing are not applicable.

It is to be understood that in this system of surveying as few as possible of the principal lines are to be actually chained, and that at the angular points triangles are fixed by observations made from one or two base lines, the remaining angles and sides being calculated by the rules given in the preceding pages; in many of the above cases it will be readily seen that to a great extent this must be unavoidably so, as for instance, in a large navigable river which cannot be chained across, but of which the deflections should be reciprocally fixed from both banks, also very likely sandbanks, shoals, or other features in the river itself; again, such a tract of land as a marsh, or any other area more or less permanently covered with water, or so intersected by dykes and cuts that an ordinary system of surveying applied alone would be insufficient; again, an extensive range of downs or steppes, without a vestige of cultivation, where the labour of chaining would be thrown away, or where so hilly or broken that good chaining is all but impracticable, and useless unless contour lines have to be surveyed.

Since many, if not all the principal angles have to be taken from this base, its position and measurement become most important objects, and whether such base be primary or secondary, as we shall presently explain, its position should be such not only to obtain in most cases a view from both ends of the most important points, but also such that the triangulation shall be as favourable as possible.

Thus much, which is almost self-evident, being admitted, the following rules may be adopted as maxims, except, however, when conditions are such as to oblige us to set them aside.

When practicable, the triangles should be as nearly as may be equiangular, as the nearer they approach to sixty degrees the more accurately they are likely to be all measured, and the more equally practical errors incident to observations will be probably divided amongst them. Generally, however, it would be very inconvenient thus to make the triangles equilateral, and practically the maxim means, that the angles should be neither very acute nor very obtuse; for instance as little as may be above 120°, and under 30° for the main triangles; this, however, should not prevent an observation being taken to a point so situated, but it should be carefully checked from other stations.

The greater the number of angles taken the greater the liability to error, and *vice versâ*, therefore the triangles should be made to cover as large spaces as possible, which should be done also in order to work from large triangles to small ones, and not to make a main triangle, or certain points of an area which should be included in one, dependent on observations taken from the stations of a secondary triangle. This number of observations taken from the base must, however, be regulated by the number of important points to be fixed.

It is not absolutely necessary to confine the stations at the angles of the main triangles within the boundaries of the survey, if favourable stations for taking observations are to be found by doing otherwise. But the stations should not be so far distant from each other as to make it inconvenient for taking observations from them on to other points; on the contrary, they should be so fixed as to make it convenient to take angles from them on to the other stations on the main triangles, and also on to points which are to become stations on the secondary triangles, or a great part of their value will be sacrificed; these stations on the secondary triangles should, therefore, be carefully thought of and considered when determining the stations of the main triangles; it is very desirable, when possible, that when the angles of the main triangles are taken, observations should at the same time be made on to those points which are to become the stations of the secondary triangles.

Every station should be so fixed as to obtain the greatest view possible of the surrounding country; so that the theodolite may be planted over it, and in such a situation that angles may be conveniently observed therefrom.

We have already alluded to the terms primary and secondary base. This secondary base may or may not be dependent entirely on the observations made from the primary one, and it may or may not be actually chained; we have alluded also to the measurement of the base, and to its position relative to the remainder of the survey.

And first, as to the measurement; that which at first sight might appear to be one of the most simple operations on a survey is often rendered by natural circumstances one of the most troublesome, and under any conditions, more particularly in a survey of this character, where everything depends on it, if the instrumental observations are all correctly made, the measurement of the base must be most accurately taken; not only any error in this will be carried *pro tanto* throughout the survey, but it may become multiplied over and over again.

In geodætical operations great pains have been taken, and the most searching ingenuity has been resorted to in order to obtain the accurate length of a base of operations, and metal and deal rods laid end to end, adjusted with the utmost nicety in troughs set horizontally, have been used for this purpose, and the greatest attention given to allow for expansion and contraction due to temperature. For the purposes of the engineer these nice minutiæ of extreme accuracy are not required; it will be sufficient to employ a good ordinary chain, of which the length has been carefully tested by a standard chain, or by a standard measure set off precisely with ten-feet rods on some level piece of ground, or better, along the coping of a wall; we should, however, for this purpose decidedly prefer a chain that had been well used and stretched, still by all means testing it before commencing operations, and again at the end of each day's work during measurement of the base. The line to be measured as a base should be carefully boned out with the theodolite, by setting up rods or other marks at every five or ten chains, and every precaution taken to set out a truly straight line, with a sufficient number of marks along its length to ensure the chain's being laid exactly along the line each time. In such a case as this all hedges, or other interruptions, must be cut through, and the banks must all be levelled. The inclinations of all undulations of the ground must all be carefully measured, and allowed for on the spot, or by the calculations made after the day's work. Particular care should be taken that there are no kinks in the chain in measuring the base line.

Where circumstances will allow of it, it is very desirable to have on the base from whence observations are taken, a third station, somewhere about the centre of the line, besides the two end stations, as such third station will become a station of verification. In measuring the base, it is well to have a third man to help to lay the chain straight and flat, more particularly if the 100 feet chain is used instead of the ordinary Gunter's chain.

Where a level piece of ground, or if not exactly level, then smooth and even, so as to ensure as much facility as possible for correct chainage, can be found situated in such a manner as to offer good positions for commanding stations, we have of course ready at hand all we can desire; such a situation will besides afford us probably an opportunity of making the length of the base some round number in chains, whereby calculations are shortened, and chances of error are diminished.

The longer the base the greater the number of chains that have to be laid in order to measure it, and the greater therefore the chances of error arising from this source, more particularly over any portion of the base running along rough or broken ground. It is therefore sometimes advisable to lengthen the base *by observation* in the following manner:—

Let A B, Fig. 106, be a portion of a base line which it is desired to produce as towards C; if there does not happen to be any object in that direction in line with A B, send forward a pole to be set up anywhere on the hill, and set it in line by planting the theodolite at D, which is made any convenient number of chains, ten, if possible, from B; and, before removing the instrument, set off D E any convenient angle, B D E, and next set off the angle B D F, equal to 60°. In both cases read both verniers, and repeat both angles. Keep the vernier plate clamped, and move to B, where plant the instrument very correctly over the centre of the station, and set off D B F, also equal to 60°, and E B F, equal to 60°. B F and D F should both measure exactly equal to B D; move the instrument to F, when the angle B F D should measure 60° also. If the nature of the ground is such as to render an angle of 60° at B and D unfavourable, then select any other that may be convenient, but 60° is better. Whilst the instrument is at F, measure the angle C F B, and at the same time set off E B F, which is B F produced, and make it intersect D E, previously laid out. Remove the instrument to E, and measure the angle C E F and F E D, the latter of which $- 180^\circ - E F D + E D F$; measure B E, and add it to

B F. Leave poles at E B F and D, under charge of an assistant, and remove the instrument to C; bring the telescope to bear on B, then let the pole B be made to lean a little on one side, when the line of sight through the telescope should also bear on D, and then, putting the pole at D out of the way, it will also bear on any distant station left on the base beyond A. If this is all correct, then the centre of the instrument is *on* the base produced, and so therefore is the station C. Now measure the angles E C B, and also E C F, the latter being equal to E C B + E C F. We have now ample means of calculating the length of C B. Supposing the ground such at B that we have been obliged to make B F or E F rather short, the work may be tested by making the angle B C G = D B F and H C B = its supplement = E B D, and measuring the angles E H C, and H C E, and B G F; this, however, can be but rarely necessary, as the former operations have been closely tested, there being six triangles, in all of which the three angles of each must be equal to 180°.

Should the nature of the ground be such on the hill B as to preclude the practicability of laying out such lines as E F and D F, we may make the angle B D F more acute, or we may carry out the operation at C, the object being to obtain a base, E F, or H G, by means of which we may calculate the length of B C; therefore this base should not be so small a proportion of the line required as to make some of the angles of the triangles too obtuse, and the others too acute.

To compute B C by means of the above triangles and the base B D, in triangle B D F, the angle B = 62° 30′, D = 54° 30′, F = 63° 0′, base B D = 20·00 chains.

Sine F : sine B : : B D : F D;

then sine 63° : sine 62° 30′ : : 20·00 : F D. Sine 63° = ·89100; sine 62° 30′ = ·88701;

then, ·89100 : 88701 : : 20·00 : 19·607 = F D.

In triangle F D E, the angle E = 39° 40′; F = 63 ; D = 77° 20′, and base F D = 19·607, as above.

Sine E : sine D : : F D : E F;

then sine 39° 40′ : sine 77° 20′ : : 19·607 : E F; sine 39° 40′ = ·63832; sine 77° 20′ = ·97566;

then ·63832 : ·97566 : : 19·607 : 29·969 = E F.

In triangle E C F, the angle C = 19° 10′; F = 52° 20′; E = 108° 30′; E F = 29·969;

Sine C : sine E : : E F : C F;

here the angle E being above 90°, we shall have to take the angle of the supplement to 108° 30′ = 71° 30′, and we shall find

·32832 : ·94832 : : E F : C F : : 29·969 : 86·563 = C F.

In triangle B C F, C = 11° 10′, F = 52° 20′, B = 116·30, C F = 86·563; then

Sine B : sine F : : C F : C B,

Or,

89493 : 79158 : : 86·563 : 76·555

This may be checked by calculating in the first triangle B D F the side B F, instead of F D, for

Sine F : sine D : : B D : B F, and in triangle B C F,

Sine C : sine F : : B F : C B.

Should the length of base now obtained be found equal to an odd number of chains and links, it may easily be reduced to a round number by shortening.

It may occur to the reader that the number of observations to be made renders this method troublesome; to a great extent this will depend on his ability to handle the theodolite; if he is familiar with the use of it, a couple of hours will do all that is required, and that much more correctly than by ordinary means; besides this, the number of inclinations to be measured by observation in such a case will be found quite as troublesome as, if not more so than, the horizontal angles to be taken; to this is to be added the difficulty and uncertainty of chaining a straight line over hilly ground. In truth, the only *practical* objection lays in the use of the trigonometrical tables by the most ordinary arithmetical means, and this is really too futile not to be at once set aside, when it is observed how insignificant it is, the object of such a base being to fix with accuracy throughout an extensive survey a given number of points, which it would be difficult and laborious, if not impracticable, to do by ordinary means and chain triangles. To this we may add, that in marine surveying this method is indispensable, and therefore the sooner practical proficiency in carrying it out is obtained the better. Where a theodolite is not at hand, a little study of the ground, and a small amount of ingenuity, will often enable us to effect our purposes with the box-sextant.

As another instance, let A and B, Fig. 107, be two positions on high ground, from which we can obtain a view of the surrounding district, and let them be far apart or near to each other, within reasonable limits, and within the survey of without it, and let the ground between be more or less difficult for correct chain admeasurement. Select any length on the ground, as C D, from which A and B are visible, if not along the whole length, then from any three points; of course the length C D must be as suitable for chaining as it is possible to find it; measure C D accurately, making it some round number, and in doing so leave a station at E, also at a round number, having first, however, boned out the line with the theodolite. From C measure the angles A C B and B C D; from E measure A E C and A E B; and from D, C D A and A D B; now move the theodolite to A, and measure C A E, E A D, and D A B; and from B measure D B E, E B C, and C B A; from these we may calculate the length of A B, by means of C D, C E, and E D, as primary bases, when A B will be the secondary one.

If we take C D as base, and begin with the triangle B C D, we shall find

Sine B : sine D : : C D : C B;

then, with C B as base in triangle C A B,

Sine A : sine C : : C B : A B.

There must be the same final result if we begin with the triangle C A D instead of B C D.

Again, if we take E D as base, and beginning with the triangle E B D,

Sine B : sine D : : E D : E B,

and in triangle E A B, with E B, last found, as base,

Sine A : sine E : : E B : A B.

The reader may readily accustom himself to these calculations, and the use of the tables by laying down a few triangles with a protractor and scale, and computing the sides, and afterwards practising in the field. The advantages to be obtained on extensive surveys are too manifest to be doubted; on many occasions it is indispensable, and the student cannot too soon become accustomed to the practice.

It will be hardly necessary to observe, that if we could not obtain a favourable line in the position of C D, we might select some other, as E F, F G, or G K, as a primary base, the last of which, G K, would be better than either of the other two, for though shorter than the first, the angles off it would be better proportioned than off either of the other two. This sufficiently instances how in many cases it may be practicable to lay down a short primary base to obtain a long secondary one, which, however, once obtained, becomes the base of the survey. This working from short to long lines may sometimes be repeated three or four times, to obtain the length of the base required, and the operation is not to be confounded with that of laying down small triangles to obtain large ones over the area of survey, which would be an erroneous method, as we have already alluded to in this chapter. As an instance, take Fig. 108, where it is required to ascertain the length of base between two headlands on which are stations O O′; set out and carefully measure the line A B, which must be very correctly done; plant the theodolite at A; measure the angle O A O′, and at the same time set up a station at C, making A C, judging by the eye, somewhat about half as long again as A B; then measure the angle O′ A E, and observe that A B E, being a straight line, is in fact an angle of 180°; then set up D, by making the angle E A D equal to O′ A E. At B measure the angles C B A and A B D, which should be exactly equal if the work is correct; also D B E and E B C, equal to one another, observing that the four angles at C are equal to 360°; from C measure A C B and A C E; from D, A D B and A D E, also equal each to each, and the three angles of each of the four triangles equal to 180, after having measured C E B and B E D; from E measure also A E O, A E O′, and C E O′. From O′, measure E O′ A and A O′ O; also from O, A O E and A O O′. From the triangle A B C, and the base A B, we obtain the length of A C, for the second base. From the triangle A C E, and base A C, we obtain the length of A E, for the third base. From the triangle A O′ E, and base A E, we obtain the length of A O′ for the fourth base; and from triangle O′ A O, and base A O′, we obtain the length of O O′, which was required.

Observe that from triangle A O′ E and base A E, the side O′ E may be calculated, which may be made the base of the triangle O E O, also to calculate the length of O O′, whereby all the former calculations are tested; observe also that A D, D E, E O′, and O′ A, form a four-sided figure of which all the angles together are equal to 360°; similarly in the quadrilateral A E, E O′, O O′, and O A, the sum of the angles is equal to 360°. Numerous other calculations may be made by means of these triangulations, but we will restrain ourselves to those above, as being all that are requisite for our purposes.

It will be unnecessary to observe that the selection of a base line is a most important operation, requiring a careful examination of the ground before it is finally determined on, to see how the triangulation will be laid down, and what advantages may be gained by selecting one station instead of another, which may often be the case by moving a station even 40 or 50 feet one way or another, though permanent objects are very desirable to select as stations when conveniently situated; it is also to be remembered that the object of the angles taken at the base is to fix stations, which are to be the centres of further observations, or for determining correctly the position of lines which may be useful on the survey, that is, in the operations of engineering field-work, which differ practically from those merely geodætic, when it is only sought to fix the stations of triangles independent of chaining. It is therefore necessary to determine how we are going to work on or from the stations determined upon, unless they are merely fixed as *instrumental* stations for angular observations. The examination thus required does not by any means result in lost time, as the engineer thereby acquires a *mental* bird's-eye view of his survey which never fails to be of great service throughout the remainder of his operations. It has already been shown that besides the stations at the ends, there may be other stations on the base, to which observations should be taken when the base is being fixed, so as not to rely merely on measurement only, for portions of the base between intermediate stations, any more than for the whole of the line.

Very little reflection will now point out the advantages to be gained in many localities by pursuing the system of triangulation above-mentioned, and how many points may thus be accurately fixed, which by any ordinary method it would scarcely be possible to effect even with considerable labour. Even when surveying by traverse, which is the best ordinary system, we must depend on the accuracy with which a multitude of angles is taken, which will not merely depend on the observer, but also on the instrument and the diameter of it; we must also rely on the chainage of *every* line. By the above system each triangle is fixed by three angles, two of which are taken off *one* measured line. When the stations on the triangles have been thus fixed, the remainder of the survey may be completed by traversing lines run from station to station, as circumstances may require, and however numerous the sides may be, the amount of practical error will be ascertained; neither will it be carried beyond the terminal stations on the lines of triangulation.

Station Poles.—As it is often necessary to distinguish the triangulation station poles several miles off, and as it should be done readily and clearly, it will follow that they should be conspicuous; straight clean natural spars will serve the purpose very well; they should be from 15 to 20 feet long; the longer ones should be placed of course where the shorter ones would not be visible, or would not so readily catch the eye. To the top of these are fastened flags of white or coloured buntin about 3 feet long, and about 2 feet wide at one end and pointed at the other; or the flag may be red and white, which will be found to attract the attention better, the one colour coming out where the other would not be perceived; dark blue and white flags are also very useful, more particularly on the survey of an estuary or a navigable river, keeping the red and white on one side and the blue and white on the other, the advantage of which is sometimes very great, more particularly in situations where the banks are invisible. The most secure way of fastening the flag to the pole is to lay a stout piece of cord along one end of the flag, and to nail through cord and flag together up to the pole, leaving the cord outermost to lay against the heads of the nails. When the flag and the observer lay with the wind, there is often considerable inconvenience in catching sight of the flag, though we may be often assisted in doing so by means of a pocket-glass, which will always be found very useful on such work. A brush of twigs fastened up to the pole is also very useful. An iron ring may also be fastened up to within a foot of the top of the pole, and above a collar to play freely round the pole; round this collar should be fastened, at equal distances from each other, tin vanes about 9 inches long, painted black on one side and left bright on the other, so that there will always be one bright and one black side presented to the eye of the observer; ingenuity may devise many other plans equally good, but generally, station poles at long distances from the observer should have some such conspicuous mark, for it will often be found very troublesome to distinguish a mere spar a long way off. Station poles should be sunk some four or five feet into the ground; they should be fixed perfectly vertical, by means of a line and plummet carried quite round the poles, and let it be remembered that unless they keep perfectly vertical throughout the survey, they are better out of the way altogether; the filling in should be thoroughly well rammed or punned; this demands much more careful attention than might at first be supposed. It has already been observed that where convenient permanent objects can be selected it should always be done. When station poles require to be removed for the purpose of planting the theodolite over the centre, this spot should be previously marked by driving in four stumps round the pole at a few feet off, so that lines strained across them may show the centre by their intersection. In some situations it will be found necessary to adopt three poles with a wide-spread, triangle fashion, and a fourth fastened above, a plummet fastened from the bottom of which will denote the centre of the station.

The details previously given, in conjunction with the observations made in this chapter, will sufficiently guide the reader as to the application of this system of surveying on particular occasions, and we will now turn our attention to another part of the subject.

There can be no possible doubt that when it comes to be well understood, the student will see how much time may be saved, and greater accuracy ensured, than by common systems which are sufficient on many occasions merely to show the relative positions of a few hedges. Indeed, if a surveyor begins by making a few observations of this kind, any intelligent man who has proved a good chainman, and can read and write, is quite competent to do a great deal of the filling in.

CHAPTER XIII.

Harbour and Marine Surveying.—Preliminary Observations.—Outline of Coast.—Starring out.—Buoys.—Station Pointer.—Stations.—Buoying out Harbour.—Triangulation.—Selection of Base.—Advantages of the System.—Triangulation Station Poles.—Instrumental Observations off the Coast.—Soundings.—The Hand-lead and the Plunger.—Holding Ground.—Anchoring.—Tidal Observations.—Tides of the Irish Seas.—British Channel and North Sea.—Mean Tide Level.—Indian Ocean.—Tide Gauges.

AMONGST the principal requirements of a harbour, the following conditions will have to be considered, and obtained as nearly as circumstances will allow.

The access should be easy for a disabled ship to enter during violent gales, and there should be facilities for working out of harbour with contrary winds.

The harbour should afford shelter and security for vessels which otherwise must be driven on a lee shore, but in harbour they should ride at anchor in safety, and without danger of being driven against each other.

There should be a considerable area of deep water for large ships, and the greater the depth the better, because some accumulation is inevitable in a harbour where the water is quiet.

There should be good holding ground for anchorage, that is, there should be sufficient tenacity in the ground at the bottom of the harbour to bring a ship up which may enter in a gale of wind.

The nature of the shore should be such as to offer beaching ground, to allow of crippled vessels being hauled up thereon and secured.

Breakwaters should be so disposed that as little as possible of injurious effects should result by the depth of water being decreased.

These conditions and general rules—and it must be remembered that there is no rule without an exception—will be met more or less by the natural advantages of the spot selected, by works to be constructed, by prevailing winds, the nature of the bottom, depth of water, tides and currents, &c., which will be the objects of research in the survey, or of future study when it is completed.

The depth of water will have to be ascertained by numerous soundings, all of which will have to be reduced to ordinary low water spring tides, and the depths figured on the plan will indicate the depths below this low water level. The exact situation at which these soundings are taken will have to be ascertained by observations made with angular instruments, in order that such situations may be correctly shown on the plan; and as necessarily these must be very numerous, some very ready and practical means must be found to effect the object.

It will also be necessary to compute the areas enclosed at different depths, that is, how many acres there will be of water at 2, 3, 4, 5, or 6 fathoms, &c., as the case may be. These will be calculated from the plan, so that unless the soundings are correctly entered on it, the computations must be incorrect.

As the nature of the ground all over the bottom of the harbour will have to be ascertained, instrumental observations will have to be made also wherever the investigation is made; for inasmuch as we may in different places find clay, gravel, sand, rock, or boulders, &c., the sites of these must be correctly shown on the plan, also the extent of area over which these deposits are spread; their depth, if merely thinly superficial; whether they be natural or merely accidental deposits; if a rock bottom, whether it be the natural or geological formation, or whether it may not be simply an accumulation of stones accidentally imbedded, The tenacity of the holding ground will have to be investigated—what portions may be good, bad, or indifferent, with the areas occupied by each; what portions may be improved, as for instance, by breaking up, or by removing boulders or lumps of rock, or by other means; where moorings may be required, &c.; in short, the most careful attention will have to be given also to this subject, for its complete investigation will generally be more or less of vital importance to the project.

Wherever works of construction are proposed, more particularly if they will require to be of a large or expensive description, an examination of the materials on the spot, or in the immediate vicinity, will be of the greatest importance; for instance, whether there be shingle and sand for making concrete, or blocks of "béton,"* or rock for the construction of breakwaters, piers, quay-walls, &c. It will be very necessary to examine very closely into the nature and construction of such rock, as to whether it be granite, sand, or limestone; whether the rock be of a laminated or homogeneous description; how it bears the action of waves or batter of the sea; how it will blast or quarry; the average size of the blocks; what ashlar and what rubble may be produced from it—of what hardness and density; the depth of baring before the rock is reached; as close an approximation of the cost of the stone as can be obtained; also what use may be made of materials at hand, what quantity will have to be brought to the works, from where, and by what means. Where the works are to be of a heavy nature, these details will all prove very important elements in the engineer's estimate.

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* "Béton" may be said to be French for concrete, though somewhat differently manufactured; it is made into large blocks, and lowered when hard into the spot where it may be required.

The sets of the tides will have to be investigated, and often the currents; as also the rise of ordinary, as well as of the highest and lowest spring tides, and of neaps.

A few observations as to the tides will be required for those who have never paid any attention to this subject. The periodical rise and fall of the sea known as the "flood" and "ebb" of the tide, are due to the combined attractions of the sun and the moon; the tide rises and falls twice during the return of the moon to the meridian, and on the coast generally takes about as much time rising as falling.

The tides though constant are not equal, but greatest at the time of the new and full moon, and least between these periods, or greatest when the moon is in conjunction or opposition to the sun, and least when in quadrature to it. The tides of greatest rise are called spring tides, and those of least rise are called neap tides. The lowest as well as the highest water occurs at the spring tides, but it does not rise so high, nor fall so low, at the neap tides; and lastly, the greatest spring tides are those which occur at the time of the equinoxes, after the autumnal, and before the vernal. The spring tides do not happen precisely at the new and full moon, nor the neap at the quarters, but a day or two after.

Some idea of the movement of the tidal wave may be formed from the circumstance of the tide being considered to occupy twenty-four hours in travelling from the southernmost point of Ireland to the Thames; this amount of time being occupied in passing round to the north of Scotland, and coming round by the German Ocean to enter the Thames. A portion of the same tidal wave, on presenting itself on the south-west of our coast, diverts off into the Channel, and, proceeding in this course, joins the other tidal wave near Dover.

The regularity which would be the result of the earth being covered with deep sea is disturbed by the great shoals, narrow straits, positions of land, breadth and depth of channels. Further observations will be made on this subject towards the end of the chapter.

The regularity of all the tidal phenomena at different places is very greatly affected by storms and particular winds; so much so that neap tides sometimes rise as high and higher than springs, and this is more particularly the case on any part of the coast where there is less difference in the rise of springs and neaps. A high wind coinciding with "flood," will cause it to rise much beyond the height due to that particular tide, and the reverse of this may occur if the wind blows hard in an opposite direction during the rise of the tide. On the opposite coast of France and England, a strong south wind produces an excess in the rise of the tide on our side, and the reverse on the opposite coast.

In tidal rivers the anomalies are still more remarkable, and the rise of the tide is affected very sensibly by the banks, their contractions and bends, as well as by currents and land-waters.

Altogether these anomalies and irregularities are so considerable, that practical observations and gaugings are indispensable, and require to be carried on with the greatest attention.

The methods of investigating all these matters, and of conducting the surveys, will now become the subjects of the following remarks:—

One of the first things to be done will be to obtain charts of prior dates, if there are any, to compare them together, and make notes of discrepancies between them, and this not so much with regard to differences as to the outline of the coast—such as little projections and indentations, but as regards depths of water and bottom of harbour; for should any such exist, they will have to be carefully examined; and as regards depth, not so much as regards one isolated depth, but as to a continuous line, for this would lead us to infer either silting up the formation of shoals, or the water becoming deepened. This comparison of charts will have to be all the more carefully made accordingly as they may otherwise the better agree; lines should, therefore, be carefully laid down on the charts according to their respective scales, and the measurements should be taken from the scales shown on the charts themselves; on each map, a line or two between prominent objects should be laid down, when a few triangles based upon these will enable us to lay down the line of difference between the two charts. These triangles should be so laid down by means of the sides oearing on distinct points on land, so that they may be accurately traced on the water. These subjects are of much greater importance than they appear at first; but it will afterwards be found in the course of the survey that a good chart is of the greatest value.

The next thing to be done will be to obtain local information, which is generally to be best procured from old resident fishermen,—as to depths of water, surface of bottom, tides, currents, formation of shoals, &c. A great deal of valuable knowledge as to the matter in hand is to be obtained from them.

So much of practical detail on surveying generally has been given in previous chapters; and with regard to bases, and "Surveying by Observation" in the last chapter, that we shall not have to go over that ground again; but inasmuch as we know by experience how strong is the predilection of taking up the *practical* subjects, and neglecting what appears purely theoretical, we must warn the reader that unless he has made himself acquainted with the contents of the last chapter, he will feel at a loss in reading the following remarks.

The outline of the coast should be surveyed from a system of triangulation off a base, measured or computed from another base as already shown; and the position of this base must entirely depend not only on the form of outline, but on the facilities for obtaining a correct measurement, and on the position of objects on the coast, inland and out at sea. We infer, however, that from previous study the reader has made himself master of this portion of the subject; but we must now observe, as regards the triangles thrown up from the base, that the sides of these must be so laid down that as many as possible of the stations of the main triangles, as well as of stations fixed by observation along the sides, should be so placed as to be seen from seaward, so that from thence observations may be made upon them; and to this leading feature particular attention must be given. It is far more on account of these latter observations that a correct triangulation has to be laid down, than on account of every little bight or indentation of the coast line; for a minor error as regards the position of a station as to the coast line itself, would probably become greatly magnified when observed upon from a distance in the offing.

Two of the most troublesome cases likely to arise are where a town surrounds a harbour, or where a coast is bounded more or less by a bold and lofty cliff, the base of which is washed by the waves at every spring tide; in the first the system of triangulation we require is not practicable, and in the second the stations on the top of high cliffs could only be conveniently observed upon at considerable distances at sea.

Under such circumstances, first, as regards the town, we must have recourse to the best traverse we can lay out, that is, with the least possible number of angles and sides, and this quite irrespective of the buildings which may be attended to at some future time, with the exception, however, of those with distinct prominent features, on which careful observations should be made.

Considerable attention should be given to set out such a traverse before the angles are taken, so as to obtain the longest sides possible; to make the lines whole numbers in length, and so that minor triangles may be formed by observations, as may be seen illustrated at Fig. 109, where the side O O′ may be calculated by taking either of the sides O *a*, or *a b*, as the bases of the triangles O *a* O′, or *a b* O′; similarly from the length of *b* O′ found by calculation, may be obtained *b e* and *e* O′. In another instance the line of coast may be so bounded that the traverse will consist entirely of a number of short lines, when we may proceed in a different manner, as in Fig. 110; set out a buoy at B, on the line O O′, with the theodolite, so that from B observations may be taken to all the stations *a*, *b*, *c*, &c.; the length of B O′ may be calculated from the triangle O′ B *g*, with O′ *g* as base, all the angles being measured with the sextant, for sine B : sine *g* : : O′ *g* : B O′; this length being then set out on O O′, the position of the stations *a*, *b*, *c*, &c., may be tested from B by measuring the angles O B *a*, *a* B *b*, *b* B *c*, *c* B *d*, &c., from a boat at B, and laying them off from B on the plan with a *station pointer*, an instrument we shall explain the use of presently; exactly the same operation may be repeated at A, which will in most cases be a sufficient check; we may, however, have another buoy at C, which we may determine the position of by taking the three angles C A B, A B C, A C B, and then take bearings from C on all the stations above-mentioned. It will be observed that if we take D B O′ for another triangle, with B O′ last found as base, we may compute the length of the side *d* B; and from *d* B, in the triangle *d* A B, we may compute the length of A B; and O A + A B + B O′ should by computation be equal to the length which O O′ plots on the plan.

The examples illustrated at Figures 108, 109, and 110, are sufficient to explain and suggest the variety of methods we may adopt under various circumstances; for instance, for Figures 109 and 110 we may calculate O O′ in the manner illustrated at Fig. 76, and explained under the head of traverse.

Buoys.—If any description of buoy is used merely with line and grapnel, it will be drawn under water as the tide rises, and when the depth exceeds the length of line; if sufficient line is allowed to prevent this, the buoy at low water will shift about with wind or current, and so continue to vary from its intended position during the fall and rise of every tide; this is prevented by adopting a particular system; the following is the description of buoy given by Sir Edward Belcher in his "Nautical Surveying" as used by himself; a sketch of it is given at Fig. 111:—

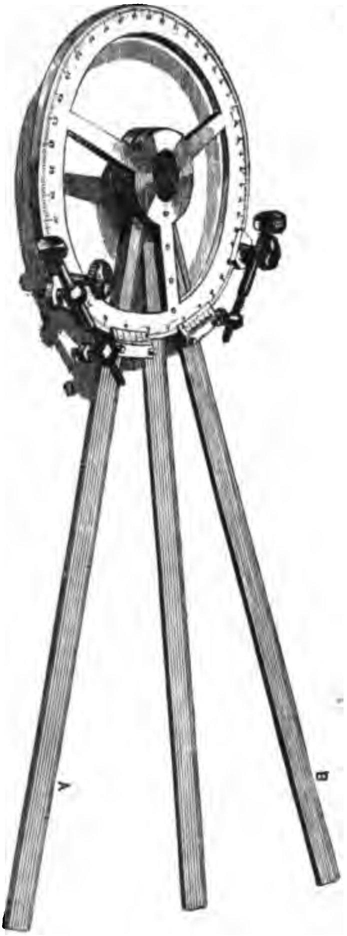
"A cask of 32 gallons is furnished with double staves and of greater width at bung and opposite, as well as double heads. Holes are bored through the bung and opposite staves to admit of the passage of a spar 3½ inches diameter.

"This spar, previously well parcelled and tarred, is driven firmly home; cleated to prevent its working out, and caulked round the cleating. Three feet project on the larger end, and nine on the smaller. The larger end is furnished with irons for a topmast, which is 22 feet in length. Close to the cask at *a* a thimble is secured, through which the mooring cable passes, as well as one similar at *b*, where the cable is clove hitched, parcelled, and its parts seized to the spar to prevent chafe. Ballast equal to 2 cwt. is attached at three fathoms below the cask. They were moored taut in the line of the stream, had latterly full-sized buntin flags, and were found to stand bad weather well, and maintain an erect position in very strong currents and tides."

Such buoys answer our purpose perfectly well, but if they are of a much less size it is quite sufficient, and six or seven feet of topmast is all we require.

Another description of buoy is shown at Fig. 112, with details at Fig. 113, in which the conical buoy B is made of zinc, about 2 feet in diameter at top, and about 18 inches deep; at top and bottom, as well as in the centre, are fixed thin pieces of wood to stiffen the zinc against any accidental blows it may receive; inside and out it is well paid over with boiled tar; through the centre of this is a tube of zinc to receive the mast M, projecting about four feet above the cone and two feet below; it is rigged in the same manner as the last. A buoy may also be constructed of pieces of 3 or 4-inch plank, as shown in elevation and plan at Figs. 114 and 115; *c* and *d* are two square pieces, above and under the star-like arms, the whole being strongly spiked together and paid over with boiled tar; through the centre the mast is passed in a hole to receive it, and fastened in the same manner as the first; it is rigged as the other two; it measures across about 2 feet, or a little more.

Station Pointer.—This instrument is so similar to the protractor, that a glance at the appended figure will explain its construction, A and B being two arms provided with verniers to work along the graduated limb, so that either any one or two angles may be set out at once. It is used in the following manner: if out at sea, *the boat being on a line fixed in position by objects on shore*, an angle is taken with the sextant, between such line and an object on shore or on the water, and shown on the plan; then this angle being set out on the station pointer between the centre bar and the arm A or B, it is only necessary to lay the first along the above line, and make the second A or B coincide with the other object, when the centre of the instrument will be on the plan in the position from which the observation was made in the boat; and it may be either pricked off through the centre of the instrument; or a line may be drawn along the arm A or B, the intersection of which with the other line will also determine the position of the station required. But we may have no line fixed, and require to determine on the plan the position of a point of observation on the water; in this case it will be required to take from the boat the *two* angles between any *three* objects on shore, the positions of which have already been carefully determined on the plan; the two angles being taken, the arms of the station pointer are opened so as to contain them,



and the instrument being laid on the plan so that the *three* arms shall coincide with the three objects, the centre will again be over the station required. In the absence of a station pointer, we may lay down the two angles on a piece of tracing paper, and produce the three lines to a sufficient length; it will be seen that these three lines may be used as a station pointer by making them coincide with the three objects between which the angles have been measured. Even a sheet of paper with the above lines partly cut out may be made to answer the purpose for once or twice.

It will be readily perceived that the station pointer, or that imitation of one which we have just described, may be used for observations on land as well as on the water, and may be made to render very good service in testing the accuracy of any plan; for if from any well-defined position we take any number of angles to well-defined objects, we have only to fix the arms of the station pointer to those angles, when if the plan is correct they will intersect the objects on it, when the centre of the instrument is placed over the point on the plan representing the station of observation on the ground.

The position of the station required may be determined geometrically in the following manner, though the process is much too tedious to be often repeated in practice. Let A, B, and C, Fig. 116, be any three stations on shore or otherwise, of which the positions have been determined; say that from a boat in any position, which we will call D, the angle A D B measures 30°, and B D C 50°; from 180° subtract twice A D B = 60, leaving 120°; from A and B lay off half the remainder = 60, as B A E and A B E; the lines A E and B E will meet at E, where the angle E will be twice the angle A D B, and E will be the centre of the circle A B D; repeat a similar operation at B and C, with regard to the angle B D C = 50, and F will be the centre of the circle B C D; the intersection of the two circles at D will be the position of the boat which was required.

It will be observed that this would be a very lengthy process, if every one of the numerous soundings required had to be thus laid down on the plan; and so indeed would be the use of the station pointer, if two angles had to be measured and laid down for every sounding also. It is for this reason that it has already been pointed out that certain lines along the shore require to be laid down in a systematic manner, for the purpose of facilitating some rapid mode for taking angles, and also for afterwards laying down on the plan the stations from which the observations have been made.

The object, in fact, to be attained is the readiest mode of measuring numerous inaccessible distances and setting these off on the plan; and there is no way more simple than that illustrated at Figures 98 and 99, in which one line is set off perpendicular to another, and therefore containing a right angle; if any number of third sides are set off *visually*, connecting the first two, and either of the other two angles in the triangles thus formed is measured, the third angle is obtained by the most simple calculation, merely subtracting the measured angle from 90°. In Fig. 117 the line G, F...E has been selected as facing the offing, from which the whole or the greater part of it is visible; the length of it has been calculated from the bases A B and A D in the manner above explained, and portions of it have been measured to verify the calculations; let it be required to set off a portion of it, as B C, equal to the round number 2500 feet, this length having been selected as suitable from a former chart, from which it has been judged whereabouts C will fall; set off B *a* at right angles to B C, and make B *a* very accurately equal to any assumed round number as 200 feet; assuming B *a* as radius, and B C as tangent, divide 2500 by 200 = 12·5000. Amongst the tabular tangents look for the nearest number to this, which is 12·5199, the tangent to 85° 26′; the next nearest is 12·4742, being the tangent of 85° 25′; the difference is 0·0457, about one-third of which subtracted from 12·5199, will be the tangent near enough for our purposes, and the angle we require will be 85° 26′, one-third of a minute, which will give 85° 25′ 40″, or as near as we shall be able to read off. If this angle be set off from *a*, as B *a* C, the intersection of *a* C with the line B C will be the station required; it being, of course, understood that whilst C is fixed with the theodolite from *a*, the assistant at C is careful to keep on the line G B C. To fix a buoy at *b*, 500 feet nearer to B; $\frac{2000}{200} = 10{\cdot}0000$; the nearest tabular number is 10·0187, the tangent of 84° 18′; proceeding as before we shall find the angle B *a b* to be (near enough) 84° 17′ 30″. To determine point *c*, 500 feet nearer again to B; $\frac{1500}{200} = 7{\cdot}500$; and proceeding as above we shall find the angle B *a c* = 82° 24′ 20″. In the same manner we may determine the positions of the two remaining stations between *c* and B. Having determined the station C, it may so occur that the line of shore prevents *b* and *c* from being visible from B; then move the theodolite to C, and set out the radius C *d*; and from *d* proceed as we have just done at *a*, for *b* and *c*, with only this difference, that we shall have to operate with 500 instead of 2000, and 1000 instead of 1500. To buoy out the lines into the offing from a boat at *b* and *c*, set out with a sextant carefully adjusted at 90°, as *b*, *e*, *f*, and *c*, *g*, *h*; and similarly with the other lines. Or from B we may set out B *a′*, perpendicular to B C, and from *a′*, *a′* C, perpendicular to B *a′*, and measure off on *a′* C′ the lengths required, and so determine the lines wanted. There will now be no difficulty in perceiving that on the line *b*, *b′*, *e*, *f*, we may take with the sextant as many angles as we may require soundings, B 1 *b*, B 2 *b*, B 3 *b*, &c., the boat being kept carefully on the line buoyed out, either by means of the buoys or the stations on shore. The stations determined by the angles thus measured may be laid down on the plan by means of the station pointer in the manner already explained; for the line *b*, *e*, *f* being set off on the plan in the same manner we have just supposed being done in the offing, it is only necessary to lay off the angle B 1 *b* on the limb of the instrument, and making one bar coincide with *b*, *e*, *f*, move the instrument along the line, until the other bar coincides with the station B, when the centre of the instrument is over the point required. We consider, however, that it may be much more readily done by keeping the note-book in the following manner, and computing the distances *b* 1, *b* 2, *b* 3, *b e*, &c.

NO. OF LINE—STATIONS OBSERVED—LENGTH OF RADIUS = 2000 FEET.

Angles measured.	Complementary Angles.	Tabular Tangent.	Distance in Feet.	
76° 46′	13° 14′	·2351	470	(= *b* 1)
61° 51′	28° 9′	·5351	1070	(= *b* 2)
56° 5′	33° 55′	·6724	1345	(= *b* 3)
49° 2′	40° 58′	·8682	1736	(= *b e*)
42° 32′	47° 28′	1·0900	2180	(= *b* 4)

The first measured angle is taken at 1; the three angles of the triangle B 1 *b* being equal to two right angles, one of which is at *b*, the remaining two angles are equal to the remaining right angle, and if from 90° we subtract the observed angle, the remainder will be the complementary angle 1 B *b*, to which *b* 1 is the tangent, B *b* being radius; at 1 the observed angle is 76° 46′, which, subtracted from 90°, leaves the complementary angle 13° 14′ = 1 B *b*; the tabular tangent of 13° 14′ = ·2351, which multiplied by radius B *b* = 2000 = 470 feet = *b* 1. Proceeding in the same manner with the next angle 61° 51′, observed at 2, we shall find $b\ 2 = 1070$ feet; and $b\ 3 = 1345$ feet; and $b\ c = 1736$, and $b\ 4 = 2180$, &c.; these distances being all plotted by scale from the point *b*, determine the position of the points or stations required. We may proceed in the above manner with any number of lines which are at right angles to another, as L, K, *k*; *l*, *m*, *n*, or any others. To fix the positions of $o'\ o''$, on the line G H produced, we shall have to set off the positions of $o'\ o''$ by means of the station pointer, the angle at H not being a right angle; the same is to be observed of the line H K, the observed angles on which are subtended by the line H I.

With regard to the line M N; to set out this line across the offing parallel to H K, both H M and K N being already set out at right angles to H K, and the distance H M having already been determined by the observation there taken, as H M B; the length of H K is known, or may be ascertained by any of the means we have already explained, and M N, being parallel to it, and between the same parallels, is equal to it, we have therefore only to set out the distance K N, or, which will be the same thing, determine the position of the buoy at N, by observation from H, which may here be done either with the theodolite or the sextant; that is, by taking H M as radius, and M N = H K as tangent of the angle M H N; or dividing H K by H M, which will give the natural tangent of the angle required; this operation we will not repeat, as it has already been just explained. The line M N being buoyed out, we may take any required observations along it on the two stations H M or K N. The point O is fixed by the two angles C O D, D O E, and laid down on the plan by means of the station pointer; we may in the same manner determine P, Q, R, or any other number of irregular stations that may be required, from which any special observations may have been made, as for instance by the diving-bell, or by boring. With a little study the reader will be able to make many more observations on this diagram.

Another method of buoying out a harbour is illustrated at Fig. 118; the stations *a*, *b*, *c*, *d*, *e*, being fixed, buoys may be laid at any convenient spots, as A, B, C, and lines through them and any or all of the stations may be *starred off*, one after another, and the required observations may be made along the lines thus buoyed out, in the manner already explained. In case any of the above stations, as *a*, *b*, *c*, *d*, *e*, are inconveniently placed, as for instance on the top of a cliff, then the positions of others may be determined from A B, and C, lower down the cliff, where they may be secured a few feet up from the beach, as for instance two or three pieces of plank knocked together about four feet square, and painted black or white, as may best show up against a dark or light cliff; the position of these may be checked by observations made upon them from two or three stations on shore.

With regard to the kind of boat required, much will depend on the locality, the extent of observations to be made, and their description as regards the soundings, as whether with the lead, the plunger, or other means; this boat must be stiff, and well manned, so as to be under command, but not crowded; the observer should, if possible, register his observations himself; he will require a man to take the soundings with the lead, as there will be quite enough for the observer to do to take angles, and enter notes; besides which he must keep his hands dry; the person sounding should be trustworthy, though still carefully watched. Sometimes a boat with two good oarsmen and a man at the helm will be sufficient, for great speed is not required; and at others such a boat as a ship's launch may be required; the reader will form a better idea of what he is likely to want when we come to the subject of sounding, and observes the conditions to be fulfilled. When he has obtained sufficient practice, the observer will often be able to make his observations without mooring, and therefore it will be necessary to get accustomed to his work by use, before he takes to it in earnest; at other times it will be necessary to anchor for every observation; this must depend on the kind of work that has to be done; it will be necessary to moor the boat when several angles have to be taken from one station; when only two have to be taken, select stations if possible, so that the two angles shall be compassed by the limb of the sextant; after having taken the second angle, repeat *back*, as this should bring the vernier to zero.

The box-sextant will often be found sufficient, but we confess our predilection for the sextant, the limb being much more legible, particularly after a little use. In using the sextant, grasp it by the handle, firm but not tight, because this will make the hand shake; let the handle be held with the four fingers of the right hand, leaving the thumb to work on the barrel of the adjusting screw of the telescope when this is in use. Place the second and third fingers of the left hand steadily against the limb and its support beyond the tangent screw, the objects to be observed being supposed to be brought already nearly in contact; the finger and thumb are now free to act steadily *together* on the tangent screw, and without any strain. The elbows should be well raised, instead of laying near the sides, so that the body may move freely from the hips, which counteracts the motion of the boat.

During fine calm weather as much boat work as possible should be got over, which the landsman will find no small comfort, merely employing an hour or two in the evening in laying down sufficient of the work to see how it progresses, and how it lays on the plan; a few hours thus passed in the evenings will be well repaid by keeping the mind easy during the progress of the work.

From the above observations we hope it will be obvious, why careful study should be given to placing certain stations and measured or calculated lines in such positions as to be conveniently seen from the offing and off the shore, and so also that there shall be as few very obtuse angles as possible. These lines and stations often require to be determined before any others, and then other lines and stations laid out subservient to obtaining the lengths and positions of the first. We hope also that it has been made evident how very valuable may be the most simple knowledge of the mere elements of trigonometry, and how very easily it may be acquired.

We shall now conclude this portion of our subject, by observing that it is not necessary to take an instrumental observation at every sounding, it being often sufficient to take these at every fourth or fifth sounding; the *intermediate distances* between the observed stations being calculated by some twenty or thirty strokes of the oars; the same number being taken from sounding to sounding, the distances between the observed stations are divided into equal spaces, according to the number of times this has been done; with care, considerable time may be often saved by this means.

With regard to the number of observed stations, much must depend on the profile of the surface at bottom, for where this is irregular or the fall rapid, a greater number of soundings will be required than where we have a regular and gentle fall.

Soundings.—The object of taking soundings is twofold; to obtain the depth of water over the proposed harbour and in the offing, *at ordinary low-water spring tides*, and also indications as to the surface of the bottom when the lead merely is used, and at some depth under it when other means are resorted to.

The Hand-lead.—The instrument most generally used for the purposes of the engineer is a chain such as that made by Messrs. Elliot, which is divided into feet by marks on which the numbers are legibly engraved, each foot consisting of *inch* links, so that the depths may be taken in feet and inches very easily; these chains are made in lengths of 50 and 100 feet; at one end is attached a leaden weight, varying from five to ten pounds, according to circumstances, a mean between the two answering the purpose best generally; it is of a long conical shape, from two to three inches in diameter at bottom, which is hollow and filled with tallow, to which any loose particles at the bottom get attached when the lead strikes it; when the lead falls on rocks or stones, it will be bruised by the blow. If the lead is allowed to drag at the bottom, whilst the chain is held in hand, jerks will be felt as the weight strikes against rocks or boulders, and where these are rough and sharp, the lead will be found torn when drawn up; when these unwelcome occupants on the surface of the bottom are encountered, careful observations of their whereabouts are required that the site all round may be carefully examined to ascertain the extent of such foul ground; the jerks will be found to subside as the lead leaves the rocky surface for one of sand or clay. The height of projection of these rocks above the surface may, of course, be easily ascertained by the chain. Care, however, is required in allowing the lead thus to drag at bottom, or it may get so entangled or wedged between two rocks, that it may be impossible to free it, and the chain gets broken; a repetition of this might become expensive; for the purpose, therefore, of dragging it is better to have one or two spare leads, one of which may be attached to a rope so protected as to prevent its getting chafed against the bottom, the chain with its weight being reserved for taking depths.

It will not fail to be observed, that this is but a very superficial way of obtaining indications as to the nature of the bottom, for a thin surface of mud, sand, or clay, may overlie a substratum of rock, the presence of which would thus remain unknown, and the whole survey, as far as the value of the holding ground, would be a fallacy; and even if the soil at the surface was of any depth, its quality as to power of holding would equally remain unindicated.

The Plunger.—In conjunction with the hand-lead, there is another instrument which is often used to ascertain the nature of the substratum to a depth of from one foot to two feet below the surface; this instrument is called the plunger, and is illustrated at Fig. 119. It consists of an iron bar, about six feet long and about 1½ inch thick, armed in the middle with a weight of from thirty to one hundred pounds; above the eye end may be attached a piece of board to act as a float, and to assist the perpendicular descent of the plunger, which is pointed at one end to enter the ground; the length between the point and the weight is jagged, as shown in the drawing, so as to bring up portions of the ground pierced through by the instrument, which being suddenly let down with such a weight enters the subsoil to some depth—the deeper accordingly as the weight is greater. A heavy weight, however, has its disadvantages, inasmuch as it is clumsy to handle, and to get in and out of the boat; it also requires a larger float; moreover, when the plunger is armed with a very great weight and strikes upon rocks, it sometimes gets bent and even broken. It is therefore well to have two in the boat, armed with two different sized weights; the lighter one being used first, if there is no indication of rock and we wish to test the quality of the ground to a greater depth, we then let down the plunger armed with the heavier weight. This instrument has been in use for very many years; it will not, however, indicate the nature of the bottom beyond a certain depth; neither will it discern between the natural rock and a mere deposit of stones imbedded in the surface; a little patience and a few trials will, however, sometimes assist us in this, but not constantly. The anchor should always be examined when drawn up, the fluke often bearing good evidence.

In very many cases the hand-lead and the plunger together will afford very satisfactory proof as to the nature of the bottom, more particularly where the shore gives no indication of rock, or where the geological formation does not give reason to apprehend anything of the kind; where the water is not of too great depth, the ground may be probed by means of a long pole armed with an iron bar.

Diving.—The next method of ascertaining the nature of the holding-ground is by the reports of a trustworthy diver walking over a portion of the ground. He is provided with an iron bar four or five feet long, attached to a buoy with a two or three-inch rope; he drives this bar into the ground with a mawl, or presses it in, and may be led some distance round the boat; if provided with a rope some twenty or thirty feet long, he may examine the ground some distance round his ladder, and even further *by sight*, which, however, will depend on the state of the water; he can also send up samples of the ground, and to test his accuracy he may be led to and from a spot examined, and back to it again, and his reports may be compared. The bar when hauled up, after being sunk in any adhesive ground, will bring up some of the earth sticking to it; and if it has been driven in to any depth will often require a windlass, and several men to haul it up. Although the diver may sling samples which may then be drawn up, his reports must be taken as he gives them to a very great extent; it is therefore indispensable that he be trustworthy, and also very desirable that he should possess sufficient nautical experience to understand the subject on which he is reporting. The points where these observations are made will, of course, have to be determined by carefully measured angles; or they must be fixed by the intersections of lines on shore, these lines being on stations, buildings, or other objects on the plan. The spots thus examined by the diver should be shown by circles equal in diameter to the distance actually examined; and these circles should be numbered in accordance with the numbers in the note-book.

Boring.—The next method of testing the bottom of a harbour is by boring down through a length of iron-piping, about three inches in diameter, and in lengths of about six feet, which is done at about the time of low-water, from a platform rigged over two boats. Under any circumstances this is a troublesome operation in deep water, where there is any swell or motion; and although it will be much less difficult to sink into clay or sand, it is very different on striking a boulder or rock, where every inch downwards has to be gained by "jumping."* To pierce through a large boulder in deep water will be a rather serious undertaking, or into the natural rock; and the boring-tool will not ascertain whether the work is being carried on a natural bed, or an accidental deposit, any more than the plunger will; but where it is only desirable to ascertain the depth of a stratum into which the auger will cut with average despatch, then boring will generally be the best and readiest means to ascertain the fact as to depth of sand, clay, mud, &c. &c.

Of the above four means, viz., the hand-lead, the plunger, the diver, and the boring-rod, any one, or all conjointly, may be used according to circumstances, and as one of these may point to the necessity of using another; the nature of the bottom of the harbour as holding-ground is a condition about which there must be no doubt, whatever the means, or the number of means, we may have to resort to before we can come to a conclusion satisfactory to ourselves or to others.

Anchoring.—In all the above operations the engineer will be more or less assisted by nautical men, and with regard to testing the quality of a holding-ground by anchoring, this assistance will become all the more indispensable; the value of this test consists in leading at once to a practical conclusion as to the actual quality of the *holding-ground.* The various means adopted to prove this by anchoring are very numerous; employing a steamer, she may be anchored in six or seven fathoms at low-water, veering out the length of cable that may be thought sufficient to hold in a gale of wind; this may be a whole cable, and afterwards less and less; the steamer may then be backed by the paddles, and as soon as the cable indicates the strain being felt, the lead may be dropped over, and it may then be observed whether the anchor starts under the power of the engines, a careful reckoning being kept of the number of revolutions made at the time; times of slack and smooth water should be sought, when convenient, for making these experiments, and consideration must be given to the condition as to whether the harbour is to be a close harbour, the amount of shelter, the part occupied by the vessel as regards prevailing winds, &c. According to the tonnage of the vessel, the weight of the anchor used is five cwt. to one ton; if in such a depth of water as that above mentioned, the anchorage is tried by letting out only some twenty-five fathoms, conditions may arise such that the anchor has no chance of getting hold of the ground, whereas were twice such quantity veered out, the anchor would get a fast hold. Whilst to let out too little cable may be one extreme, the other may be easily run into by "making arrangements" such as could not be worked out under the natural state of things in which a vessel would make her way into harbour. It is also to be remembered, that although a sailing vessel, by doing all that may be done to check her way, may assimilate to the conditions of a steamer, still it is not possible to obtain as ready a command over her sails as the engineer has over his engine; it is therefore necessary in the log to keep a correct account of all that is done in this way to try the quality of the holding ground, for should the tide set rapidly across the mouth of a harbour, a good way must be kept upon a vessel to enable her to get in; the quarter of the prevailing gales will also have to be considered.

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* The reader who has had no experience with boring tools, will find a full description of them in *Railway Construction,* by the author of the present volume.

Reducing Soundings.—During the greater part of the time that soundings are being taken, to ascertain the depth of water the tide is rising or falling, and therefore to reduce the depth to one standard or level, it is necessary to have a correct record by time of such rise or fall; it has already been observed that this level is generally "Low Water Ordinary Spring Tides." It is therefore necessary that the observer in the boat should keep a correct time as to his soundings, and that a most trustworthy person be employed to make the observations also by time of the rise and fall of the tide as indicated by a tide gauge established for the purpose. By this means the observer has on returning to land the opportunity of obtaining the rise and fall at stated times, and therefore of reducing his soundings to the standard level.

At Fig. 120, which is a slight sketch of Holyhead harbour, the manner in which the depths of soundings are entered on the plan has been shown; the dotted lines, however, are only laid down on the working or draft plan. With regard to the nature of surface at bottom of harbour, this is shown by patches or circles, or by long bands, accordingly as observations have been made in various spots, or in a continuous route; and these various marks are coloured by conventional colours, an explanation of which is carefully entered on the plan; the patches or circles are all numbered in Roman figures according to the notes previously made, and to distinguish them from the depths of water. If the nature of the bottom has only been ascertained by the lead or plunger in a continuous line, it is shown merely a strong dotted line; if by a diver, then the width tried by his probing bar is laid down on the plan, according to the widths of surface examined. Any other system may be adopted, provided it expresses distinctly the nature of the information obtained; thus small geometrical figures, as circles, squares, triangles, crosses, &c., may be used to distinguish the different features of the surface of the bottom, provided that their meaning be entered on the plan. Similar or analogous marks and signs may be used for expressing the power of the holding-ground, according to the number of revolutions made by the engine before the anchor was moved; two parallel lines drawn quite close together are generally the means of showing the length taken by steam-vessels to bring up, and along these lines is shown in "knots" the rates at which the vessels were going when the anchor was let go. The quantity of information to be thus shown is often so great, that to enter it all on one plan would result in a mass of confusion; two or three plans are, therefore, often prepared, on each of which is entered only one description of information; all this must in a great measure depend on the quantity and different natures of the information that has to be expressed on the plans prepared.

With regard to the tidal observations that have to be made in connexion with harbour and marine surveys, they may be very simple, and at times very complex, particularly if extending over any length of time, as is generally required to be the case, or if made along any long line of coast, or up an estuary or tidal river.

On the open coast there is a greater regularity in the rise and fall of the tide than in most other situations, particularly narrow channels, estuaries, and tidal rivers. In these cases the anomalies in the tides are so numerous that the tidal lines are by no means parallel during the whole period of flood and ebb, as a very little consideration will show. For when the tidal wave enters a channel, it does so with a certain velocity and volume of water; if a contraction occurs in the width of the channel, either the depth of the water or its velocity, or both, must increase according to the momentum due, so that in some situations the height of the tide may be considerably in excess of the elevation in the open sea. The greater the rise of the tide, the more irregular the longitudinal and transverse sections of the bed, the more numerous the sharp turns, bends, and contractions, the more rapid the currents, the more numerous should be the stations at which tide gauges are established.

Although we suppose the reader more or less acquainted with the nature of the tidal phenomenon, it will be necessary for many younger students that a few general observations be made on the subject in such an elementary work as this. These, however, we will endeavour to give in as few words as possible.

The tidal wave is due to the force of attraction exerted by the sun and moon on the waters on the surface of the earth. The curved surface of the water due to the progress of the tides from that part of the ocean where they are first produced, and which passes from one low water over the summit of the tide down into the next low water, has been denominated the tidal wave; it is so long that it takes 12½ hours to move its own length, but is, nevertheless, a true wave.

The force of attraction from bodies at equal distances is proportional to their masses.

The attractive forces of bodies equal in mass, but at different distances, vary inversely as the squares of the distances. The effect of the combined attraction of the sun and the moon upon any point of the earth's surface, would be to draw up the waters in a heap at that point under the sun and moon, if the earth stood still long enough; but on account of the velocity with which the earth rotates, there can only be a *tendency* to this result.

Taking the mass of the sun at 28 millions that of the moon, and its distance from the earth 400 times that of the moon, its power of attraction will be $\frac{28,000,000}{160,000} = 175$ times that of the moon.

This is on the mass of the earth, and not at different parts of the surface. Taking the distance of the moon from the centre of the earth at 60 times the radius of the earth, then the side nearest the moon will be at a distance of 59 times the radius, and the farthest at 61 times the radius; and the difference in the force of attraction at the nearest point and the centre of the earth, and at this centre and the farthest point will be $\frac{1}{30}$.

In the same manner, taking the nearest point of the earth to the sun at 23,999 times the radius, the centre as 24,000 times, and the farthest point at 24,001, it may be shown that the difference of attraction at the surface and at the centre is only

$\frac{1}{1200}$, instead of $\frac{1}{30}$, as in the case of the moon, and being, therefore, only $\frac{1}{400}$th that of the moon; but on account of the mass of the sun being 175 times that of the moon, its actual proportion of effect on the raising of the tides to that of the moon is as 1 to 2½.

The moon making her apparent circuit in about 24 hours and 48 minutes, the *moon's tidal wave* will recur at every 12 hours and 24 minutes. The sun makes his circuit in 24 hours, and his *wave* will therefore recur at every 12 hours.

The sun making about 30 waves, whilst the moon makes 29, if at the first of these 29, the summits of the two coincide, at the 15th the summit of the moon's will coincide with the hollow of the sun's, and at the 29th the summits of the two will again nearly coincide; a process very analogous to the well-known phenomenon of the recurrence of spring and neap tides, the first of which occur at about every fortnight, and the latter in the intermediate weeks.

When the moon is new she is between us and the sun, and in the south at mid-day when he is, and the two act conjointly in drawing up their respective waves.

When the moon is full she is in the south at midnight, when the sun is due north; here also there is conjoint action, because the moon's nearest wave will coincide with the sun's farthest wave.

In the moon's "*quarterings*," or intermediate weeks, she is in the east or west when the sun is in the north or south; and the moon's wave will have its summit at a point of the globe at right angles with the sun's wave, and will coincide with the position of the hollow caused by the sun.

Sir Isaac Newton calculated that the height of the tide due to solar attraction would be 2 feet at the equator; the influence of the moon taken at 2¼ times that of the sun would make this 4½ feet, and the spring and the neap tides, being each the sum and difference of the two, would be 6½ and 2½. This complete effect of the forces of attraction would, however, be prevented by the velocity of rotation of the earth, even if its surface were completely covered with water.

But on account of the many changes in the relative positions of the three bodies, there are also many modifications in their attractive force, and the following circumstances are observed.

During summer and winter, at new and full moon, the sun and the moon exert their attractive forces together upon *alternate* belts of the earth's surface, so that the consecutive waves caused by these attractions, although recurring at regular times, or nearly so, will be in themselves different; and it is found that in summer and winter *alternate* spring tides are high and low, and neap tides nearly regular.

In spring and autumn the attractive forces exert their power upon the equator each time, consequently the consecutive waves will be nearly similar; and during these seasons the spring tides are found regular, and the neap tides alternately vary.

The above modification is called the *diurnal inequality* of the tide.

Equinoctial tides are the greatest; at the equinoxes the sun and moon act in concert on the equator, where they have the greatest effect, this being the greatest circle round the earth, and where the centrifugal force exerts its greatest strength; the tide-wave whose summit is in the equator is greater than one with its summit on either side of the equator. This depends on the declination of the sun and moon being greatest when the declination is zero, and least when the declination is greatest, either north or south. On the 21st of March the sun strikes down vertically on the equator, and the sun's declination of zero. On the 21st of June the declination is 23½ degrees north; and on the 21st of December the declination is 23½ degrees south. When new or full, the declination of the moon is always nearly the same as that of the sun. In the quarters the moon's declination is greater in spring and autumn, and least in summer and winter.

With regard to the prediction of the tides, the following was the principle of the method adopted by Sir John Lubbock:—

He took a series of observations extending over 19 years; that being the period in which the moon returns into the same position relative to the sun in all respects, and consequently containing a cycle of all varieties of tides; all amounts of declination, distance, &c., for the different directions of the sun and moon.

This gave him about 14,000 observations as his materials.

He divided these 14,000 into 24 groups, the first containing all those tides which occurred on the day of new or full moon, when the moon crossed the meridian, or became exactly south, between twelve and half-past twelve; the second group contained the tides of the days when the moon was south between half-past twelve and one; the third between one and half-past, and so on for the 12 hours.

Each of these 24 groups contained about 580 observations.

Striking an average of all the heights in each group, he found the height to which, *on an average*, all tides which occur on the day of a certain time of transit of the moon rise, which is as much as to say, that when the sun and moon have their mean declination, and are at their mean distance from the earth, such will be the height of the tide at the different periods of the moon's age.

His next process was to place against the date of each tide, in each group of 580, the declination of the moon when in the meridian on that day. He then divided each group into 11 subdivisions, each subdivision consisting of those tides corresponding to a declination of 0° to 3°, 3° to 6°, 6° to 9°, &c., up to 27° to 30°. He then took the averages of the heights in each of these subdivisions, and so found how much the tide at any given period of the moon's age would be affected by the moon's declination. With the moon on the equator the height would be so much; and the difference would be so much for each three degrees of declination.

Next he took the same 24 groups of 580 tides, and subdivided them according to the moon's distance from the earth at the time each occurred, making a subdivision for each minute of parallax.* From this he found how much the moon's distance made the height of the tide vary from the mean height at any given age of the moon.

The same process was gone through with the sun's declination and parallax; and also the declination tables were again subdivided into north and south declinations, or day and night tides, to show how much inequality there was between them for each 3 degrees of declination.

Thus 7 tables were obtained as follows:—

I. The average height of the tide corresponding to any half-hour of the moon's transit.

II. The variation from this average caused by the moon's declination.

III. The variation caused by the moon's parallax.

IV. The variation caused by the sun's declination.

V. The variation caused by the sun's parallax.

VI. The inequality between two consecutive tides caused by the moon being alternately north and south of the equator.

VII. The same inequality dependent upon the sun.

With these tables therefore, a set of which may be found in the *Companion to the British Almanac for* 1837, the height of the tide may be predicted for any future day for any place to which the tables are applicable in the following manner.

The *Nautical Almanac* gives the time of the moon's transit for the day in question. Table I. gives the average height corresponding to such time of transit. The *Nautical Almanac* further gives the moon's declination for that day. Table II. gives a correction of a few inches to be made to the height given in Table I. In the same way Tables III., IV., and V. give further small corrections according to the parallax and declination for the day as given in the *Almanac*, and then a height is found which is half-way between the morning and evening tides of that day. Tables VI. and VII. give the correction to be made according as the higher or lower tide of the day is wanted.

This mode of calculation having been compared with observation, it was found to have indicated with remarkable precision the changes which did actually follow in the various heights of the tides. Such discrepancies as appeared were made the groundwork of renewed study, and some corrections were made, especially that in calculating the tide for a certain day, the time of the moon's transit of two or three days previous was taken as the starting-point of the calculation.

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* Astronomical tables give the distance of the sun and moon, not in so many miles, but by their *parallax*, that is, the size which half the diameter of the earth would appear to be to a person standing on the sun or moon. This would evidently be larger as the sun or moon became nearer. The moon's parallax varies from 54 to 61 *minutes*; the sun's from 8·66 to 8·94 *seconds*.

In this manner tides have been predicted for every day of the year for all ports at which a sufficient number of observations had been made to found the tables upon, and the predictions have been verified in a most remarkable manner.

* * * * * *

The following table will illustrate the effect of the sun in accelerating the tide from springs to neaps, and in retarding it from neaps to springs, and how at neaps as well as springs it is coincident with the lunar wave.

It gives the time of high-water at Liverpool in the month of December, 1853, from the full moon on the 16th to the new moon on the 30th.

Column I. is the time of the lunar tide, varying from 44 to 51 minutes daily.

Column II. that of the actual combined solar and lunar tides.

Column III. the difference from day to day of the time of the actual tide.

Column IV. the amount which the actual tide is *before* the lunar tide.

Column V. the amount which the actual tide is *behind* the lunar tide.

Day.	I.	II.	III.	IV.	V.	
○15	11·6	11·6		0·0		Spring.
16	11·33	11·40	34		0·7	
17	12·23	12·13	33	0·10		
18	1·13	12·50	37	0·23		
19	2·03	1·26	36	0·37		
20	2·51	2·8	42	0·43		
21	3·38	2·48	40	0·50		
22	4·24	3·35	47	0·49		
☾ 23	5·08	4·27	52	0·41		Neap.
24	5·54	5·33	56	0·21		
25	6·41	6·39	66	0·02		
26	7·31	7·47	68		0·16	
27	8·26	8·46	59		0·20	
28	9·26	9·41	55		0·15	
29	10·30	10·33	52		0·03	
●30	11·36	11·28	55	0·08		Spring.

Now here it will be observed, that from two days after spring tide till two days after neap tide, the differences are in Column IV., or the actual tide is in advance of the lunar tide. After that, the differences are for four days in Column V., or the actual tide is in arrear of the lunar tide. The irregularities are consequent upon the changes of declination and parallax which take place within the fortnight. Now it is clear that these variations in the times, being dependent upon the same causes as the variations in the heights of the tides, can be treated in the same manner as the observations of height. Each observation being referred to its group, tables may be formed from which the time of any future tide may be predicted.

This has been done, and the result has been equally successful with the result of the predictions of height.

With regard to the calculations that might be made to predict the time and height of the tidal wave at any particular spot, from the position of the earth and the heavenly bodies, and applied to a globe covered with a uniform thickness of water, it will at once be seen that the earth we live on is totally different from such a globe; and the tidal wave instead of moving on steadily and regularly, is on our earth obstructed and distorted in various ways by the various forms of the land, such as promontories and peninsulas, islands, inland seas, reefs, rivers, &c.; and the obstructions from these relate not only to the immediate configuration of the land between high and low water, but to the shelving shores stretching out to a distance into the sea, where it forms probably a kind of submarine country with its hills and valleys. When the main body of the travelling tidal wave approaches the shores of islands and projecting continents, it becomes checked on the diminished depth of water long before it is perceptible on any shore. The direction of what may be termed the branch of the wave, as it curves round the shores of an island or peninsula, may assume that of a line at right angles to the line of the summit of the wave. This alteration will be in height and time as well as in direction. Not only it may assume this right-angled direction, but it may, and even does assume a rotatory or retrograde motion in some instances.

Even along a straight line of shore parallel to the direction of the tidal wave, the different depths of water will produce different ratios of speed in the tide as it approaches the land; and this approach may be in a series of curves of greater or less curvature, according to different depths of water off the shore, the sharper curvature probably corresponding to a less depth of water. Supposing the tidal wave to travel along the Atlantic from south to north, at a rate of 700 miles per hour, it was supposed the sea was there five or six miles deep, which has since been verified.

The impediments formed by extensive coral reefs round the shores of islands, are often such as to afford inlets to the tides only at certain places; the direction and time of the tide-wave may thus be entirely changed, and therefore give no idea of the tidal movements outside these bars, and variations are thus caused to the amount of several hours, and when they extend to twelve hours, it is doubtful whether the tide under observation is due to one transit of the moon or to another.

The "*age of the tide*" represents the time elapsed since the heavenly bodies were in such position as to form it, and includes both the time in forming the tide, as well as that during which it has been rolling about at sea. At places where the tides are, for instance, two or two-and-a-half days old, the highest spring-tides occur two and two-and-a-half days respectively after new and full moon.

Enough has been said to impress the necessity of great caution being used as to the presumed cause of the tidal wave on any part of the globe, and to show that there is a vast difference between the tidal wave in reality and in theory.

Professor Airy treats the tide from its formation under the attractive forces, as a wave, using the horizontal current as the basis of the calculations, the same as La Place, who instead of taking as primary cause the tendency of the water to rise under the attractive forces, took the horizontal movement of the water, and considered the rise due to the meeting of two horizontal currents under the sun or moon.

This movement is caused by the tendency which all the water between E and A, and E′ and A, Fig. 121, has to move towards A, and all that between E and A′, and E′ and A′, to move towards A′, being the resultant force of the combined attractions of the moon and of the earth. This theory represents the tide from its first formation as the tidal wave subject in motion and magnitude to fixed laws, after it has become independent of the attraction of heavenly bodies.

Depth of Water in feet.	Length of Wave in feet.							
	1	10	100	1,000	10,000	100,000	1,000,000	10,000,000
	Corresponding velocity in feet per second.							
1	2·26	5·34	5·67	5·67	5·67	5·67	5·67	5·67
10	2·26	7·15	16·88	17·92	17·93	17·93	17·93	17·93
100	2·26	7·15	22·62	53·39	56·67	56·71	56·71	56·71
1,000	2·26	7·15	22·62	71·54	168·83	179·21	179·33	179·33
10,000	2·26	7·15	22·62	71·54	226·24	533·90	566·72	567·10
100,000	2·26	7·15	22·62	71·54	226·24	715·43	1688·3	1792·1

From which it appears that—

1st. When the length of wave is not greater than the depth of water, the velocity depends (sensibly) only on its length, and is proportionate to the square root of its length.

2nd. When the length of the wave is not less than one thousand times the depth of the water, the velocity of the wave depends (sensibly) only on the depth, and is proportionate to the square root of the depth. It is in fact the same as the velocity which a free body would acquire by falling from rest through a height equal to half the depth of water.

3rd. For intermediate proportion of length of wave and depth of water, the velocity can only be got by the general equation.

The wave originally produced by the action of the sun or moon, may be called the *Free Tide Wave*. The semi-diurnal tide wave is of this character, and may be taken to have a period of 12 hours 24 minutes; now by the foregoing table we see that a wave proceeding 10,000,000 feet, will travel with a velocity sensibly independent of its length; on this principle, therefore, is calculated the following

Table for the Semi-diurnal Free Tide Wave.

Depth of Water in feet.	Velocity of free tide wave per second, in feet.	Length of free tide wave, in miles.	Space described by free tide wave per hour, in miles.
1	5·67	47·94	3·86
4	11·34	95·89	7·73
10	17·93	151·62	12·28
20	25·36	214·42	17·29
40	35·87	303·24	24·45
60	43·93	371·38	29·95
80	50·72	428·88	34·58
100	56·71	479·46	38·66
200	80·20	678·05	54·68
400	113·42	958·91	77·33
600	138·91	1174·4	94·71
800	160·40	1356·1	109·36
1,000	179·33	1516·2	122·27
2,000	253·61	2144·2	172·92
3,000	310·62	2626·1	211·78
4,000	358·67	3032·4	244·55
5,000	401·00	3390·2	273·41
6,000	439·27	3713·8	299·50
7,000	474·47	4011·4	323·50
8,000	507·23	4288·3	345·84
9,000	538·00	4548·5	366·82
10,000	567·70	4794·6	386·66
20,000	802·00	6780·5	546·82
30,000	982·25	8304·4	669·71
40,000	1134·2	9589·1	773·32
50,000	1268·1	10721·	864·59
60,000	1389·1	11744·	947·11

The diurnal and other tidal waves, so far as they are free, may be all considered as travelling with the same velocity, but the column of lengths of the wave must be doubled for the diurnal wave.

Until lately there was considerable uncertainty as to the movement of particles of water in the waves. The wave itself moves forward, but the particles of water in it do not move forward with the wave, but merely move up and down, or rather, *nearly so*, whilst the wave continuing its identity must have its component water constantly changed.

The oscillatory motion produced in ripples, or waves, by the wind blowing on the water, extends but little beyond the surface, and even in a storm there is but little motion some 15 or 20 feet down.

If instead of this action of the wind on the surface, we can imagine some mechanical means driven down through a depth of water, and then pushed forward, there will be an onward movement imparted to the whole body of water before it; a wave will be raised consisting of the body of water thus disturbed. The waves produced by the horizontal force of the wind on the surface, come after one another, and the oscillations of the one increase the oscillations of the next, and the whole surface is brought into a condition of ripples, or waves, as we so often observe.

But in the second case, that of the mechanical means acting with uniform force throughout the depth of water, there is only one wave, which travels onward and leaves all tranquil behind it. This is what Mr. Scott Russell has denominated the *great primary wave*, or of the *first order*, the wind-waves of the sea being of the second order only.

In the wave of the *first order*, the motion of the particles of water is uniform from top to bottom.

In the wave of the second order, the force and its effects are on the surface, and gradually decrease down the depth of the water.

The following extract is from a report of Mr. Scott Russell to the British Association:—

> That the velocity of this wave in channels of uniform depth is independent of the breadth of the fluid, and equal to the velocity acquired by a heavy body falling freely by gravity through a height equal to half the depth of the fluid, reckoned from the top of the wave to the bottom of the channel.
>
> That the velocity of this primary wave is not affected by the velocity of impulse with which the wave has been originally generated, neither do its form or velocity appear to be derived in any way from the form of the generating body.
>
> This wave has been found to differ from every other species of wave in the motion which is given to the individual particles of the fluid through which the wave is propagated. By the transit of the wave the particles of the fluid are raised from their places, transferred forwards in the direction of the motion of the wave, and permanently deposited at rest in a new place at a considerable distance from their original position. There is no retrogradation, no oscillation; the motion is all in the same direction, and the extent of the transference is equal throughout the whole depth. Hence this wave may be descriptively designated THE GREAT PRIMARY WAVE OF TRANSLATION. The motion of

translation commences when the anterior surface of the wave is vertically over a given series of particles, it increases in velocity until the crest of the wave has come to be vertically above them, and from this moment the motion of translation is retarded, and the particles are left in a condition of perfect rest, at the instant when the posterior surface of the wave has terminated its transit through the vertical plane in which they lie. This phenomenon has been verified up to depths of five feet.

That the elementary form of the wave is cycloidal; when the height of the wave is small in proportion to its length, the curve is the prolate cycloid, and as the height of the wave increases, the form approaches that of the common cycloid, becoming more and more cusped until at last it becomes exactly that of the common cycloid with a cusped summit; and if by any means the height be increased beyond this, the curve becomes the curtate cycloid, the summit assumes a form of unstable equilibrium, the summit totters, and falling over on one side forms a crested wave, or breaking surge.

* * * * * *

That in the rectangular channel the velocity is that of gravity due to half the depth. In the sloping or triangular channel the velocity is that due to one-third of the greatest depth. In a parabolic channel the velocity is that due to three-eighths or three-tenths of the greatest depth, according as the channel is convex or concave; and finally that the velocity of the great primary wave of translation of a fluid is that due to gravity acting through a height equal to the depth of the centre of gravity of the transverse section of the channel below the surface of the fluid.

The height of a wave may be indefinitely increased by propagation into a channel which becomes narrower in the form of a wedge, the increased height being nearly in the inverse ratio of the square root of the breadth.

If waves be propagated in a channel whose depth diminishes uniformly, the waves will break when their height above the surface of the level fluid becomes equal to the depth at the bottom below the surface.

The great waves of translation are reflected from surfaces at right angles to the direction of their motion without suffering any change but that of direction.

The great primary waves of translation cross each other without change of any kind, in the same manner as the small oscillations produced on the surface of a pool by a falling stone.

The WAVES OF THE SEA are not of the first order—they belong to the *second or oscillatory order* of waves—they are partial displacements at the surface, which do not extend to considerable depths, and are therefore totally different in character from the great waves of translation, in which the motion of displacement of the particles is uniform to the greatest depth. The displacement of the particles of the fluid in the waves of the sea is greatest at the surface, and diminishes rapidly. There are generally on the surface of the sea several coexistent classes of oscillations of varying direction and magnitude, which by their union give the surface an appearance of irregularity which does not exist in nature.

When waves of the sea approach a shore, or come into shallow water, they become waves of translation, and obeying the laws already mentioned, always break when the depth of the water is not greater than their height above the level.

A tidal bore is formed when the water is so shallow at low water that the first waves of flood tide move with a velocity so much less than that due to the succeeding part of the tidal wave as to be overtaken by the subsequent waves, or wherever the tide rises so rapidly, and the water on the shore or in the river is so shallow that the height of the first wave of the tide is greater than the depth of the fluid at that place. Hence in deep water vessels are safe from the waves of rivers, which injure those on the shore.

The identity of the tide wave and of the great wave of translation, show the nature of certain variations in the establishment of ports situated on tidal rivers. Any change in the depth of the rivers produces a corresponding change on the interval between the moon's transit and the high water immediately succeeding. It appears from the observations in this report, that the mean time of high water has been rendered 37 minutes earlier than formerly by deepening a portion of about 12 miles in the channel of a tidal river, so that a tide wave which formerly travelled at the rate of 10 miles an hour, now travels at the rate of nearly 15 miles an hour.

It also appears that a large wave or a wave of high water of spring tides travels faster than a wave of high water of neap tides, showing that there is a variation on the establishment, or on the interval between the moon's transit and the succeeding high water, due to the depth of the fluid at high water, and which should, of course, enter as an element into the calculation of tide tables for an inland port derived from those of a port on the sea shoré. The variation of the interval will vary with the square root of mean depth of the channel at high water.

The report suggests that "these results give us principles, 1st, for the construction of canals; 2nd, for the navigation of canals; 3rd, for the improvement of tidal rivers; 4th, for the navigation of tidal rivers; 5th, for the improvement of tide tables.

The following experiments were made for the purpose of determining whether the velocity of the so called great primary wave were not affected by the initial velocity given to the fluid at its generation by the moving body. The velocity of genesis, or of the vessel by whose displacement the elevation of fluid was produced, is given in miles per hour, and the time occupied by the wave in describing 700 feet is given in seconds.

	Velocity of genesis.	Space described by the wave.	Interval of time.
(1.)	5 miles an hour	700 feet	62· seconds
(2.)	3 "	700 "	61· "
(3.)	10 "	700 "	61· "
(4.)	7 "	700 "	62· "
(5.)	7 "	700 "	62· "
(6.)	4 "	700 "	61·5 "

From this it is manifest that the velocity of the propagation of the wave does not vary with the velocity of its genesis.

To determine whether the height of the wave produced any variation in its velocity, the following experiments were made:—

	Height of the wave above the level.	Space described.	Interval.
(7.)	6·0 inches	700 feet	61·50 seconds.
(8.)	5·0 "	700 "	61·75 "
(9.)	3·5 "	700 "	62·50 "
(10.)	2·0 "	700 "	63·50 "

It appears from these examples that, in a given reservoir of fluid, the higher wave moves more rapidly than the lower; and it was afterwards found that the increase in height was equivalent in its effect on the velocity to an equal addition to the depth of the fluid in the reservoir.

To determine whether the depth of the fluid affected the velocity of the wave, the following experiments were made in the same channel filled to different depths :—

	Depth of fluid.	Space described.	Velocity of wave.
(11.)	5·6 feet	486· feet	9·594 miles an hour.
(12.)	3·4 „	150· „	7·086 „

The former of these observations is exclusive of the height of the wave, and adding six inches to the depth of the fluid in this case, the height of the wave being already added to the depth in (12), we find that the velocities are nearly proportional to the square roots of the depths, and are nearly equal to the velocities that would be acquired by a heavy body in falling through heights equal to half the depth of the fluid.

In the last case the channel was rectangular, and consequently the depth of the fluid was uniform across the whole depth of the channel; it was next of importance to ascertain what law held in those cases where the depth diminished towards the edges of the channel. For this purpose two channels were selected having the greatest depths in their middle, and diminishing towards the sides. The following are the results :—

	Greatest depth in the middle of the channel.	Space described.	Velocity of wave.
(13.)	5·5 feet	1000 feet	7·84 miles an hour
(14.)	4·0 „	820 „	6·09 „

In these instances the diminished depth at the sides has diminished the velocity of the wave below that due to the greatest depth in a ratio in the first example nearly of 9·5 to 7·8, and in the second of 7· to 6·. See Experiments (11) and (12).

The following three experiments are instructive as having been made on channels in which the maximum depth was nearly the same in all; but in (15) the depth remained constant to the side which was vertical, in (16) the sides had a slope of nearly 20°, and in (17) a slope of nearly 40°, so as to diminish the depth towards the sides.

	Maximum depth.	Form of channel.	Space described.	Velocity.
(15.)	5·6 feet	Rectangular	486 feet	9·59 miles
(16.)	5·5 „	Slope of 20°	2038 „	8·83 „
(17.)	5·5 „	Slope of 40°	1000 „	7·84 „

From these it is manifest that the depth of the channel, while it modifies the depth of the fluid, affects the velocity of the wave. It was not found that the breadth of the channel produced any similar effect.

The report contained some experiments made on the river Clyde. The stations extended from the Bromielaw to Port Glasgow.

	Diff. of level at H.W.		Diff. at L.W.	H. W. time.	
Station I. Glasgow	10·1 inches.	Higher than at Port Glasgow.	33 inches.	83 minutes.	Later than at Port Glasgow.
Station II.	9·1 „		31 „	76 „	
Station III. Clyde Bank	7·0 „		27 „	61 „	
Station IV.	6·1 „		25 „	43 „	
Station V. Bowling	5·2 „		12 „	24 „	
Station VI.	2·2 „		5 „	6 „	
Station VII. Port Glasgow	0·0 „		0 „	0 „	

From a laborious discussion of the observations, it appeared that the wave of high water travelled

From IX.	to VIII.	in 6 min.	14 miles	80 miles an hour.
From VIII.	to VII.	in 9 „	6 „	
From VII.	to VI.	in 6 „	3·75 „	20 „ „
From VI.	to V.	in 18 „	4·25 „	
From V.	to IV.	in 19 „	2·5 „	8·1 „ „
From IV.	to III.	in 18 „	2·5 „	
From III.	to II.	in 15 „	2·75 „	15 „ „
From II.	to I.	in 7 „	2·75 „	

These results show that in the deep water being between 40 and 60 fathoms, or between 240 and 360 feet deep, the wave travels at the enormous rate of 80 miles an hour; that on reaching water from 20 to 30 feet deep, the velocity is diminished to 20 miles an hour; and from V. to II. where the river is wide, shelving, and shallow, the velocity of the tide wave is retarded to 8 miles an hour; while on ascending further up, where the banks are nearly upright, and the contracted width gives an increase of mean depth, the velocity has a corresponding increase to 15 miles an hour.

It appeared by the plans that the average depth of the river from I. to III. was 15 feet. From III. to V. the river is wide and shallow, spreading over extensive banks, where there are not 2 feet of water, for which we may take a third part of the greatest as a mean depth, or about 5 feet. In the division from V. to VII., both depth and breadth increase very rapidly to about 35 and 37; taking 25 feet as the mean depth, we have

Velocities of the Tide wave as observed.	Mean depth.	Velocity due to depth.
80 miles an hour.	240—360 feet.	60—80 miles.
20 „ „	25 „	19·3 „
8·1 „ „	5 „	8·6 „
15 „ „	15 „	14·9 „

The force of attraction having commenced its operations without having time, through change of position, to complete, a wave of the *first order* has been raised. This wave moves onwards over the sea, setting in motion the water before it, and leaving all still again behind. This wave is the great *primary wave*, raised at some distant period by the force of attraction on the sea, and moving onwards towards that point of the earth's surface the nearest to the moon, as directly as the obstructions formed by the land will admit of.

Professor Airy has calculated the original height of the tidal wave at two feet for spring tides, and nine inches for the neaps, which has been considered very nearly correct for the open sea; but terrestrial influences, more particularly the bed of the sea as it approaches the land, give in the enlarged dimensions it attains on our shores.

Tides of the British Seas.—On this subject nothing can be more valuable than the observations of Admiral Beechey, and the following remarks are from his reports in the *Philosophical Transactions*, March, 1848, and May, 1851.

Tides of the Irish Sea.—Up to the period of these observations the set of the tides in the Irish Sea had been greatly misunderstood, owing to the association of the turn of the stream with the rise and fall of the water on the shore; and it was generally understood that as Holyhead was three hours later in its tides than places at the entrance of the channel, a vessel starting with the first of the flood would carry nine hours' tide in her favour in her run up channel, and *vice versâ.*

This was an error sufficiently great in itself, but it was liable to be increased by an entire ignorance as to the time when a vessel would take any particular tide; for the times of high water by the shore were very various on different sides of the channel.

The observations have shown that, notwithstanding the variety of times of high water throughout the channel, the turn of the stream is simultaneous; that the northern and southern streams in both channels commence and end in all parts (practically speaking) *at the same* time, and that time happens to correspond with the time of high and low water on the shore at *Morecambe Bay*, an estuary rendered remarkable, as being the point where the opposite tides coming round the extremities of Ireland finally meet. So that it is necessary only to know the times of high and low water at Morecambe Bay to determine the hour when the stream of either tide will commence or terminate.

By these observations,

Taken in connexion with the very valuable series of observations which were carried round Ireland by the Ordnance at the suggestion of Professor Airy, we are made acquainted with several curious facts:—First, that whilst it is high water at one end of the channel, it is low water at the other; that the same stream makes both high and low water at the same time; that there are two spots in the channel, in one of which the stream runs with considerable velocity without the water either rising or falling, and in the other, that the water rises and falls from 16 to 20 feet without having any visible horizontal motion of its surface; and that during the first half of the flowing, and last half of the ebbing tide-wave, the stream in the south channel runs in a contrary direction to the wave, and goes up an ascent of about one foot in four and a-half miles.

* * * * * * * *

Plate II.* shows the lines of direction of the stream with the rate of the tide at its greatest velocity on the day of syzygy, all being reduced to the same standard.

An inspection of the Plate will show that the tide enters the Irish Sea by two channels; of which Carnsore Point and Pembroke are the limits of the southern one, and Rathlin and the Mull of Kintire the boundaries of the northern.

The stream in the southern channel (as before stated) has been ascertained to move *simultaneously in one vast current throughout;* running six hours nearly each way, at an average rate of from two to three knots per hour at the height of the springs, increasing to four knots and upwards near the banks and at the pitch of the headlands; its *times of slack water* corresponding sufficiently near for all practical purposes, with *the times of high and low water for the day at Morecambe Bay*, or more correctly *at Fleetwood*, which is twelve minutes earlier than Liverpool.

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* For plans, see Admiral Beechey's Charts in the *Phil. Trans.;* in the absence of these, consult the Admiralty or the Ordnance maps.

The *central portion* of the stream of *flood* or *ingoing stream* runs nearly in a line from a point midway between the Tuskar and the Bishops, to one six miles due west of Holyhead; beyond which it begins to expand eastward and westward, but its main body preserves its direction straight forward for the Calf of Man, which it passes to the eastward with increased velocity as far as Langness Point, and then at a more moderate rate on towards Maughold Head. Here it is arrested by the flood or southern stream from the north channel coming round the Point of Ayre, and is first swayed round to the eastward by it, and then goes on with it at an easy rate direct for Morecambe Bay.

The *outer portions* of the stream are necessarily deflected from the course of the great body of the water by the impediments of banks on the Irish side of the Channel, and by the tortuous form of the coast on the Welsh. The eastern portion passing Linney Head rushes with great rapidity between the Smalls, Grassholm, and Milford Haven, towards the Bishops, which it passes at a rate of between four and five knots; sets sharply round those rocks in an E.N.E. direction, right over the Bass bank, and into Cardigan Bay; makes the circuit of that bay; and sets out again towards Bardsey at the other extremity of it; then sweeping to the N. by W. past the island and through the sound, it gradually takes the course of the shore, round Caernarvon Bay, filling the Menai Strait as far as Bangor; but the stream still continuing outside towards the South Stack, which it rounds, setting towards the Skerries at a rate of upwards of four knots; and finally, turns sharply round those rocks for Liverpool and Morecambe Bay; completing in its way the high water in the Menai, and filling the Dee, Mersey, and Ribble.

The *western* portion of the stream, after passing the Saltee, runs nearly in the direction of the Tuskar, sets sharply round it, and then takes a N.E. ½ N. direction, setting fair along the coast, but over the banks skirting the shore. Abreast of the Arklow is situated that remarkable spot in the Irish Channel, where *the tide neither rises nor falls*. The stream, notwithstanding, sweeps past it at the rate of four knots at the springs, and reaches the parallel of Wicklow Head. Here it encounters an extensive bank recently known; and whilst the outer portion takes the circuit of the bank, the inner sweeps over it, occasioning an overfall and strong rippling all round the edge, by which the bank may generally be discovered; beyond this point the streams unite and flow on towards Howth and Lambay, growing gradually weaker as they proceed, until they ultimately expend themselves in a large space of still water situated between the Isle of Man and Carlingford, where occurs the phenomenon of the water rising and falling without having any perceptible stream. This space of still water is marked by a bottom of blue mud.

In the north channel the stream enters between the Mull of Kintire and Rathlin *simultaneously* with that passing the Tuskar into the southern channel, but flows in the contrary direction. It runs at the rate of three knots at the springs, increasing to five knots near the Mull, and to four near Torr Head on the opposite side of the channel. The eastern branch of this stream turns round the Mull towards Ailsa and the Clyde, a portion passing round Sanda up Kilbrannin Sound and Loch Fyne.

The *main body* sweeps to S. by E., taking nearly the general direction of the channel, but pressing more heavily on the Wigtownshire coast; off which it has scooped out a remarkable ditch, upwards of twenty miles long by about a mile only in width, in which the depth is from 400 to 600 feet greater than that of the general level of the bottom about it. Near the Mull of Galloway the stream increases in velocity to five knots, the eastern portion turns sharply round the promontory towards the Solway, and splits off St. Bee's Head; one portion running up the Solway, and the other towards Morecambe Bay.

The *central portion* from a midway between the Mull of Galloway and the Copeland Islands, presses on towards the northern half of the Isle of Man, and while one portion of it flows toward the Point of Ayre, the other makes for Contrary Head, and is there turned back at a right angle nearly to its early course. Passing Jurby it reunites with the other portion of the stream, and they jointly rush with a rapidity of from four to five knots round the Point of Ayre, and directly across all the banks lying off there, and catching up the stream from the south channel off Maughold Head, they hurry on together towards that great point of union, Morecambe Bay. This bay, the grand receptacle of the streams from both channels, is notorious for its huge banks of sand heaped up in terrible array against the mariner unacquainted with its locality, and also remarkable for a deep channel scoured out by the stream, and known as the Lune Deep, which, to the wary navigator, is the great hidden beacon of his safety, and serves him, alike in fog or in sunshine, as a guide to his position, and to a harbour of safety in case of need.

We have now only to speak of the *western limit* of the stream, which we left off Torr Head running at a rate of four knots off the pitch of the Point. Hence it strikes directly towards the Maidens, boiling over the Highlander and Russell rocks, and other reefs in the vicinity of that dangerous group; and takes the direction of the coast again from Muck Island to Black Head, at the entrance of the Lough of Belfast, which it fills.

The portion of the stream which sets up the Lough splits again off Grey Point; one portion flowing up towards Garmoyle, while the other bends back along the shore of Bangor, Grimsport, and Orlock, and blends with the general stream which has come on from the Maidens and Black Head, and passes with it through the sounds of the Copeland Islands. Hence it proceeds along the coast, brushes the South rock, and runs on towards St. John's Point; off which, the stream, like that coming from the southward, expends itself in a large space of still water, which remains undisturbed although pressed upon by streams from various quarters.

Such is a general description of the streams in both channels which attend the *flowing of the water*, or which, for the purpose of distinction, we may designate the *ingoing stream*.

The ebbing or *outgoing* streams do not materially differ from the reverse of these, except that in the southern channel they press rather more over towards the Irish coast.

This is a general idea of the course of the streams throughout the Irish Sea, represented in Plate II.; but besides these there are (as usual) at all the points and headlands, when abrupt, counter streams or eddies beginning at about two hours after the offing stream, increasing with the strength of the tide, and occasioning races and overfalls at the places marked on the chart. In the direction of the offing stream there is as little variation of the current at the different hours of tide as will be met with in any sea of similar extent, and indeed it is only with the slackening of the tide that the variations occur, which happens from about forty minutes before to about forty minutes after high or low water at Morecambe Bay.

In the Irish Sea it was found that the place of the ***water at the half-tide interval*** did not correspond with that of a mark at ***the half-range of the wave***, but that it was always below it, showing that the upper half of the wave rose and fell more rapidly than the lower. It was also found that the curve of the Irish Sea tide did not correspond with that of the Bristol Channel tide; that neither followed the law of the sines to corresponding arcs of tidal intervals.

In connexion with the range of tide is that of the apparent mean elevation of the water. All the observations confirm the remark of Professor Airy (*Philosophical Transactions*, 1845, Part I. p. 31), viz., that this mean level is higher at the springs than at the neaps. The mean place of the water, however, for an entire lunation, during the summer months at least, is tolerably constant, and affords a fair standard to which the reductions used in our nautical surveys may be referred in the event of the gauge being removed by which the observations were made; annexed is the result of observations made at Holyhead during nearly four entire years.

Apparent Mean Place of the Water, at Holyhead.

Month.	1838.		1839.		1846.		1847.		Mean of Months.		
	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.	
January...	11	3¼	10	6	......		10	7	10	9⅓	
February..	10	6½	10	3	......		9	10	10	2⅓	
March.....	10	1	10	2	......		10	4	10	2⅓	
April......	10	1	9	10	......		9	11	9	11⅓	Summer months.
May	10	2	9	9	10	0½	9	11¾	9	11¾	
June......	......		10	1	10	4	9	11¾	10	1½	
July	10	1	10	1½	10	1¾	9	10¾	10	0¾	
August ...	10	6	9	10	10	1	9	10¾	10	0¾	
September.	10	4	10	7	11	1	10	3¼	10	6¾	
October ...	10	6¾	10	3⅓	10	8	10	10	10	7	
November.	10	7	10	2½	10	7¾	10	9¾	10	6¾	
December .	10	7	11	2	10	6	10	11	10	9½	
Mean of the year.....	10	5	10	2¾	10	5½	10	3¼	10	3⅚	

All the tides of the Irish Sea partake of the nature of river tides in having their ebb longer than their flood, except those of Tuskar and Holyhead, which are the reverse. The respective intervals are given in the order in which the places occur.

Duration of Tide.

	Rising.			Falling.	
	h.	m.		h.	m.
Tuskar	6	27	...	6	8
Bardsey	5	24	...	6	52
Holyhead	6	18	...	6	0
Peel, Isle of Man	6	0	...	6	15
Ramsay, Isle of Man...	5	48	...	6	35
Fleetwood	5	46	...	6	39

All these are the mean of many observations.

The change at Holyhead is remarkable, and if we follow the durations up to Ramsay, we shall see that Peel also, an intermediate station, is affected. The cause of this may possibly be connected with the effort of the water to maintain its level; for in projecting the curve of the wave on paper, this peculiarity, in connexion with the very short flood of Bardsey, has the effect of reducing the curve from what it would assume were Holyhead similarly influenced with other places.

It seemed evident that the water was influenced by forces acting in opposition nearly to each other, and that there was a tide in the offing whose streams of ebb and flood did not correspond with those of the channels. By applying this idea first to the English Channel, the observations responded to it; and carrying it to the offing of the Irish Sea, and considering that channel as comprising the Bristol Channel within its limits, as the English Channel does the Gulf of St. Malo, the idea was confirmed so far as the observations themselves extended. This offing stream appears to be of great extent, setting to the north and south along the coast of Biscay and the British Isles, running six hours nearly each way, and exercising an influence with more or less effect over all the waters of the channels and estuaries it passes in its progress, diverting their courses, and in some cases, when the streams oppose, wholly overpowering or reversing their direction. From the connexion of the observations of the Irish Sea with those of the British Channel, it is clear that the whole of the ebb or outgoing stream of the eastern half of the Irish Channel runs into the Bristol Channel, and forms the flood or ingoing tide of the northern half of that great estuary; and *vice versâ* the ebb or outgoing stream from the northern half of the Bristol Channel, forms the flood of the Irish Sea, each tide passing to and fro with great rapidity round St. Gowan's Head. The centre and southern half of the Bristol Channel receive their waters from the offing and the English Channel, the coast stream bringing the waters up from the Land's End and the English Channel, as the stream on the northern half did those of the Irish Channel, and *vice versâ*.

The great offing stream at the entrance of the English Channel extends its influence as far up as Cape La Hague, beyond which, owing perhaps to the sudden contraction which there occurs in the Channel, the stream suffers no interruption, but, as in the Irish Sea, passes up and down the Channel six hours nearly each way as far as a line joining Dungeness and Cape Grisnez, the apparent *virtual head* of the tidal channel. Here the influence of the North Sea stream begins to be felt, and here, as in the Irish Channel, again *the time of high and low water at the virtual head of the tide regulates the turn of the up and down stream along the whole channel* as far as the contraction. Beyond this the offing stream being governed by its own high water, and that occurring at about six hours earlier than that of the head of the channel, the offing stream either butts against the returning streams from the channels, or withdrawing its water, solicits their streams and thus alters their course, making them for the most part set across the Channel in curves more or less bent as the spot is more or less removed from the offing; so that there seems to be but one hour's tide each way that passes clean down the Channel from Beachy Head to Scilly, and round the Land's End to Bristol. The outgoing stream from Beachy Head encounters the ingoing stream of the offing tide somewhere about the Start Point, and both are turned down into the great Gulf of St. Malo, which seems to receive the accumulated waters of these opposite tides.

Whether or not this influx is instrumental in raising the water here to the extraordinary height of forty-seven feet perpendicular range at springs, or whether it be owing to its form and position as regards the advancing tide wave, is a problem; but it is a coincidence that cannot escape observation, that this spot, like the Bristol Channel, is the concentration of streams from opposite directions; that it has its waters raised to the same extraordinary elevation nearly to a foot, and that its time of high water is nearly the same.

On the change of tide, this great bay, like the Bristol Channel, as it received so it returns its waters in opposite directions, the tide splitting somewhere between Alderney and the Start; but here especially, as also in a similar locality in the Irish Channel, we are in want of observations.

In tracing these streams, it was impossible not to be impressed with the many coincidences which assimilate the tidal phenomena of the two channels, so much so as to render it probable that they are subjected to precisely the same laws.

Considering the Irish Channel to extend from a line joining the Land's End and Cape Clear to the end of the tidal flow, which is either at Morecambe Bay or Peel, in the Isle of Man; and the English Channel as reaching from a line connecting Ushant with the Land's End, to the end of its tidal flow, or to Dungeness. We shall then see that the English Channel, from its outer limit to the end of its tidal stream, is 262 geographical miles, and that the Irish Channel, from its western limit to the end of its tidal stream, is nearly the same; being about 265 geographical miles. In both channels the stream enters from the south-west, and flows up until stopped by a counter stream. In both channels there is a contraction of the strait almost midway, by the promontories of Cape La Hague in one instance, and St. David's Head in the other, and at very nearly the same distances from the entrance. This contraction is, in both cases, the commencement of the regular stream, which flows six hours nearly each way, the *turn of the stream throughout coinciding with the times of high and low water at the virtual head of the channel*, situated in both cases about 145 miles above the contraction, and that time being very nearly the same, viz., 10h. 50m. at full and change; below this contraction, away from the land, the stream in both cases varies its direction nearly every hour, according to the force exerted upon it by the opposing offing stream.

In both cases, between the contraction and the southern horn of the channel, there is situated a deep estuary, the Bristol Channel and the Bay of St. Malo, in which the times of high water coincide, and where, in both cases, the opposing streams meeting in the channel pour their waters into these gulfs, and where the tides in both places rise to the extraordinary elevation of forty-seven feet at the syzygies. From the Land's End to the meeting of these streams in the Bristol Channel is seventy-five miles, and from Brest to the meeting of the streams off Guernsey the same. A still further coincidence is apparent between the phenomena of these channels. In one, at a place called Courtown, a little above the contraction of the strait, and at 150 miles from Cape Clear (its entrance), there is scarcely any rise or fall of the water; and in the other channel (about Swanage), situated also a little above the contraction of the strait, and just 150 miles from the Land's End, there is only five feet rise of the water at a spring range. In both cases these points of small range of tide are situated on the opposite side of the channel to that of the high elevation above-mentioned, and in both cases these spots are the node of the tide-wave (on either side of which the times of high and low water are reversed). And again we trace a similarity in an increased rise of the water on the south-east sides of both channels abreast of the virtual head of the tide: at Liverpool in one case, where the range amounts to thirty-two feet, and at Cayeux in the other, where it is thirty-four feet.

It may also be shown that the progress of the tide-wave along the side of the channels opposite the node is not very dissimilar. Reckoning in both cases from the line which we have before drawn, as the outer limits of the channel, we find that in the English Channel, from this line to Cherbourg, opposite the small range of tide,—

	Miles per hour.		Miles.
The wave travels	50	and if complete would be in extent	616
In the Irish Channel, from a similar line to Bardsey, it travels	52		649
From Cherbourg to Havre	32		397
From Bardsey to Holyhead	16		193
From Holyhead to end of tide	78		959
Dieppe to the end of the tide	75		922

These numbers are given roughly, merely for the purpose of showing the general resemblance in the character and motion of the wave; and it is probable a more judicious selection of positions and numbers would give a still nearer coincidence. Besides which we are somewhat uncertain as to the establishment at our starting-point. As a comparison, however, the numbers run fairly together. In both cases the retardation of the tide-wave about mid-channel, and the great elongation of the wave towards the end of the strait are remarkable, especially in the Irish Sea.

Lastly, we may notice a singular coincidence in more respects than one, indeed, between the situation of the node placed by Professor Whewell in the North Sea, and a corresponding point of small range and inversion of tide at the back of Kintire. The node or hinge of the tide in the North Sea is curiously enough situated as nearly as possible at *the same distance from the head of the tide off Dungeness, as the node at or near Swanage* is on the opposite side of it; and the node at Kintire communicated by Captain Robinson, is about the same distance from the meeting of the tide in the Irish Sea as the North Sea node is from the meeting of the waters off Dungeness, and is similarly *situated with respect to the node of Courtown as the North Sea node is with regard to Swanage.*

*Report of further Observations upon the Tidal Streams of the North Sea and English Channel, with Remarks upon the Laws by which those Streams appear to be governed.** By Captain Beechey, R.N.

The method pursued in making these observations was to anchor the vessel at each of the stations for twelve hours and upwards, and to observe the direction of the tide, flood and ebb, every half hour. The rate of the stream, in addition to the usual method by the common log, was detected by current logs, constructed for the purpose by Mr. Massey, and which registered feet; and that it might be ascertained whether the stream was confined to the surface or extended to the depth of a vessel's hull in the water, another of these logs was occasionally sunk twelve feet, and registered simultaneously with the one which was two feet beneath the surface.

The time of slack water, as well as the times of the cessation and commencement of the stream, were noted as closely as such observations are capable of being made; and to render the times more certain, I have taken the mean interval between the time of the cessation of one stream and the time of commencement of the next.

Instead of these channels having, as has been hitherto supposed, a stream which turns progressively later as the tide advances up the strait, these observations have shown that the progressive changes of stream cease at a certain point near the mouth of the Channel, and that beyond that spot there is a tide peculiar to the channel, and quite distinct from that of the seas on either side of it; so that there may be said to be two distinct streams on each side of the Strait of Dover,—the stream of the Ocean or outer stream, and the stream of the Channel, or that which is contained between the Oceanic stream and the Strait of Dover. These streams are always running in contrary directions, and in the North Sea meet between the Texel and the estuary of Lynn; and in the English Channel between the Start and the Gulf of St. Malo.

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* *Philosophical Transactions*, May, 1851.

In the locality where these streams meet, the tide is ever varying its direction according as the strength of one stream prevails over that of the other, giving to the water a rotatory motion, and scarcely admitting of any interval of slackwater, whilst in the spaces between these rotatory tides and the point of meeting of the tides in the Strait of Dover, the stream is free from all rotatory motion, and sets steadily throughout the tide in a direction *towards* Dover, while the water is rising there, and *away* from it while it is *falling at* that place.

I have designated the last-mentioned *the true Channel stream*, and its extent is, as nearly as it can be measured, 180 miles in either direction from the point of union of the tides in the Strait of Dover to the region of rotatory tides off Lynn, and off the Start and St. Malo.

As the true Channel streams are always running in opposite courses, there is necessarily a point where they meet and separate, and this occurs in the Strait of Dover; but in this strait the stream, although it first obeys one tide and then the other, does not slack with the Channel streams, but is found to be still running at high and low water on the shore, at which time those streams are at rest, so that the Strait of Dover never has slack water throughout its whole extent at any time. I have in consequence called this "*an intermediate tide*."

The limits of neither of these streams appear to be stationary, but *range to and fro as the tide rises and falls at Dover*, travelling to the *eastwards* on *both* sides, and at high and low water suddenly shifting sixty miles to the westward to re-commence their easterly courses with the next tide; and although so far apart, they possess the remarkable peculiarity of shifting together; so that the Channel streams preserve as nearly as possible the same relative dimensions.

In the Strait of Dover this line of meeting and of separation oscillates between Beachy Head and the North Foreland, a distance of about sixty miles. When the water on the shore at Dover begins to fall, a separation of the Channel streams begins off Beachy Head. As the fall continues, this line creeps to the *eastward*. At two hours after high water it has reached Hastings; at three hours, Rye; and thus it travels on until at low water, by the shore, it has nearly arrived at the North Foreland on one side of the strait, and at Dunkirk on the other. At this time the Channel streams on both sides slack, but in that portion which I call the "intermediate stream" in the Strait of Dover, the water is still running to the westward; and when the new channel streams make as the water rises on the shore, this intermediate point is found to unite with, or to oppose one or the other of these streams, according as it was before the reverse; so that, as before mentioned, the line of meeting at low water appears off Beachy Head to recommence its easterly course. This intermediate stream forms a remarkable feature in the tidal system of the Channel;—it is well established, as the line of meeting and of separation occupies a very limited space, and it seems to be entirely due to the contracted form of the Channel in this immediate locality, preventing the free escape of the water.

Captain Bullock, in order to test the point of separation, anchored two vessels a mile apart between Beachy Head and Dungeness, and found both vessels at the same moment to ride with their heads in opposite directions, in obedience to the streams which were then running opposite ways.

The Channel stream which I have described as running between the intermediate stream, and the rotatory or mixed streams at the outer extremities of the Channel, pursues a steady course along the main trunk of the strait, slacking only towards high and low water at Dover, when it is preparing to invert its course; and, contrary to the generally received opinion of a progressive slack water in a strait having a progressive establishment, this stream has the peculiarity of slacking throughout its whole extent at *nearly the same time*, and this time, as was anticipated in my former paper, corresponds nearly with the time of high and low water on the shore at Dover, the site of the combined wave, and of the *vertical head of the tide*.

* * * * * * * *

It will be seen on a very cursory inspection of the charts which accompany this paper, that for a period of six hours after high water at Dover, and for five hours before that time, the stream maintains a steady direction from and towards Dover; so that any delay which may exist between the change of the stream and the time of high water, is shown by these charts alone to be confined within very narrow limits, but a still closer agreement will be found on referring to the observations themselves, especially if we take those which were made on board the light vessels at a distance from the shore, which are particularly valuable from the circumstance of their being spread over intervals of several days at each station.

Referring, then, to these observations, and taking them in the order in which the light vessels are placed with respect to the progress of the tide wave, we shall have the intervals between the turn of the stream at those places and the time of high water at Dover (the standard) as follows :—

At Newarp, L.V., the stream is slack 22 minutes before Dover.
At the Cockle, L.V., the stream is slack at the same time as Dover.
At the Stanford, L.V., the stream is slack 9 minutes before Dover.
At two stations nearer Dover, the stream is slack 24 minutes after Dover.
At the Shipwash, L.V., the stream is slack 24 minutes after Dover.
At the Galloper, L.V., the stream is slack the same time as Dover.
At the Kentish Knock, L.V., the stream is slack 9 minutes after Dover.
Lastly, off the North Foreland, L.V., the stream is slack same time as Dover.

* * * * * * * *

To the eastward of 3° of longitude, the times of slack water get considerably later in that part of the channel which lies between the northern limit of the Thames and the Texel, and the phase of the stream in all this portion of the North Sea appears to be inverted; the intervals also to get longer as the Dutch coast is approached. The occurrence of this inversion of the phase of the stream with the inversion of the phase of the establishment on the coast of Holland, and its position occupying the site of the node of the tide in Professor Whewell's chart of cotidal lines, is a coincidence which ought not to be overlooked, although I am not able to discover any connexion between the phenomenon, which is evidently the cause of the unusual retardation of the stream above-mentioned, and the phenomena of the cotidal chart.

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If, then, we except such observations as are shown to be due to the disturbance from the Texel tide, we shall find that in *the whole extent of channel from Cromer to the North Foreland, there is not half-an-hour of retardation in the time of slack water from the time of high water at Dover, while in the establishments there is an increase of at least five hours.* That we may appreciate this very small difference of the time of slack water throughout the channel and the time of high water at Dover, let us inquire what takes place in parts of the North Sea, which exhibit the ordinary phenomena of tides under the influence of the progressive wave, such, for instance, as are comprised in the space between Kinnaird's Head and the estuary of Lynn. There is in this space an increase of tidal establishment of five hours, an amount about equal to that which exists between Lynn and Dover. By the Admiralty tide tables it will be seen that in this space there are nearly five hours' retardation in the time of slack water in the offing, making at least *hour for hour in the change of the stream with the change of establishment;* whilst in the North Sea, in the same change of establishment, there is scarcely *half-an-hour difference of time in the change of the stream.* If we now take the English Channel, we shall find that half-way between the Start and Alderney the stream *turns with the high water at Dover.* Off Portland, 9 *minutes before;* off St. Alban's, 3 *minutes before;* between the Isle of Wight and Cape Barfleur, 15 *minutes after;* and so we run on, the differences *varying from* 8 *minutes one way to an extreme of* 42 *minutes the other.* In the Baie of la Seine the turn of the stream is earlier in the western part than towards its eastern extremity, but the formation of the coast about Cape Barfleur will fully account for this.

* * * * * * * *

Upon the whole, therefore, judging from the observations that have been collected, there seems to be sufficient evidence to warrant the conclusion, that with the exceptions arising from extraneous causes, which are now known, and for which an allowance can be made, the streams of the channel turn sufficiently near to the times of high water on the shore of Dover to be considered as simultaneous, without occasioning any error that can be considered of the smallest consequence to shipping."

With regard to the rotatory motion of the stream in the English Channel and the North Sea, Admiral Beechey observes that there can no longer be any doubt that the rotatory tides are the exceptions to the general courses of the stream, and not the rule.

With regard to the time at which the stream attains its maximum rate in the Channel, he is of opinion from numerous observations, that the stream attains its greatest strength at about the time of *half tide*, Dover, and not, as supposed, at high and low water on shore.

I have now only to mention a popular error which still exists with reference to the streams of our channels; viz., a belief in a "*tide and half tide,*" by which it is understood that the stream runs half a tide longer near the centre of the channel than it does near the coast. So general an opinion requires substantial proof to invalidate it, and I must refer once more to the diagrams which accompany this report. It will be seen then, that in no part of either channel does the change of stream in the centre differ even an hour from the change of stream, as near the land as a vessel would be safe in navigating, except in that part of the North Sea which is affected by the tide sweeping round the Texel, and then at the most it amounts only to a quarter tide. It is not difficult to discover that this error has arisen from the confusion of high water on the shore with the time of slack water in the offing.

* * * * *

"Under ordinary circumstances, such as those which attend the passage of a single tide wave up an estuary or deep gulf, or even along an open coast, we find a progressive turn of the stream to accompany a progressive increase of establishment. But in the channels under discussion the stream is found to turn nearly simultaneously throughout the strait, wholly regardless of the order of its establishments, and to reverse with the time at which a wave peculiar to such channels is matured.

This wave, which exercises so singular an influence over the streams of these channels is occasioned by one portion of the Oceanic tide wave of the Atlantic passing round the north end of the British Islands, while the other finds its way up the English Channel and meets in the Strait of Dover. In consequence of this meeting or opposition, the character of the wave is changed; its dimensions become diminished just one-half in extent, its rate of travelling is reduced a like quantity, and its streams are almost always in opposition to those of the outer or parent wave."

* * * * * * * *

Referring to "Plan 4," he observes:—

"At V. hours the crests of two waves are seen to be formed a little beyond the Deadman on one side and at the Spurn Point on the other, and the foot of the wave or hollow which they create is resting at Dover. Following the progress of these two crests down the page, we find them gradually approaching each other and wearing out the depression in advance of them, until between VIII. and IX. o'clock the hollow has vanished; at X. o'clock there is a decided convexity of surface, which gradually increases until at XI. o'clock the crest has attained its zenith, the combined wave is perfected, and it is high water at Dover.

If we now follow the wave as the tide falls, we find the crest to become more oblate, and finally to be obliterated without there being any material progress of the wave in either direction."

Keeping in sight some of the principal features on which this eminent practical philosopher has observed, it will be seen that this meeting of the two waves occurs in both the Irish and British Channels, and they have the appearance rather of a stationary than a travelling wave, and the great rise of the tide compared with the small depth of the seas, makes the currents very important features.

There will be a similar result in a wave travelling in a gulf; the current will change its direction at half tide at the head of the gulf. So long as the current is running up, the tide must rise, for there is no escape for the water; but as soon as the current turns, the tide will fall, from the receding of the water.

Some of the effects caused by the converging forms of a gulf may be noticed in the Bristol Channel; a branch of the tidal wave of the Irish Sea is cut off by Pembrokeshire, and is diverted up this channel; for ordinary spring tides, at St. Gowan's Head, the southern point of Pembrokeshire, the rise is 24 feet; at Padstow, in Cornwall, 22 feet; at Lundy Island, a little up the channel, 27 feet; at Worm's Head, Carmarthenshire, 28 feet; Swansea Bay, 30 feet; Nash Point, 33 feet; Minehead and Bridgewater Bay, 35 feet; Cardiff, 38 feet; and Portishead, 44 feet.

Mean Tide Level.—With regard to mean tide level, it is to be observed, that this mean tide level is a point half-way between high and low water on the *coast;* the tide is an oscillation equally above and below this level. It has been ascertained that the level of half tide in the Bristol Channel, where, as we have just seen, the range of the tide is so considerable, is the same as on the south coast of Devon, where the range is 13 feet; it has also been ascertained that on the coast of Ireland the mean tide level is the same everywhere, though liable, however, to be affected by such causes as the winds.

The reader who is desirous of obtaining knowledge as to the tides in the English estuaries and tidal rivers, can refer to Mr. Beardmore's hydraulic tables, in which work he will find many pages devoted to much valuable and original matter on this very interesting and practically useful portion of Marine Surveying.

We must close this portion of our subject with an extract from *The Tides*, referring to the Indian Ocean.

> We now pass to the west side of the Island of Sumatra. Here, as at our starting-point in the Pacific, we have the equator passing from the land to an open ocean, and we might reasonably suspect a new origin for the tide wave.
>
> There are, however, great differences between this region and that of the American coast. In the first place, the northern part of the tropical belt lies in great measure upon the continent of Asia, so that the moon, when in north declination, passes over land, and the formation of a tide wave by attraction would be much obstructed. The southern part is more free, but the extent of ocean under the direct influence of the sun and moon is still much less than in the Pacific. The circumstances, therefore, are not so favourable for the formation of a tide wave by direct attraction.
>
> On the other hand, this sea, by its geographical position, is much more accessible to the tide wave of the Pacific, than the Pacific is to that of the Atlantic; the southernmost point of Van Dieman's Land being 10 degrees further north than Cape Horn, while there are also passages, though intricate ones, among the islands of the Asiatic Archipelago.
>
> We may expect to find, then, that the tide of the Indian Ocean has rather the character of a wave which has travelled from the Pacific by these various routes, than of the direct wave raised by attraction in its own waters.
>
> We have seen that the direct wave, even in the most advantageous circumstances, is very small, while the derived, or travelling wave, has mounted in certain cases to the height of 15, 20, or even upwards of 30 feet. It is to this latter, then, that we must look as the main source of the tides of the Indian Ocean.
>
> If we suppose the main body of the wave to enter the Indian Ocean from the south-east, we shall find that its features on the eastern shores are very much affected by the channels between the islands of the Archipelago, which at once permit the exit of branches of the Indian Ocean wave, and the entrance of branches from the waves of the Pacific and China Seas.
>
> The tides, therefore, all along the shores of Western Australia, of Java, Sumatra, Malacca, and Burmah, will necessarily be much affected by the combination of these various branches, and we can scarcely expect to find the means of tracing the main wave by the shore tides.

The tides on the coast of Northern Australia, between the meridians of longitude 120° and 130° we stated to have been high. We have no information of the tides north of this nearer than the south-west coast of Sumatra, where the wave is only 2 to 3 feet high, and it vanishes altogether in the northern part of the Straits of Malacca. At Singapore, the southern extremity of these straits, it has peculiarities, described in the last chapter.

We can now trace some kind of a regular progress for the wave. At the Nicobar Islands it is high water at new and full moon at a quarter past nine, and the tide rises 8 or 9 feet. At the Andaman Islands, due north, the tide is a few minutes later. About the islands of the Merqui Archipelago, off the coast of Tenasserim, and on the main coast itself, the arrival of the wave is somewhat later, being probably retarded by the shallowness of the ocean on approaching the coast. Its height, however, is increased to 18 or 19 feet, and at the head of the Martaban Gulf it rises as much as 24 feet, the greatest tide being here two days after new or full moon, making the tide two days old.

At Cape Negrais, the western extremity of the Gulf of Martaban, the rise is 10 feet, little more than at the Andaman Islands. The time is about an hour later. It takes another hour to get to Cheduba, 200 miles north, where the rise is about the same. The range increases to 12 feet at Chittagong, at the head of the Bay of Bengal. Hence it flows up the various mouths of the Ganges. The tide wave exhibits no regular progress along the eastern shore of Hindostan, and the rise is small.

At the Chagos Islands, lying due south of Hindostan, in latitude 6 degrees south, the tide rises 5 to 6 feet, and it is high water at new and full moon at half-past one, which is nearly five hours later (making allowance for the difference of longitude) than the tide on the coast of Australia.

At the Maldive Islands, lying 10 degrees farther north, the tide exhibits a curious feature. The group of islands (which lie in a line due north and south, extending over a space of 300 miles) is surrounded by a coral reef, which only admits the tide wave to enter freely from the north. Here the time of high water at new and full moon is half-past nine, and it becomes successively later in going southward, till at the southernmost islands the time is one o'clock. This direction is exactly contrary to the general progress of the tide wave. The range is about 5 feet.

At Cochin, on the opposite shore of Hindostan, the range is only 3 feet. The tide wave from thence travels northward. At Goa the time is eleven hours forty-five minutes, and the range is 5 feet. Beyond this the features strikingly resemble those of the coasts of Malacca and Burmah. The tide becomes later as we go north, and the range increases. At Bombay it rises from 14 to 17 feet, at Surat 21 feet, while at the head of the Gulf of Cambay it rises 30 to 36 feet. Passing on, however, to the mouth of the Indus, the range is again reduced to 9 feet. It will be seen upon the map that the conformations of these two coasts are very similar, the Gulf of Martaban being represented by the Gulf of Cambay. In each case the tide in the gulf becomes very large, while the portions of the waves which pass clear of the gulfs retain their original dimensions.

At Bassadore, near the entrance to the Persian Gulf, spring tides rise 9 feet 6 inches, and neaps 3 feet 6 inches, and here a peculiarity occurs similar to that which we noticed on the western coast of Australia. In the Gulf of Cambay there is a diurnal inequality of 7 or 8 feet in height, but the two tides arrive at their regular intervals; there is no inequality of time. At Bassadore the two tides are of the same height, but the one arrives proportionally two hours earlier than the other.

We may remark here, as we did of the Australian tides, that this difference in the manifestation of the diurnal inequality may easily be accounted for, by supposing the two waves, having been originally formed in different latitudes, to take somewhat different routes. One being perhaps a little more easterly than the other, may find a less obstructed path in the ocean than that which is more directed along the shore; it may thus arrive sooner, and be smaller when it arrives at Bassadore. In its route to the Gulf of Cambay, it may meet with circumstances which retard it and increase its dimensions, in an equal degree with those which by a different route have affected the other.

This is a mere conjecture, which more perfect knowledge may refute or confirm, but we mention it to show how terrestrial causes may affect differently tides formed on different parts of the earth's surface. The tide is here also two days old.

The tide wave flows up the Persian Gulf, and at the head there is a rise of 7½ feet.

The progress of the wave along the southern coast of Arabia is generally to the westward, but it is stated to be irregular; probably its continuous movement is interfered with by meeting a more direct portion of the wave travelling from the south-east. The general range is from 6 to 8 feet; the time, at new and full moon eight to ten o'clock, and at the Straits of Babelmandel twelve o'clock.

There are but few observations of the tide in the Red Sea, and the range is small. It is probable that there is a peculiar movement of the wave, of the exact nature of which we cannot judge without more observations. The time of high water at Suez appears to be nearly the same as at the Loheia River, a little within the straits. It is scarcely likely that the wave travels over this space in twelve hours, and the small rise and irregular time near Mecca would seem to indicate a movement somewhat similar to that which we find on our own coasts, and which we shall describe in the sequel.

The tide wave appears to fall directly upon the east coast of Africa, making high water generally along the coast at nearly the same time, between one and two o'clock by Greenwich time; except in some cases, where either the wave is retarded by the conformation of the ocean bed, or by some peculiar undulation. The range is pretty large in the Mozambique Channel and for some distance to the northward, being from 10 to 12 feet.

We have been compelled to extend our remarks on the subject of the tides much beyond what was our intention at first, but we have found it necessary, in order to impress upon our readers the importance of all the observations they may have to make on the subject, and at the same time to afford some clue as to that which may be required, either in the tidal survey of two or three spots at some distance from each other, or along the whole length of a line of coast, or up a tidal river or estuary.

In the United Kingdom, and in Europe generally, there is so much to be obtained from public documents, such, for instance, as the Admiralty Charts, that the surveyor has only as it were to fill up gaps occasioned by the want of details; but where he has also to find out the leading features as well as the details as to the tides and currents, the work is often laborious, and must necessarily extend over a considerable period of time.

One, two, or a much greater number of tide gauges may be required, not only to ascertain the total rise and fall of the tide at different places, but the intermediate heights between high and low water, and often particularly half tide by time.

One step that may be first taken, is a careful set of levels to different spots near low-water mark, and to ascertain by observation the total ranges of the tide at each place, both as regards springs and neaps; we shall then know where the gauges will be required.

In the surveys of tidal rivers, and of estuaries, the numerous bends, contractions, and expansions of the shores, and often of projecting bluffs under water, or any marked changes in the bed, either longitudinally or transversely, will all be found to affect more or less the parallelism of the tidal wave; and here again sets of tide gauges, all referring to one carefully ascertained level will be required, for the bulks or velocities of the tides will be interfered with at such spots, and consequently the parallelism of the wave; the greater the rise of the tide, and the more rapid the currents, the more numerous the tide stations will often have to be.

The tide-staff erected at each station will require to be from six to nine inches wide, and steadied in a strong frame for its support and safety, unless it can be fixed against some rock or bank, having its foot reaching to low-water mark; the foot of the staff should be sunk some two or three feet below the lowest reputed low-water mark; in length the staff should be some four or five feet higher than the highest reputed spring-tide; it should be painted white and divided into feet, and the numbers of these should be shown in large bold figures, that they may be seen at some distance; each foot may be subdivided by lines into halves and quarters, but no figures shown for inches.

On a *long* shelving bank, or where the rise of the tide is great, the staff may be divided into short equal lengths, and carried along, as it were, in steps, the foot of each length being carefully adjusted by levelling, to correspond with the top of the length of staff below. A careful, trustworthy man should be appointed to each staff, and his observations should be taken every ten minutes; he should, therefore, be provided with a watch. With regard to high and low water, he should not only watch the highest rise and greatest fall, but also the time of the change of the current. A set of tidal observations, to be of any value, must extend over a considerable period of time, and will of course be expensive; unless great care be observed, numerous errors will creep in; the whole may therefore be invalidated, and so much money thrown away.

The staff above described will be sufficient, where it can be so placed as to be protected from the undulations of waves; but for harbours and coast work, it will be necessary to shelter the staff by means of a piece of stout glass, between which and the staff the water may rise and fall without being affected by the undulation of the wave. A large hollow copper ball has been used, rising and falling within a casing; to the ball is attached a chain, working by a kind of clock-work on a dial-plate.

For measuring the velocities of currents moving at the rate of above two miles an hour, Massey's patent log will give all that is required. Below this velocity, Elliott's Current Meter, which we shall describe in another place, is preferable.

It need not be added that, in taking the velocity of a current, its direction must at the same time be observed, as also the spot from whence the observation has been made. To perform this, ample instructions have already been given.

An illustration has been given of the method adopted to show the rise and fall of the tides on paper. On the horizontal line equal distances denote equal times, and the reverse; they may represent hours, days or months, or fractions of them. The rise or fall is plotted on the vertical lines corresponding to relative time.

CHAPTER XIV.

Survey of Gathering Grounds for Water Supply.—General Observations.—Gathering Grounds.—Objects of the Survey.—Rainfall.—Mean annual depth of Rain, with Notes.—Observations on the Causes of Differences.—Rain Gauges; the Gauge Glass.—Variations in depth of Rainfall.—Position of Rain Gauges.—Discharge from Rivers and Streams, and from large Districts.—Stream Gauging.—Theoretical Discharge.—Contracted Vein.—Rules for Weirs and Overfalls.—Coefficients.—Initial Velocity.—Establishment of Gauges.—Surface and Mean Velocities.—Current Meters.

PUMPING from wells sunk to a certain depth, from rivers with or without storage reservoirs, and a system of gravitation from reservoirs supplied from extensive gathering grounds; these singly, or combined, and modified in various ways, are the principal methods adopted by engineers for supplying towns with water; often also for canals, and on a smaller scale they are likewise adopted to obtain a supply for mills and extensive manufactories where large quantities of water are required.

With very few exceptions, more particularly with regard to wells and reservoirs supplied from gathering grounds, the surveying engineer has not only to ascertain the quantity of water that may be obtained from the above sources; he has also to define the quantity of water already appropriated by existing interests, how they will be affected by the quantity proposed to be withdrawn, and how compensation can be made in the best manner for all parties. Wells are generally sunk through some overlying impermeable stratum, as clay or rock, down to some water-bearing bed from which the fluid will rise; or rather the well is sunk to a certain depth to receive the uprising water, which is then tapped by means of a bore-hole, which may be six or even eighteen inches in diameter, as may be required, and deep enough to reach from the bottom of the well or store to the water-bed; the only way to ascertain the quantity of water obtainable by this means is by direct experiment.

A gathering ground, properly so called, is any area of land the rain-fall on which may be collected; it is always more or less affected by certain causes, such as geological formation, the dip of the strata, the altitude, geographical position, the character of surface-soil, or of the substratum, &c.

The storing of water from a gathering ground into a reservoir, and sometimes a chain of reservoirs, is effected in the most economical manner by embanking across one or several valleys; it will therefore be readily seen that in most cases the land survey required will be more or less extensive, for all the valleys leading to that to be embanked, will have to be shown on the plan, and very often also those separated from it by hills on one or both sides; a scale of from one to three inches to one mile is sufficient for this purpose, and in England the Ordnance maps are generally used for the purpose of showing the lines of watershed, their areas, and the position of reservoirs; the lines of watershed show the boundaries of the lands on which rain falling, may by gravitation be expected to flow towards the streams and rivers by which the waters of the valleys are discharged.

It will not be difficult to perceive why the parallel valleys have often to be surveyed, when it is considered that a short heading driven through the intermediate hills into such valleys may often be the means of leading considerable quantities of water into the reservoir; similarly, feeders may be laid to effect a like object by bringing rain or spring waters into the reservoir valley, at a point much higher than that they would otherwise flow to.

High lands are much more favourable to the system of storage from gathering grounds than a low lying country, where not only the rainfall is smaller, but where also the ground is much less favourable for the construction of reservoirs. The consequence of this is, that whilst low flat districts are mostly supplied by pumping from rivers, towns in more elevated regions are supplied by gravitation from reservoirs constructed in valleys collecting water from gathering grounds.

The dip of the strata, in many hills surrounding a valley, is often such, that the rainfall due to the reservoir valley may be appropriated by another, by reason of the strata on one or both sides dipping in opposite directions, or away from the valley where it is proposed to impound the water; for evidently in such a case the percolating water may be carried away in an opposite direction, unless caught or brought back in the manner above suggested. Or we may have the reverse of this, by reason of the geological stratification being such, that a much greater quantity of water is brought within the drainage area, than that due from mere rainfall; in this latter case, however, the water will make its appearance in the shape of one or more springs, sometimes at a low level, and at other times at a much higher level than might be at first expected.

The above remarks will be sufficient to indicate that the following features will all have more or less to be carefully investigated, and the following points kept in view.

1. It is desirable to obtain as much information as possible as to the nature of the geological formation and its structure as regards material, inclination, and faults, as bearing on the quantity and quality of the water, as regards at least the supply of towns.

2. The nature of the surface soil; as meadow, grass, barren lands; decomposing peat, lands richly manured or thickly wooded, such as are thickly populated, or more or less appropriated by unhealthy manufactories, or such as may affect the quality of the water proposed to be supplied for any town; where the areas of any of the above descriptions of land have any extent, such areas often require to be shown on a general map, such as the Ordnance, and also to have their contents computed.

3. The area of the whole watershed.

4. The rain-fall per annum and during the dry months.

5. The quantity of water discharged by streams.

6. The compensation already alluded to for withdrawing water, more or less appropriated by existing and competing interests.

7. To these have sometimes to be added the fall in the streams, as water power, and its value.

Finally, a close investigation will have to be made of that most important question, which has already been pointed out in another portion of this volume as being of such vital consequence to any engineering project, namely, the kind and quality of the materials on the spot, or to be obtained, and from whence; and more particularly in the case of reservoirs, as regards clay or loam for puddle; the materials of which embankments are to be formed, and as to whether the reservoir will require lining to make it water-tight; for reservoirs connected with "towns' supply," we must not neglect to look into the question of sand and gravel for the construction of filtering beds. Many other incidental matters will arise, in the course of the survey, but the above are amongst the most important.

There are three questions principally which it is here proposed to assist the student in investigating; viz., the depth of rainfall, the quantity of water discharged by streams, and that from rivers.

The annual depth of rainfall varies geographically, locally, and periodically; in tropical regions it is five or six times what it is in some parts of our temperate climate; even in our northern and elevated districts it is often three or four times as great as what it is about London; one side of a hill will often have a much greater depth of rainfall than another; even in two adjoining fields, circumstances may be such as to cause a difference. Year by year the depth of rain varies in the same place; it has been recorded that at Blackstone Edge, in Lancashire, the rainfall in 1826 was only 19·6 inches, whilst in 1836 it was 41¼ inches; it varies also with the season, one portion of the year being much more wet than another.

The following table, taken from various authorities, and extending over a number of years, will show some of the anomalies that have to be met with on this subject.

Mean Annual Depths of Rainfall, in inches, at the undermentioned places.

Notes.	Height above Sea. Feet.		Inches.
1...		Abbot's Hill, Herts.	26·61
	1300	Allenshead, near Newcastle-upon-Tyne	47 90
		Applegarth, Dumfries	33·80
		Aylesbury, Bucks...	28·40
		Barrowby, Yorkshire	27·50
		Bath, Somersetshire	32·40
		Belfast	34·96
2...	1200	Blackstone Edge, Lancashire	36·29
3...	320	Bolton, „	49·00
	30	Boston, Lincolnshire	23·10
		Bridgewater, Somersetshire	29·30
		Bristol	29 20
4...	300	Bury, Lancashire...	41·72
		Carlops, near Edinburgh	48·82
		Castlecomer	37·80
		Chapel-en-le-Frith, Derbyshire	43·00
		Chatsworth	27·80
		Chiswick, Middlesex	24·00
		Cobham Lodge, Surrey	24·50
		Cockermouth, Cumberland	45·40
	720	Coombs, Lancashire	45·80
		Cork	41·00
		Crawley, near Edinburgh	37·00
5...		Crawshawbooth, Lancashire	60·00
		Cuba	141·00
		Dickleborough, Norfolk	25·00
		Derry	31·12
6...		Dover, Kent	37·50

Notes.—No. 1, Near King's Langley, 1837, 21·10; 1841, 32·10. No. 2, On roof of house at Reservoir of Rochdale Canal, six miles N.E. of Rochdale, near summit of mountain, ranging N.E. and S.W., 1844, 24·80; 1833, 47·37. No. 3, 1844, 34·63; 1832, 53·77, staff gauge. No. 4, 1844, 28·65; 1833, 50·61; staff gauge. No. 5, near Haslingden, said to be the highest market-town in England. No. 6, 1789, 16·30; 1792, 51·40.

Mean Annual Depths of Rainfall—continued.

Notes.	Height above Sea. Feet.		Inches.
		Dublin	26·41
7...		Dumfries	37·00
8...		Edinburgh	23·40
		Epping, Essex	26·60
	141	Exeter, Devonshire.	29·20
9...	220	Fairfield, Lancashire	34·84
		Falmouth	40·72
		Felthorp, Norfolk	22·60
10...		Fell-Foot, Westmoreland	55·70
		Fyfield, Hampshire.	25·90
		Garsdale	52·30
	326	Gatesgarth, Westmoreland	117·20
		Genoa	56·00
	600	Gilmourton, Lanarkshire	47·70
	8	Glasgow	33·60
	734	Glencorse, Pentland Hills	36·11
	800	Goodamoor, Devonshire	56·80
	30	Gosport, Hampshire	30·20
	180	Grasmere, Westmoreland	107·50
	2925	Great Gable, do.	89·40
		Greenock, Frith of Clyde	65·00
	143	Greenwich Observatory	23·93
		Hastings, Sussex	31·98
		Honiton, Devonshire	33·20

Notes.	Height above Sea. Feet.		Inches.
	320	Hungerford, Berkshire	26·58
	320	Hyde, Lancashire	35·20
11...		Kendal, Westmoreland	58·10
12...	258	Keswick, Cumberland	67·50
		Kimbolton, Huntingdonshire	25·00
13...		Kirkmichael	40·78
		Lille, North of France	29·00
14...	41	Liverpool	34·40
		London (Howard's average)	24·80
		Ludguan, Cornwall	41·00
		Lyndon, Rutlandshire	24·30
		Lyons, France	37·00
		Manchester	36·50
15...	500	Marple, Lancashire	36·56
		Milan, North Italy	37·50
	510	Moss Lock, Lancashire	27·24
	121	Newcastle-upon-Tyne (2 years)	17·60
		Norwich	25·50
		Nottingham, Highfield House	27·00
		Paris, France	26·60
		Paisley	56·00
		Pencarrow, Cornwall	45·30
	40	Penzance „	43·10

Notes.—No. 7, 1788, 26·42; 1789, 48·09. No. 8, Roof of Mr. Adie's house, 1826, 15·27; 1827, 32·59. No. 9, Three miles E. of Manchester, 1844, 26·35; 1847, 40·75, staff gauge. No. 10, South end of Winandermere, at foot of enclosing hills; 1788, 42·06; 1789, 66·52. No. 11, 1788, 39·25; 1792, 84·88. No. 12, 1788, 34·30; 1792, 84·60. No. 13, eight miles N.E. of Dumfries, near foot of high mountains. No. 14, On roof of house; 1780, 24·84; 1792, 54·24. No. 15, Four and a half miles S.E. of Manchester; 1844, 29·40; 1847, 43·70; staff gauge.

Mean Annual Depths of Rainfall—continued.

Notes.	Height above Sea. Feet.		Inches.
		Piza, Italy	49·00
	30	Plymouth, Devonshire	35·70
16...		Preston, Lancashire	... ...
		Reading, Berkshire	25·40
17...	500	Rochdale, Lancashire	46·75
		Rome	33·50
		Seathwaite, Westmoreland	140·60
		Selborne, Hampshire	37·20
18...	850	Sharples, Lancashire	53·92
		Southampton	30·34
19...	268	Sowerby Bridge, Lancashire	28·80
	1900	Sparkling Tarn, Westmoreland	124·00
20...	364	Stubbins, Lancashire	32·83
		St. Breock, Cornwall	41·00
	1290	Styehead, Westmoreland	92·80
		Swaffham Balbeck, Cambridge	23·80
	50	Tottenham, Middlesex	24·80
		Townley, Lancashire	41·00
		Upminster, Essex	19·50
		Venice, Italy	31·50
		Viviers, S.E. of France	34·00
21...		Waith-Sutton, Westmoreland	46·00
	276	West Denton, near Newcastle-upon-Tyne	36·80
22...		Whaley, Cheshire	... ...
	90	Whitehaven, Cumberland	47·00
		York	22·28

Mr. Beardmore's table enumerates 55 different places. Dividing the year into three periods, as Winter, Spring, and Summer, and calling November, December, January, and February, Winter; March, April, May, and June, Spring; and July, August, September, and October, Summer, his table shows that for the mean, in 8 different places, the rainfall is greatest in Winter; and that in 21 places it is greatest in the Summer months; and

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Notes.—No. 16, *a*, *b*, and *c*, at Rivington; *a*, 410 feet, 48·83; *b*, 710 fect, 46·48; *c*, 750 feet, 45·96; *d*, *e*, and *f*, in Boddlesworth valley; *d*, 550 feet, 50·22; *e*, 700 feet, 57·10; *f*, 900 feet, 52·53; all in 1847. No. 17, S.W. of Rochdale, 1844, 34·41; 1836, 61·11. No. 18, Belmont Spring-water Reservoir, near Bolton; 1846, 49·8; 1843, 63·4. No. 19, Seven miles N.E. of gauge, at Blackstone Edge, *see ante*, at considerable distance from mountain. No. 20, Near Todmorden, in a narrow tortuous valley. No. 21, Seven miles S.W. of Kendal. No. 22, Todd's Brook, Brinks, three and a half miles S.W. of Whaley, 1500 feet above level of sea, 29·5; Todd's Brook Reservoir, by Whaley, bottom of hill, 620 feet, 38·39: Coomb's Ridge, 3½ miles S.E. of Whaley, 1670 feet above level of sea, 35·85; Coomb's Reservoir, two miles S.E. of Whaley, bottom of hill, 720 feet, 51·30, staff gauge; all in 1847.

The above observations are principally due to Mr. Bateman, Mr. Beardmore, Dalton, Mr. Homersham, &c.

that the falls are equal in these two periods, or nearly so, in the remaining places; in most instances the Spring period is the dry season.

If the student will study the above table with the help of the Ordnance maps, he will be enabled to make observations by himself as to many of the above anomalies.

In England it is not often that a day's rain exceeds two inches in depth, and rarely three inches, except in some few very hilly districts particularly exposed to wet weather.

There are a great many circumstances which will cause the depth of rain to vary very greatly in some seasons in many parts of the same district, such as proximity to the sea, difference of levels, prevailing winds, and exposure to or shelter from them; it is therefore necessary to have a number of rain gauges over any extensive district, always as much as possible in the most open and unsheltered situations, and this in the widest sense of the word; it is also necessary that they be situate so as to give as nearly as possible the result of modifying causes.

As we approach hilly districts and mountainous countries, we almost always find a larger quantity of rain than in a flat country, and the same may be said of situations nearer the sea than of others farther distant from this influence. More rain, however, as a general rule, is collected at the feet of hills than on the summits, where the evaporation is greater. These subjects require mature consideration in fixing rain gauges, if we wish to come near the truth; it is not sufficient to depend upon the observations of a year or two, notwithstanding their value as an independent guide and check upon the observations of others; it is also necessary to know the amount fallen during a dry year, and also during a wet one. It is considered that more than half the annual depth of rain falls during the latter part of the year, and that the proportion is about as 3 to 2, and that the largest proportion of rain falling in the year comes down in gluts, varying from 0·4 to 2 inches in depth in less than 24 hours, and in steep hilly districts large quantities may therefore be collected in short periods, or a considerable proportion of the annual fall will probably be lost.

The proportion of the depth of the annual rainfall to be allowed to evaporation and absorption varies in different districts, and is affected by various conditions; it is greater on high than in low lands, both being in the same part of the country; but mountain-streams and torrents, which we see so rapidly flooded and again so suddenly dried up, winding their way through gorges and along the bases of steep hills, will often carry off in a few hours as much water as flows down their courses in many summer weeks during an ordinary state of things, the steep fall of the hills favouring a rapid flowing off of the water down their precipitous sides. On the high flats of table lands, particularly if covered with an absorbing material, and exposed to the action of sun and wind, much will at first be detained and then evaporated, unless means be taken to drain it off rapidly; again, of the rain falling in such situations, and thus absorbed into the surface soil, a portion will very probably gravitate through it down to cracks, fissures, channels, and chasms, until it meets an impermeable stratum, along which it will percolate until it finds its way out in the shape of springs at the sides or feet of the hills, at a greater or less distance according to the dip of the bed, the more rapid inclination of which favours the escape of the water.

The Rain-Gauge.—The instrument, by means of which the depth of rainfall is measured, is well known as the "rain-gauge," and the following is its principle of construction: if any portion of the surface of the ground on which a shower has just fallen were perfectly smooth, level, and impermeable, and also surrounded by some wall or barrier, so that no portion of the rain should escape, it would soon form a shallow pool, the depth of which could be measured with a foot-rule; this then would be the depth of rainfall *at that particular spot*, and such is the principle of the rain-gauge.

There are various descriptions of rain-gauges, the most simply constructed of which is the "kettle-gauge," which consists of a funnel with a square aperture of given area, from which the rain water flows into another vessel or reservoir beneath, where it is stored until measured, which is done by means of a graduated glass, which will be presently described; the aperture of this kettle-gauge is generally about 6 inches square.

Crosley's rain-gauge is a self-registering instrument; from the mouth of the gauge the water falls into a vibrating bucket; as soon as one side is full, the bucket oversets and another compartment is presented for the reception of the water, which being received is similarly discharged; during a fall of rain the bucket is thus kept in a state of vibration; attached to the instrument is a description of clock-work which carries a hand upon a dial-plate, by means of which the depth of rain is registered in inches, tenths, and hundredths.

In some gauges a float and "staff" are elevated by the water, and show by means of a scale the depth of water in the receiver; it is called a staff-gauge.*

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* This gauge is defective, inasmuch as the staff, when much elevated above the funnel mouth, catches more rain than is due.

The rain-gauge known as Dalton's gauge is intended, where used, to ascertain the depth of rain evaporated rather than the depth of rain fallen, and modified, we believe, has been used by Mr. Dickinson, of King's Langley, Herts. It consists of the usual funnel mouth, which conveys the water into a cylinder some three inches in diameter, about three or four feet deep, and supported on an inverted funnel answering the purpose of a stand; the cylinder is filled with two or three feet deep of earth, resting on a perforated bottom, through which the water reaching to that depth passes off into a short horizontal pipe, a little below the perforated plate, and connected with a vertical glass tube, graduated and bearing a definite proportion to the mouth of the rain-gauge. By means of a tap the water may be drawn off when required. It was by a gauge similar to this that the following results were obtained by Mr. Dickinson, at King's Langley, some years since. The *entire* depth of rain was measured by the usual funnel-gauge.

Year.	Depth of rain.	Filtration.	Evaporated.
1836	31·00	17·65	13·35
1837	21·10	6·95	14·15
1838	23·13	8·57	14·56
1839	31·28	14·91	16·39
1840	21·44	8·19	12·25
1841	32·10	14·90	17·91
1842	26·43	11·76	14·67
1843	26·47	8·10	15·37

Although a set of gauges of this description made in a single locality cannot be looked upon as a general guide, and under certain conditions is even liable to exception, still such experiments are very interesting, as tending to throw light to a certain extent upon this important subject.

The proportion of the annual rainfall, or the mean average allowed for evaporation and absorption, is very commonly taken at one-third of the whole depth; about the south of England often more, sometimes exceeding a half; to the north generally one-third is ample, and often in excess. We shall have further remarks to make upon this subject hereafter.

Rain-gauges may be constructed of japanned tin, or better of zinc, and are mostly made circular, so that the rim of the mouth, the *defined area* of which it is most important to have correct, may be turned in a lathe; this rim should be stout, so as not to be easily deformed, and may be made of tinned copper; it may be about half-an-inch deep, and should project inwardly over the body of the funnel, so as to prevent the rain in heavy showers from splashing out of the funnel. The area of the rim is generally made of 6, 9, or 12 inches in diameter, as the observer pleases.

The Gauge Glass.—As already observed, this glass is used to measure the quantity of rain fallen; it is made cylindrical, and we have ours of such form as to go into a case like that of a telescope, slung over the shoulder by means of a strap, as being the most safe and convenient mode of carrying it from gauge to gauge, accordingly as these may be disposed over the face of the district under survey; always, however, having at least one glass in store in case of accidents. Let us now suppose, as an example, that the internal diameter of the rim of the rain-gauge is nine inches; the square of any diameter of a circle multiplied by 0·7854 is equal to the area of the circle; in our example 81 × ·7854 = 63·6174 square inches; and one cubic inch of distilled water at a temperature of 62° Fahrenheit is a standard of weight according to legal enactment, and has been determined to weigh 252·458 grains, of which 437·5 are equal to one ounce avoirdupois.

One inch in depth of rain on the gauge will give 63·6174 *cubic inches.*

Then,

$$\frac{63{\cdot}6174 \times 252{\cdot}458}{437{\cdot}5} = 36{\cdot}710 \textit{ ounces avoirdupois.}$$

Therefore any cylindrical glass holding this quantity of water marked on it in depth, and such depth divided into tenths, and these subdivided each into ten other equal parts, will show one inch, or tenths and hundredths of an inch of rain on the area of the gauge of nine inches diameter supposed circular.

Then, as a general rule, let A be the area of the mouth of the gauge in inches, and D any depth in inches also of rain falling on it, and we shall have,

$$\frac{A \times D \times 252{\cdot}458}{437{\cdot}5} = \text{weight of water in ounces avoirdupois.}$$

It is well to remember this rule, because circumstances may arise when we have to attend to the making of these divisions on the glass ourselves, and as a truly cylindrical glass is not always to be obtained, we may have to pour in small quantities at a time and graduate the glass accordingly, for it is only on the assumption that the glass *is* truly cylindrical that equal quantities will occupy equal depths; in large towns we have only to give the total quantity of water, and the divisions we require, to a maker of mathematical instruments, or a manufacturer of chemical apparatus, and we get the thing done to our hand.

If we wish to define the diameter of the cylindrical glass, and to ascertain the depth that shall hold one inch of rain over the gauge, the rule will be: multiply the area of the glass by 252·458, *and divide by* 437·5; *then the weight in ounces, of one inch depth of rain over the mouth of the funnel, divided by the quotient last found, will give the depth of the glass of the given diameter.*

These calculations may be shortened by making use of the constant 0·577, instead of 252·458 ÷ 437·5.

The above rules will now be as follow:—

1. *With a given diameter of rain gauge, and any given depth of rain over its area, to find the weight of the rain water: square the diameter in inches, and multiply by* 0·7854 *to obtain the area* A; *multiply the area by* D, *the depth in inches also, and multiply the product by* 0·577, *which will give the weight required in avoirdupois ounces.*

2. *With a given diameter of cylindrical glass, to find the depth that shall hold a given weight of water. Multiply the area of the glass in square inches by* 0·577, *and divide the weight of water in ounces avoirdupois by the product; the quotient thus found will give the depth required in inches. This depth in the glass being then divided into tenths and hundredths as above, will give the means of ascertaining the depth of rainfall on the gauge, and thousandths may be read off by approximation.*

The following tablet will in most cases save the trouble of making the necessary calculations.

Diameter of rain gauge. Inches.	Weight of one inch in depth of rain in oz. avoirdupois.	Diameter of measuring glass. Inches.	Depth of glass to hold one inch deep of rain over the area of gauge. Inches.
6	16·31	1¼	16·00
8	29·00	2	15·989
9	36·71	2¼	16·00
12	65·25	3	16·00

A *heavy* rainfall may be measured in two or three times.

Position of Rain Gauges.—We have already observed that they should be placed in the most exposed, that is, the least sheltered places, and at different altitudes, as well as at the different principal spots where circumstances are such as to be likely to interfere with the rainfall, as on or near the summits, and also the lower parts of hills, on table-land, and in valleys, &c. If the reader will study the table we have given of the rainfall in different parts of Great Britain, and the appended notes, with some of the Ordnance maps before him, he will be able to form a very good idea of how circumstances may interfere as regards quantities of rain in different parts of the same locality. In all cases rain-gauges should, if possible, be placed on the ground, and where not likely to be interfered with; they should be watched to prevent undue interference; every day they should be examined, or the rainfall at least should be registered; particularly after a heavy fall this should be done with as little delay as possible, for if dry windy weather should ensue directly after, it is surprising, under some circumstances, how great the effect of evaporation may be. It is very desirable to note also the direction of the wind every day, and keep a systematic register.

However valuable a series of observations made by the above means may be when obtained with care, it will not fail to be remarked, from a slight study of the table already referred to, that the greatest uncertainty prevails as to the quantity of rain that falls from year to year, and this more particularly in hilly districts; therefore observations made during a series of years are very desirable, and should be obtained when possible.

We have already said enough, however, to show that, setting aside evaporation and absorption, very great difference exists as to the proportion of rainfall over the drainage area which may be discharged by the streams and rivers by which valleys are drained. The following very valuable remarks and tables by Mr. Beardmore,* will further tend to elucidate this portion of our subject.

"Districts may greatly vary in their general slope and geological character; graüwacke, granite, and the volcanic districts generally throw water in great rapidity, and are equally liable to great drought in summer time, unless they are capped by moss beds, which act as sponges not always the most pure; some of the newer rocks, on the other hand, such as the old and new red sandstones, have great power of storing water; the latter rocks from their flatness, generally holding it, as indicated by the wells, which are always plentiful in this formation; the former, on the other hand, generally give out the purest spring-water when occurring on mountain slopes, rising above the plains occupied by our numerous coal-fields.

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* Hydraulic Tables, by Nathaniel Beardmore.

"In the chalk districts this porous material absorbs a great portion of the rain that it receives, collecting it in great underground sheets represented by the numerously-interlaid flint beds, and pouring out almost rivers at places that have no indication of a feeder; so strongly is this marked, that the chalk districts may be always identified upon the Ordnance maps by the absence of streamlets on its surface, a characteristic likewise of some of the mountain limestones and oolites.

"In this latter formation we lately had occasion to examine springs which, although most copious, could be scarcely recognised to have any area of drainage beyond them; the rock, having a very flat underlie, was fetching water far away from the outcrop, and pouring it out at a point not forty feet below the summit level of the hills whence it proceeded.

"These instances are familiar to all who have studied the water-bearing properties of the hills of Great Britain; watching the progress of agriculture and drainage, we find the hill pastures scored in all directions with sheep drains, while in agricultural districts thorough draining steadily advances; all these operations are rapidly contributing to pour out floods on the dwellers in the plains, the inhabitants of the rich levels at the mouth, or on the lower course of our rivers, who, by this simple but incontrovertible order of events, find themselves forced into improvement, which the natural resources of their soil had too long delayed. Frequently in this state of affairs, the march of improvement has commenced in the harbour at the mouth, and tidal volume is sent up vastly higher and sooner than known before; dredging machines are set to work, and bold piers or long river walls are constructed; and although the landowner may find his outfall better, he also discovers that a concurrent high tide and upland flood has topped the walls and robbed his meadows of their burthen, or swept down his ripening corn.

"In the estimate of floods, at pages 26-9, we have endeavoured to sketch out a few examples, with which the engineer should expect to deal, in constructing outfalls, or improving lowland rivers. In the following table we give a few examples of actual discharge from considerable districts, where the total flow of water has been gauged for the whole year round. The Bann Reservoirs, and some of those near Manchester, are from Mr. Bateman's paper in the *Philosophical Memoirs* of that town.

TABLE OF ACTUAL DISCHARGE FROM LARGE DISTRICTS,

with the amount given per square mile, the amount of water run off in depth over the surface, and the storage in reservoirs where existing or intended.

	Height above Sea.	Drainage Area.	Total Discharge for the Year.	Discharge per Square Mile.	Representing Rainfall per Annum.	Registered Rainfall per Annum.	Reservoir Room per Square Mile.
	ft. ft.	square miles.	cubic ft. pr. min.	cubic ft pr.min.	inches.	inches	cu. ft. in millns.
Bann Reservoirs, 1837–8, moorland	400 to 2,800	5·15	1092·6	210·2	48·0	72·0	...
Greenock, 1827-8, flat moor	512 to 1,000	7·88	1416·6	197·7	41·0	60·0	38·0
Bute (*a*), 1826, low country	...	7·80	819·0	105·0	23·9	45·4	...
Glencorse Pentland Hills (*b*)	734 to 1,600	6·00	600·0	100·0	22·3	37·0	7·66
Belmont, 1843, moorland...	850 to 1,600	2·81	630·4	224·3	50·7	63·4	26·8
„ 1844 „ ...	...	...	412·8	146·4	33·3	50·0	...
„ 1845 „ ...	...	...	511·2	181·9	41·2	55·0	...
„ 1846 „ ...	...	...	411·3	146·3	33·2	49·8	...
Rivington Pike (*c*) 1847-8.	1,545	16·25	1752·0	107·8	24·25	55·5	29·6
Turton and Entwistle, 1886	500 to 1,300	3·18	576·7	181·3	41·0	46·2	31·43
„ „ 1837	...	...	548·2	172·3	39·0	48·2	...
Bolton Waterworks, moor...	800 to 1,600	·80	100·2	125·2	32·7	...	25·6
Sheffield, since enlarged ...	...	1·42	...	...	...	...	36·5
Ashton, 1844	...	·59	40·7	65·5	15·5	40·0	21·0

(*a*) This year's rain was about 12 inches less than an average.

(*b*) Glencorse discharge is only the amount exclusive of floods; the reservoir supply totally failed in the drought of 1843; it is now in course of enlargement. The Glencorse drainage is generally precipitous.

(*c*) Rivington Pike Reservoirs are not yet made. The amount running down for two years was gauged. The country is moor, partly flat, and partly precipitous.

"In 1846-7 the author conducted an experiment by accurately gauging for four months the water, flowing from 3,800 acres, into the Glencorse Reservoir, in the Pentland Hills, belonging to the Edinburgh Water Company, as follows:—

	Cubic Feet.	Inches.
1846, December.—Supply into Reservoir...	19,762,000=fall of rain	1·43
„ „	Registered ditto	1·02
1847, January.—Supply into Reservoir ...	14,524,200=fall of rain	1·05
„ „	Registered ditto	·750
„ February.—Supply into Reservoir ...	18,637,100=fall of rain	1·34
„ „	Registered ditto	1·56
„ March.—Supply into Reservoir ...	9,662,520=fall of rain	·69
„ „	Registered ditto	1·02
The total rain passing into the Reservoir being		4·53
Ditto registered in rain gauges at level of Reservoir being ...		4·35
„ „ „ on the Hills being		4·71

"Again, in the Bonally district adjoining, but about 500 feet higher than Glencorse, we found the mean run of the streams to be 112 cubic feet per minute, or equal to a fall of rain over 879 acres of 4·55 inches, the registered fall being as before 4·71 inches.

"As a contrast to the foregoing table, we now offer a second :—

"TABLE OF ORDINARY SUMMER DISCHARGE

of various Rivers, Streams, and Springs, as uninfluenced by any immediate rain ; with the drainage area, and amount run off the surface represented in depth of rain.

RIVERS.	Height above Sea.	Drainage Area.	Total Discharge.	Discharge per Square Mile.	Representing Rainfall per Annum.	Total average Rainfall per Annum.
	Valley. ft. Hill. ft.	square miles.	cubic ft. pr. min.	cubic ft pr. min.	inches.	inches
Thames at Staines—chalk, greensand, Oxford clay, oolites, &c.	40 to 700	3,086	40,000	12·98	2·93	24·5
Severn at Stonebench—silurian...	400 to 2,600	3,900	33,111	8·49	1·98	...
Trent at its mouth—oolites and Oxford clay	100 to 600	3,921	...	...	...	...
Loddon (Feb. 1850)—greensand.	110 to 700	221·8	3,000	13·53	3·01	25·4
Nene, at Peterborough—oolites, Oxford clay, and lias...........	10 to 600	620·0	5,000	8·45	1·88	23·1
Mimram, at Panshanger—chalk.	200 to 500	29·2	1,500	51·4	11·58	26·6
Lee at Lee Bridge—chalk (*Rennie*, April, 1796)	30 to 600	570·0	8,880	15·58	3·53	...
Wandle, below Carshalton—chalk	70 to 350	41·0	1,800	43·9	9·93	24·0
Medway, driest seasons (*Rennie*, 1787)—clay	...	481·5	2,209	4·59	1·04	...
Ditto, ordinary summer run (*Rennie*, 1787)	...	481·5	2,520	5·23	2·19	...
Verulam, at Bushey Hall—chalk	150 to 500	120·8	1,800	14·9	3·37	...
Gade, at Hunton Bridge—chalk	150 to 500	69·5	2,500	36·2	8·19	...
Plym, at Sheepstor—granite......	800 to 1,500	7·6	500	71·4	15·10	45·0
Woodhead Tunnel—millstone grit	1,000	...	139	...	...	46·0
Glencorse Burn ....(*e*)	750 to 1,600	6·0	130	21·6	4·9	37·4
Crawley Spring—felspar and porphyry— Summer (*e*)	556 to 1,600	·6	54	90·0	20·2	...
,, Winter (*e*)	,, ,,	·6	77	128·3	29·0	...
Blacksprings—felspar—Sum. (*e*)	1,000 to 1,600	·1	30	300·0	70·0	(*c*)
,, ,, Winter (*e*)	,, ,,	·1	40	400·0	92·0	(*c*)
Bavelaw—sandstone—Summer(*e*)	900 to 1,600	1·42	120	84·5	19·0	(*c*)
Colzium ,, ,, (*e*)	,, ,,	4·20	113	29·9	6·87	(*c*)

"The examples here given have generally been corroborated by our own observations; the rain at the places marked (*c*) has not been kept, but that at Glencorse represents the rain for the lowest point of the district; the rain on the hills is much higher.

"The Blacksprings indicate a drainage from a district far beyond the water shed; they occur at a protrusion of basalt; these springs and others marked (*e*) are from gaugings made at different periods, under the provisions of Acts of Parliament, for the protection of parties having water privileges.

"The Wandle, Verulam, and Gade, all flowing out of the chalk towards the metropolis, are from Mr. Telford's gaugings made in 1833, a period which he terms "the driest season known for the last half-century;" but there is reason to believe that this was not the time of the minimum discharge.

"The table gives the run of the several streams per square mile of drainage area, which is an excellent measure of their productiveness, and we have likewise reduced the amount (as supposed to run uniformly during the year, irrespective of floods) into the quantity distributed over the drainage area, as fed by rain; for example, the summer run of the Thames is equal to 12·98 cubic feet per minute for each square mile, and represents 2·93 inches of rain falling over its drainage area, while the summer run of the Mimram is 51·4 cubic feet per minute for each square mile, representing 11·58 inches of rain.

"In generalizing on the discharge of districts in ordinary and flood time, the observer will be well aware, that there are seasons of drought when the most certain streams are seriously affected in their water-bearing properties; the table exhibits the very high power of the Pentland Hills, yet we have it on record that almost the whole of these sources were dried up in the summer of 1843; there appear to be, indeed, rare occasions when hill-country suffers more from drought than lower districts—the lake district of Cumberland was similarly affected at about the same time.

"In hotter countries these facts are proverbial; in the great tertiary plains of Central Spain, which are 1,200 feet and upwards above the sea, large rivers shrink into dry gravel beds, while at the same time the seaward face of the mountains which fringe the Bay of Biscay are clothed with verdure caused by perpetual rains; this grand escarpment is broken up into the wildest and most precipitous glens, where the vapours rolling from the sea are caught and poured down with astonishing rapidity and volume. The character of this great condensation, as it were, is so marked that severe droughts are experienced on the inland side of the escarpment (5,000 to 7,000 above the sea) while within a very few miles rains are daily pouring down. These effects occur in a similar manner along the escarpment of the Bombay peninsula, where the rainfall is from 80 to 150 inches in the year, while it gradually recedes to 10 or 15 inches in the

Deccan or dry country; the same is to be traced on the coast of Arracan, and in California on the West Coast of America. The rainfall of the Western Highlands is remarkable, likewise that of the Lake district, as shown by Mr. Millar, of Whitehaven, in his papers published by the Royal Society."

We have already remarked on the various circumstances and conditions which will form features in the description of survey now under consideration, and it will not fail to be observed that, added to the observations made with the help of rain gauges, it will be necessary to ascertain the quantity of water discharged by streams and rivers from a catchment basin which may be one or several valleys, generally bounded by ridges or high lands, and of a few or very many square miles in extent. This may be done by measuring the velocity of the current and the sectional area of the water, or by means of an overfall. The first method is generally adopted for considerable bodies of water, and the second for streams, across which it may be found convenient to construct a kind of dam or weir, in the shape of a notched board, which it will be our business to describe, after making a few general observations on this branch of hydraulic surveying.

A heavy body dropping from a state of rest and from a height, *in vacuo*, falls through a space of 16 feet 1 inch in *one second* of time, at the *end* of which it has acquired a *velocity* of 32 feet 2 inches; in the next second it has fallen through 48 feet 3 inches, so that in two seconds it has fallen through 64 feet 4 inches.

If the time or seconds be represented thus	1"	2"	3"
The *spaces* fallen through in *each* second will be, as	1	3	5
And the spaces fallen through in the *whole* time, as	1	4	9

Although, strictly speaking, this bears only on a heavy body falling *in vacuo*, it may be taken as sufficiently near for the fall of a body of water for practical purposes. The spaces fallen through are thus as the squares of the times, and *the velocities as the square roots of the heights*, and the height *in feet* through which a body falls × 64·3, will give *the square of its velocity* in feet per second. If the fall be *one foot*, then $\sqrt[2]{64{\cdot}3}=8{\cdot}02$, is the *velocity* of the body through *such space*.

If we take a foot as the unit of measurement, and a second for the time, and if we put v for the velocity, and g for the force of gravity, we obtain the following algebraical formula:—

$$v=\sqrt{2g}=\sqrt{32'-2'\times 2}=8'{\cdot}02;\text{ or,}$$
$$v=\sqrt{64'-4}=8'{\cdot}02;\text{ or,}$$
$$v=2\sqrt{g}=2\sqrt{16{\cdot}08}=8'{\cdot}02;$$

and putting h for the height of fall, we get

$$v = 8{\cdot}02\sqrt{h} \text{ (per second) and } h = \frac{v^2}{64{\cdot}4}$$

The force of gravity increases with the latitude, and decreases with the altitude above the level of the sea; and, according to the latest experiments, it is now considered more accurate to write in our latitude—

$$2\sqrt{g} = 8{\cdot}03.$$

If we now put a for the sectional area of a body of water falling through an orifice or opening, and Q for the quantity, we shall have—

$$Q = 8{\cdot}03\, a\sqrt{h},$$

or in words, *the sectional area multiplied by* 8·03 *multiplied by the square root of the height is equal to the discharge in cubic feet per second.*

And multiplying by 60, we shall get *per minute,*

$$Q = 481{\cdot}8\, a\sqrt{h},$$

or in words, *the sectional area multiplied by* 481·8 *multiplied by the square root of the height equal to the discharge in cubic feet per minute.*

For measures in inches, *per second*—

$$v = 27{\cdot}8\sqrt{h}, \text{ and } h = \frac{v^2}{772{\cdot}8},$$

and per minute,

$$v = 1668\sqrt{h}, \text{ and } h = \frac{v^2}{2770222{\cdot}8}$$

Velocities in inches per second may always be translated into velocities in feet per minute by multiplying by 5.

Example.—*Let the width of an aperture be* 2 *feet, and the depth* 1 *foot; then the sectional area* A, *equal to* 2; *let h, the height from the central* point of the aperture be* 9 *feet, the square root of which is* 3: *then* $8{\cdot}03\, a\sqrt{h} = 8{\cdot}03 \times 2 \times 3 = 48{\cdot}18$ *cubic feet discharge per second; or,* $481{\cdot}8\, a\sqrt{h} = 481{\cdot}8 \times 2 \times 3 = 2890{\cdot}8$ *cubic feet discharge per minute.*

The discharge obtained by the formula $8{\cdot}03\, a\sqrt{h}$ is called the theoretical or natural discharge, and can only obtain when there is no friction amongst the particles of falling water, or against the sides of the opening; no reaction from the adjacent fluid, or when no *contraction of the fluid vein* arises from the water passing through the aperture. This last element plays a most important part as regards the quantity of water discharged through any opening, and in some cases diminishes it very considerably. It is well known that after passing through an orifice or opening, a vein of water undergoes a contraction at a certain distance outside the orifice, and that the correct computation of the quantity of water discharged depends on the accurate value of the coefficient, which follows a law enforced by the contracted vein; that which renders the correctness of this value most difficult to attain, is that not only with any change in the conditions or form of the orifice, the coefficient varies more or less, but that it does also with any important alterations of the quantities of which it is to become a function.

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* In order to obtain the mean velocity.

The following proportions between the contracted vein and the orifice were obtained by Poncelet and Lesbros, during their experiments at Metz, during the years 1827 and 1828:—

Taking the English foot as unit, the side of the orifice was $= 0{\cdot}656$; area $= 0{\cdot}43$; the mean head (*charge moyenne*) above the centre $= 5{\cdot}51$. The contracted vein, measured at nearly a foot from the orifice, a distance of 1½ the length of the side, gave an area of 0·268, the square root of which is equal to a very little more than ¾ of the side of the orifice. By the expression "mean head," Poncelet means the depth of water measured exactly in the centre of the orifice, and not anywhere in the sides, over which it differs from that at the centre.

For a long time past it has been a subject of anxious investigation by experimentalists and philosophers, foremost amongst whom we may reckon the names of Michelloti, Venturi, Bossut, Dubuat, D'Aubuisson, Prony, Castel, Eytelwein, Young, Smeaton, Brindley, Rennie, and more lately, Poncelet, Lesbros, Boileau, and Blackwell. The large scale on which the experiments of Mr. Blackwell were conducted, have rendered them of the greatest value in practical application.

Now suppose a board with a rectangular notch cut in it to be set up across a stream, so as to dam it back; in some short time afterwards the water will flow over the notch; setting aside for the present all other considerations, the velocity of the issuing water will be that due to its fall, or to the difference of level between the centre of the notch and the surface of still water some short distance up the stream; this velocity per second we obtain from $v = 8{\cdot}03 \sqrt{h}$, h being the difference of level or fall; the issuing water assumes the form of a parabola, the sectional area of which is two-thirds of the circumscribing rectangle; then if we take ⅔ of the width of the notch multiplied by h, the height, we obtain a, the sectional area of the issuing body of water; and thence we get,

$$\text{Cubic feet discharge per second} = 8{\cdot}03 \times \tfrac{2}{3}\, a \times \sqrt{h};$$
$$\text{or } 481{\cdot}8 \times \tfrac{2}{3}\, a \times \sqrt{h} \text{ per minute.}$$

And if, instead of a notch open to the surface, the aperture were in the condition of the opening in a sluice, then under our present assumption we should take the whole of a instead of $\tfrac{2}{3}$, as the opening would be filled by the issuing water.

But on account of the contraction of the fluid vein after its issue from the aperture, friction, and other causes mentioned above, it has been fully established that the above formula must be modified by being multiplied by some coefficient, which considerably reduces the quantity shown as the theoretical discharge. As to the practical value of this coefficient, there is some difference in the opinions of engineers, a difference which is warranted by various conditions under which the best experiments have been made. This coefficient is called the *coefficient of discharge.*

If we now return to the formula $Q = 481{\cdot}8 \times a \times \sqrt{h}$, and put C for the coefficient of discharge, we shall have,

$$Q = 481{\cdot}8\, a \sqrt{h}\, C,$$

or, in words the quantity discharged in cubic feet per minute is equal to 481·8 multiplied by the area, multiplied by the square root of the fall or height, multiplied by the coefficient.

The following table from Mr. Blackwell's experiments will show how much the coefficient varies with the conditions attending an overfall:—

EXPERIMENTS ON THE KENNET AND AVON CANAL.

Table showing the variations in the coefficient for different heads of water in overfalls, the head being measured to still water.

	Description of Overfalls.	Head in inches.	Coefficient C.	481·8 × C
No. 1.	Thin plate 3 feet long	1 to 3	·440	211·99
	" " "	3 " 6	·402	193·68
No. 2.	" 10 "	1 " 3	·501	241·38
	" " "	3 " 6	·435	209·58
	" " "	6 " 9	·370	178·26
No. 3.	Plank 2 inches thick, with notch 3 feet long	1 " 3	·342	164·77
	" " "	3 " 6	·384	185·01
	" " "	6 " 10	·406	195·61

	Description of Overfalls.	Head in inches.	Coefficient C.	481·8 × C
No. 4.	Plank 2 inches thick, with notch 6 feet long	1 to 3	·359	172·96
	” ” ”	3 ” 6	·396	190·79
	” ” ”	6 ” 9	·392	188·86
	” ” ”	9 ” 14	·358	172·48
No. 5.	Plank 2 inches thick, with notch 10 feet long	1 ” 3	·346	166·70
	” ” ”	3 ” 6	·397	191·47
	” ” ”	6 ” 9	·374	180·19
	” ” ”	9 ” 12	·336	161·88
No. 6.	Plank 2 inches thick, with notch 10 feet long with wings	1 ” 2	·476	229·34
	” ” ”	4 ” 5	·442	212·95
No. 7.	Overfall with crest; 3 feet wide sloping 1 in 12, 3 feet long, like a weir	1 ” 3	·342	164·77
	” ” ”	3 ” 6	·328	158·03
	” ” ”	6 ” 9	·311	149·84
No. 8.	Overfall with crest; 3 feet wide sloping 1 in 18, 3 feet long, like a weir	1 ” 3	·362	174·41
	” ” ”	3 ” 6	·345	166·22
	” ” ”	6 ” 9	·332	159·96
No. 9.	The same sloping 1 in 18, 3 feet wide and 10 feet long	1 ” 4	·328	158·03
	” ” ”	4 ” 8	·350	168·63
No. 10.	Overfall with level crest; 3 feet wide and 6 feet long	1 ” 3	·305	146·95
	” ” ”	3 ” 6	·311	149·84
	” ” ”	6 ” 9	·318	153·21
No. 11.	Overfall 6 feet long; crest level 3 feet broad	3 ” 7	·330	158·99
	” ” ”	7 ” 12	·310	149·36
No. 12.	Overfall 10 feet long; crest level 3 feet broad	1 ” 5	·306	147·42
	” ” ”	5 ” 8	·327	157·55
	” ” ”	8 ” 10	·313	150·80

It is to be carefully remembered that the coefficient C in the above table refers to the formula,

$481{\cdot}8\sqrt{h} \times a \times C = Q$, discharge in cubic feet per minute,

and not to,

$481{\cdot}8\sqrt{h} \times \frac{2}{3}a \times C =$ ” ” ” ” ”

To take one example for the application of this rule, let us suppose an overfall similar to that of No. 5 in the above table, consisting of a plank 2 inches thick, with a notch 10 feet long for the flow of the water, and let h, the *head*, measure 4 inches, equal to 0·3332 of a foot. Then we shall have

$$a = 10 \times 0{\cdot}3332 = 3{\cdot}332\,;$$
$$h = 0{\cdot}3332, \text{ and } \sqrt{h} = 0{\cdot}577\,;$$

Taking $C = 0{\cdot}397$, we shall have,

$$Q = 481{\cdot}8\,a\sqrt{h}\,C = 481{\cdot}8 \times 3{\cdot}332 \times 0{\cdot}577 \times 0{\cdot}397 = 368 \text{ cubic feet per minute,}$$

or,

$$Q = 191{\cdot}47\,a\sqrt{h} = 368 \text{ cubic feet per minute.}$$

If we now for one moment set aside the effects of friction and other minor retarding causes, and attribute the decrease from the theoretical to the actual discharge, to the *contracta vena*, or contracted vein of the fluid, we shall see how necessary it is in many instances to apply a coefficient proper or suitable to the conditions of the overfall, and more particularly to the head of water, for had we in our example applied the coefficient ·346 instead of ·397, we should have had only 320, instead of 368 cubic feet. According to Mr. Beardmore's tables, we should have about 402 cubic feet; he observes, however, in his "remarks on the use of the tables," that "causes which may render the observation liable to error must be treated with judgment according to circumstances."

If in the above table we compare No. 1 with No. 3, both the gauges being three feet long, we shall perceive that in No. 1 the coefficient decreases as the head h increases, and attributing the difference to the contracted vein, or *contracta vena*, we may possibly be led to conclude that this contraction is much more perfect or complete for the lesser depths. In No. 3, the coefficient increases with the head, from one inch up to ten inches; here the aperture, or mouth, for the discharge of the water, is two inches thick instead of a thin plate, and as the pressure increases with the head, or weight of water, this two-inch tube, as we may term it, does not appear to have the same effect in perfecting the contraction of the fluid vein as the thin plate. If we now look at No. 2, being an aperture of *ten feet* in length instead of three, and in a thin plate, we also remark this perfecting of the contracted vein as the head of water decreases; we also observe that for heads of from one to three inches, the coefficient is greater in No. 1 than in No. 2, and also for heads from three to six inches; this we may probably attribute to the effects of the sides of the notch on the issuing water. The plate in No. 2 being much longer than that in No. 1, there would be, with the same heads of water, a greater proportion of contraction arising from the effects of the sides due to No. 1 than to No. 2.

If we now revert again to No. 3, the plank two inches thick and three feet long, we shall see that the coefficients for heads from one inch to three inches, and from three to six inches, are very considerably less than those for the same heads of water in the thin plate gauge (No. 1) of the same length; the two-inch plank being a short tube instead of a sharp edge, there would be a greater area for friction to produce retardation. Are we to attribute this important diminution in the coefficients in No. 3 entirely to this cause?

Let us now take Nos. 3, 4, and 5, being the plank notches two inches thick, but of three, six, and ten feet respective lengths. By Nos. 3 and 4, we perceive that for *heads up to nine inches* the coefficient increases with the head; for greater depths than nine inches the coefficient decreases, at least in No. 4, and we may conclude the same for No. 3. It would appear to be the same for No. 5.

Comparing Nos. 1 and 2 with 3, 4, and 5, there is not in the last three so great a difference of coefficient to be attributed to the difference of the lengths of the gauges as in the first two.

We now come to No. 6, a plank two inches thick with notch ten feet long, with WINGS converging towards the overfall. This last addition appears to play a most prominent feature; the coefficients are much greater than for the other plank gauges, but instead of increasing with the heads of water they diminish.

The student is now in a position to be able to compare the remaining results of the experiments by himself.

It will not fail to be remembered that in the foregoing pages we have always supposed h the head as measured to still water a little above the overfall, the velocity obtained being considered as due entirely to the fall from this point. We must request the reader to keep this in view, as we shall have particular occasion to recur to it presently.

Before referring to Poncelet and Lesbros' experiments, we have now to add another table from experiments made by Messrs. Blackwell and Simpson, at Chew Magna, and in which the coefficient C refers to the same formula as that mentioned above.

Experiments at Chew Magna.

	Description of Overfall. Overfall bar, 10 feet long.	Head in inches.	Coefficient C.	481·8×C.
No. 13.	2 inches thick	1 to 3	·437	210·54
	„ „	3 „ 6	·499	240·42
	„ „	6 „ 9	·505	243·31

If these coefficients be compared with those at No 5, in the former table, where the overfall is similar, a very great difference will be perceived.

We now subjoin two tables of experiments on overfalls, by Poncelet and Lesbros. The metre is equal to 3·2809 English feet, the decimetre = 0·328, and the centimetre = 0·0328. The litre = 0·1 of a metre.

The total head, or the height above the fall to the surface of still water, was measured at *a distance* of 11·48 feet above the overfall; the notch cut to a sharp edge lined with metal, and bevelled at an angle of 45°, the sharp edge being towards the overfall; h, the head to still water; h', the depth of water immediately on the sill or edge of the notch; the depths of water on the sill vary throughout the length, and the mean depth is their mean. This mean is found at the centre of the notch, $o = h - h'$; o is equal therefore to the depression from the surface of still water to that of the issuing vein, and is termed the mean head; E, the actual discharge; d is the discharge in the formula of *theoretical discharge*, or $d = l\,h\sqrt{2\,g\,h}$, or a $\sqrt{2\,g\,h}$; $\frac{E}{d}$ is put for the ratio of the actual to the theoretical discharge. All the notches are two decimetres long, being very little short of eight inches.

HEAD.						
Total head h Metres	Mean head $o = h - h'$ Metres	Ratio of $\frac{o}{h} = \frac{h - h}{h}$ Metres	Velocity due to the total head h, or to $\sqrt{2\,g\,h}$ Metres	Discharge per second by the formula d Litres	Actual discharge per second Litres	Value of the coefficient or ratio of $\frac{E}{d}$
0·2205	0·20321	0·9215	2·0800	91·728	34·810	0·380
0·2075	0·19072	0·9191	2·0176	83·730	32·079	0·383
0·1815	0·16579	0·9134	1·8874	68·513	26·276	0·384
0·1795	0·16387	0·9129	1·8768	67·377	25·873	0·384
0·1770	0·16148	0·9123	1·8637	65·975	25·427	0·385
0·1753	0·15985	0·9118	1·8542	65·008	24·861	0·382
0·1305	0·11714	0·8976	1·6000	41·760	16·751	0·401
0·0735	0·06341	0·8627	1·2008	17·652	7·234	0·410
0·0290	0·02259	0·7790	0·7538	4·372	1·806	0·413
0·2079	0·1912	0·9195	2·0195	83·971	32·643	0·3887
0·1631	0·1478	0·9063	1·7888	58·351	22·929	0·3929
0·1029	0·0909	0·8833	1·4208	29·240	14·528	0·3943
0·0605	0·0514	0·8491	1·0894	13·182	5·277	0·4003
0·0446	0·0368	0·8253	0·9354	8·344	3·356	0·4052
0·0235	0·0176	0·7468	0·6790	3·191	1·324	0·4149

In the above experiments, the greatest care was taken to obtain, as nearly as practicable, still water in the reservoir above the overfall, the horizontal and vertical sides of which were, as already stated, bevelled to an angle of forty-five degrees, to obain a sharp edge, the sill was 1·77 of a foot above the bottom of the reservoir, and the sides at 5·70 of a foot from those of the canal leading to the overfall; considering, therefore, the length of the overfall, 0·656, it might be considered as entirely isolated, being only about $\frac{1}{18}$ of the side of the reservoir in which it was constructed.

With regard to the experiments on the overfall on the Kennet and Avon Canal, Fig. 123 will give an idea of the circumstances attending it. The area of the reservoir was 2A. 1R, 30P.; the head was measured from still water in it, which remained unchanged from the beginning to the end of each experiment; the width at A B was about 32 feet, and at C D 13 feet, below which the approach suddenly widened, and then contracted to the overfall at O; depth in front of the dam about 3 feet, and on the dam next the overfall about 2 feet, and the depth on the sunk masonry M, about 18 inches. "Altogether, the circumstances were such as to increase the amount of resistances between the reservoir from which the head was measured, and the overfall, particularly for the larger heads, and we accordingly see that the coefficients become less for heads over six inches, with few exceptions."

As to the experiments at Chew Magna, Fig. 124 is a sketch of plan and section of the overfall bar 10 feet long, which consisted of a cast-iron plate two inches thick, with a square top. The heads were measured from still water at the side of the reservoir and at some distance up in it. The reservoir, averaging about 4 feet deep, and of irregular figure, contained 21 perches. The supply came from an upper reservoir by a pipe two feet in diameter, with a fall of 19 feet; the distance between the supply and the overfall was about 100 feet; the width of the reservoir near the approach to the overfall was about 50 feet; from the difference of the coefficients obtained here, with the coefficients from the Kennet and Avon canal experiments, it seems probable that there was some irregularity in the velocities of approach caused by the increase of supply. A comparison of the results with those obtained from the canal experiments will show the importance that should be attached to the velocity of approach, friction over the bed, resistances at the sides, and to forms which more or less isolate the overfall.

The experiments by Poncelet and Lesbros were made on orifices closed at top in the manner of sluices, as well as on overfalls; the following table of coefficients relate to the former.

Table of coefficients, from Poncelet and Lesbros, for orifices in thin plates sunk below the surface. Width of the orifice = 0·20 *metre.*

	Height of the Orifices.					
Head on the centre of the orifice.	*m* 0·20	*m* 0·10	*m* 0·05	*m* 0·03	*m* 0·02	*m* 0·01
m 0·01	...	...	...	...	...	0·709
0·02	...	...	...	...	0·660	0·698
0·03	...	...	...	0·638	0·660	0·691
0·04	...	...	0·612	0·64	0·659	0·685
0·05	...	...	0·617	0·64	0·659	0·682
0·06	...	0·59	0·622	0·64	0·658	0·678
0·08	...	0·6	0·626	0·639	0·657	0·671
0·10	...	0·605	0·628	0·638	0·655	0·667
0·12	0·572	0·609	0·630	0·637	0·654	0·664
0·15	0·585	0·611	0·631	0·635	0·653	0·660
0·20	0.592	0·613	0·634	0·634	0·650	0·655
0·30	0·598	0·616	0·632	0·632	0·645	0·650
0·40	0·600	0·617	0·631	0·631	0·642	0·647
0·50	0·602	0·617	0·631	0·630	0·640	0·643
0·70	0·604	0·616	0·629	0·629	0·637	0·638
1·00	0.605	0·615	0·627	0·627	0·632	0·627
1·30	0·604	0·613	0·623	0·623	0·625	0·621
1·60	0·602	0·611	0·619	0·619	0·618	0·616
2·00	0·601	0·607	0·613	0·613	0·613	0·613
3·	0·601	0·603	0·606	0·607	0·608	0·609

Variations of Coefficients.—It will be observed from the above table, how and to what extent the coefficients increase with the ratios of the heads and of the sides to the width of opening. Some experimenters have found the increase to continue uninterrupted for all orifices up to the surface, but this seems to hold only for depths taken at or near the orifice when it is square or nearly so. It has also been found that the coefficient increases as the orifice approaches to the sides or bottom of a vessel: as the contraction becomes imperfect the coefficient increases. The lateral orifices A, B, C, D, E, F, G, H, I, and K, Fig. 125, have coefficients differing more or less from each other. The coefficient for A is found to be larger than either of those for B, C, E, or D; that for G or K larger than that for H or I; that for H larger than that for I; and that for F, where the contraction is general, least of all. The contraction of the fluid on entering the orifice F removed from the bottom and sides is complete; it is termed, therefore, *general contraction;* that at the orifices A, E, G, H, I, K, and D, is interfered with by the sides; it is therefore incomplete, and termed *partial contraction*. The increase in the coefficients for the same sized orifices at the same mean depths may be assumed as proportionate to the length of the perimeter at which the contraction is partial, or from which the lateral flow is shut off; for example, the increase for the orifice G is to that for H as $c\,d + d\,e : d\,e$; and in the same manner the increase for G is to that for E as $c\,d + d\,e : c\,d$. If we put n for the ratio of the contracted portion $c\,d\,e$ to the entire perimeter, and, as before, c_a for the coefficient of general contraction, we shall find the coefficient of partial contraction to be equal to

$$C + {\cdot}09\,n$$ [*]

for rectangular orifices. The value of the second term $\cdot 09\,n$ is derived from experiments.

These remarks, and what has already been said, will be sufficient to assist the judgment in the selection of coefficients to obtain correct results as to the discharge from an overfall, so far at least as they are to be obtained, for the whole subject is for all of us still surrounded by many difficulties.

When the spot from which the heads to the still surface are measured, is approached by the water with any degree of velocity, so that the surface is no longer still water, but in fact a current, it will be necessary to ascertain the head due to the velocity, and add the head thus found to the measured head or depth of water at the spot above mentioned. This occurs so often in stream gauging, when there is any considerable addition to the body of the stream, that it requires to be attended to. Reference to Neville's Tables will give the head corresponding to certain surface velocities per minute, which will have to be added to the measured heads before any correct computations are made; putting h for the head due to such velocity, the formula is,

$$\text{Discharge in cubic feet per minute} = 481{\cdot}8 \times C \times a \times \sqrt{h + h'}$$

Example.—Overfall 5 feet long; h, the head measured at 5 feet above the overfall $= 0{\cdot}666$; velocity of the current at from 5 to 10 feet above the overfall, 20 feet per second; coefficient assumed at $0{\cdot}505$; h' the head due to the velocity of approach.

$481{\cdot}8 \times 0{\cdot}505$ or $C = 243{\cdot}31$; $a = 5 \times 0{\cdot}666 = 3{\cdot}330$; from the table of head corresponding to velocity, h' the head due to a velocity of 20 feet per second = 0·0624; $h + h^1 = 0{\cdot}7284$; $\sqrt{h + h^1} = 0{\cdot}8532$, and

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[*] Neville's Hydraulics.

$$243{\cdot}31 \times 3{\cdot}33 \times 0{\cdot}8532 = 691{\cdot}28 \text{ cubic feet per minute.}$$

The table of discharges by overfalls gives the quantities according to the coefficient employed, for one foot in length of overfall, and from one tenth of a foot to one foot and a half of head. For closer approximations consult Neville's work,* which should be in the hands of every student of hydraulics, being the most complete work on the subject in the English language.

The means of measuring the velocity of approach, when sufficient to be taken into consideration, will be mentioned hereafter.

We will now say a few words as to the erection of the overfall boards for gauging streams. It is very desirable, in the first place, unless there should be preponderating reasons for acting otherwise, to look for a run of the stream, where the banks are straight and even, as also the bed of the stream, and if there be a spot where the banks widen out so as to form a kind of basin, then just below will probably be a good spot to construct the overfall or notched board, which should consist of 2-inch planking carefully jointed, with joints caulked, if necessary, so as to prevent leaking; when of any length it will require strong framing; in this boarding a notch is to be carefully cut, perfectly rectangular, and with dimensions carefully ascertained; we prefer the length of this notch to be small rather than large in proportion to the width of the stream, but not sufficiently so as to *create* a velocity in the approaching water; the edges of the notch to be carefully bevelled off to an angle of 45 degrees; at the back of the upper angle, along the sill and vertical sides should be fastened a plate of smooth iron, filed so that its edges may coincide with the bevelled sides of the notch, and so that the whole notch may be smooth and sharp; the metal not only completes the sharpness of the edges, but preserves it so for any length of time; if omitted it will be found in a few months, more particularly after floods and frosts, that the edges of the unprotected timber become ragged, and interfere with the contraction of the fluid vein, more particularly with low heads. The depth of the whole of the planking must be sufficient to reach from the top of the banks, unless they happen to be very high, down into the bed of the stream for a depth of about a foot and sometimes more, according as this happens to be a sound loam or clay, or loose sand or gravel; this is necessary to prevent leakage; and it should be long enough from end to end to reach well into the banks of the stream.

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* Neville's Tables, Coefficients, and Formulæ.

The setting up of a dam of this kind across a stream may appear at first sight a very easy matter, but in some cases it gives a good deal of trouble to get it properly done. Care must be taken that it be fixed at right angles across the stream, and that the centre of the notch coincide with the centre of the current. The boarding being prepared and its position determined on, narrow trenches are to be cut down the banks of the stream and along the bed to receive it, and it will generally be found advisable to have a heap of earth, clay if obtainable, well tempered and worked, ready at hand before attempting to fix the dam in its place, so that immediately it is done the earth may be thrown in, and trampled down in the fashion of puddle, before the stream dammed back has time to overflow the notch, or work its way in any quantity round the sides of the boarding; a rough kind of timber apron should also be in readiness to *fix*, in the bed of the stream in front of and well below the dam, to receive the fall of water as soon as it overflows the notch; where there are any sluices above the overfalls, so that the water may be temporarily turned off, even if partially, this portion of the work may be considerably facilitated; these sluices might often be employed for gauging a stream, but they are generally badly built, and often ruinous in condition. When we have not been able to procure clay or loam for puddling in a sandy district, we have employed bricks and cement, but everything must be at hand and the work quickly done, and the apron laid on before the notch overflows.

With regard to the level at which the sill of the notch is fixed; the higher this is placed the better will *still water* be secured to measure the heads from; but there are one or two conditions to be considered in connexion with this. Where the stream runs along between steep banks, the sill of the overfall may be raised to a considerable height above the level of the surface of the water when the dam is set up; but in meadow land, where the banks are not so deep, if the sill were raised too high, the first heavy rains would cause the water to overflow the banks above the overfall; the level of the sill must therefore be established accordingly; it is, however, very desirable that the sill be sufficiently raised to prevent its being easily submerged by any increase of water in the stream.

A few feet above the overfall, a square post must be driven in the centre of the stream with the top level with sill of the proposed weir, which had better be done before setting the weir; a sharp feathered edge rule, some two feet long, divided into tenths and hundredths, should then be nailed to the post, with its zero exactly level with the edge of the sill of the overfall; this will have to be done with the spirit level; divisions on the rule should be distinctly marked that they may be correctly read a few feet off.

This rule gives the means of ascertaining the total head of water above the sill, minus that due to the velocity of the stream, when there is any sufficiently great to deserve consideration, as above mentioned.

It will be better that the foot be adopted as the unit of measurement, as being much more convenient than inches; where there are perhaps a dozen overfalls, each to be gauged a great many times occasionally in the course of a day, as for instance during and after wet weather, it will be perceived that every facility that can be obtained towards readily effecting the calculations is an object of importance.

MEASURING THE VELOCITIES OF STREAMS.

Mean Velocity.—The greatest velocity in rivers and streams is along the surface of the greatest current, and gradually decreases towards the bottom and sides of the bed of the channel; if we call the maximum velocity V, and the mean velocity v, then in many ordinary cases,

$v = 0{\cdot}81$ V for small channels,

and $v = 0{\cdot}835$ V for large channels, but not rivers formed by nature.

Hydraulic Mean Depth.—In pipes and streams the perimeter in contact with the water is called "*the border;*" the hydraulic mean depth is equal to the sectional area of the water divided by the border; in rivers and streams the border consists of the bed and wetted sides of the channel; with reference to pipes, the hydraulic mean depth is termed the mean radius, and for full pipes is found by dividing the area of the pipe by its circumference; for pipes flowing full "*the mean radius*" is equal to *half the radius of the pipe.* Where there are many aquatic plants, Girard considers the border should be multiplied by 1·7 before dividing the area.

Hydraulic Inclination.—This for *streams and rivers* will be found with sufficient accuracy for all practical purposes by dividing the *head* by the length of channel, or, which is the same thing, taking the natural sine of the angle of inclination of the surface water.

Another rule for finding the mean velocity per second, is to take $\frac{10}{11}$ of a mean proportional between the fall in feet in two English miles, and the mean hydraulic depth; putting r for the mean hydraulic depth, and h for the fall in one mile, then

$$v = \frac{10}{11}\sqrt{2 r h};$$

or multiply twice the fall in feet per mile by the mean hydraulic depth in feet; extract the square root, and take $\frac{10}{11}$ of it for the mean velocity per second. This rule, however, rather gives an excess of velocity, and besides, it involves the necessity of extremely careful levels to find the inclination of the surface of the water, or the fall.

Unless under very favourable circumstances, the coefficients 0·81 and 0·835 given above will also be found to give too great results in many cases. Where a stream is shallow, when r, the mean hydraulic depth, will be small, where the bed is very stony or covered with weeds, and the banks crooked, 0·81 may be reduced to 0·7; so that for small channels the coefficient will range between 0·81 and 0·7.

The velocity of water in streams and rivers is ascertained by floats or by the current meter; we shall mention the floats first.

Supposing that the mean velocity of the water is to be made the means of ascertaining the quantity discharged by a stream some few feet in width; select as straight and clean a run as the stream affords, and drive in a couple of stakes some distance apart, the length of which has been measured, and observe the velocity by a float carried down by the stream from stake to stake; this should be repeated a few times to obtain a mean; when the float is laid on the water it should be a few feet above the highest stake, in order that it may receive the impetus due to the current at the first stake; some persons prefer very light floats, barely touching the surface, but they are very likely to be affected by the least breath of wind. For such a case as the present we prefer a hollow tin ball about 1½ inch in diameter, painted white, and with a small hole that may be stopped with a peg of wood, so that sufficient water may be admitted to sink the ball to the surface of the water; pieces of white wax have also been recommended. The surface velocity being obtained, the mean velocity is deduced from it in the manner we have mentioned above; if the surface velocity be thus ascertained in three places across a stream, the one in the centre, and the others half way between it and the banks, then the central mean velocity may be multiplied by the greater coefficient for small channels, unless very shallow and with rough stony bottom, and the side velocities by lesser coefficients, according to the state of the banks; a mean transverse area of the water multiplied by the mean velocity gives the quantity discharged; or, for the three mean velocities take the mean areas due to each velocity; judgment will be here quite as useful as rules; a length of 15 or 30 feet will be quite sufficient to gauge at a time; calm weather is indispensable; to check the calculations, repeat the operation on another length above and below, carefully observing, however, that there are no drains running into them so as to increase the quantity of water, which would in all probability increase the velocity, unless some remarkable change occurred in the bed of the stream; we have sometimes seen drain-pipes running with such a full bore and impetus into a stream that the quantity of water was thereby considerably increased.

The system of gauging streams by mean velocity is not so accurate as that by overfalls, but where time is precious or other circumstances make it advisable to resort to it, a very tolerable approximation to the true quantity may be obtained if proper care is bestowed upon it; where the proportion due to each of two streams forming a junction is to be obtained, observations must be made on each above the junction, and on the united stream below; the results obtained from the latter should of course be equal to the two former, unless there happen to be springs or drains to affect the quantity of water in the united stream; many persons do not consider the measure of surface velocity satisfactorily obtained until the distance between the stakes has been repeatedly traversed by the float in equal times.

In gauging the discharge of a river, large or small, by the system of mean velocities obtained from surface velocities, we have to observe the same principle as above mentioned, but we shall have to proceed into more details accordingly as the importance of the river or its volume of water increases.

Having selected as straight and fair a run as there is to be found, stake off equal distances on both sides of the river, and at right angles to its course; at each of the stakes set up a rod.

Levels for cross sections of the river, some 15 to 30 feet apart, should then be taken on the lines marked out by these stakes, and another midway between; and we always plumb the water first and stake out where the levelling staff has to be planted. From these cross sections the areas of the waterway may be calculated, and, if required, the area of the waterway may be divided into sections at *a*, *b*, *c*, *d*, *e*, Fig. 126. The object of the three cross sections is to ascertain that there are no material differences between them as regards the form of the bed of the river, to obtain an average of the waterway from the three, and also averages of the areas due to the sections into which each cross section is divided. From the cross sections on paper the lengths of the divisions *a*, *b*, *c*, *d*, and *e*, are taken, and these are staked off in the river; iron-shod poles about 10 or 12 feet long are used for this purpose, which a boatman will soon drive into the ground; midway between these the surface velocities are taken, by means of white tin balls, as we have just mentioned as regards streams, only rather larger in diameter, about three inches, so that they may be readily seen. We have thus the surface velocities for each of the sections into which each cross line is divided; with regard to the coefficients by which each of these is to be multiplied to obtain the mean velocities, it is to be observed of the coefficients ·81 V, and ·835 V, mentioned above, that they are due for the surface velocities in the centre of the stream, and those by which the mean velocity of the whole of a stream, from the surface water to the bottom, and from the centre to the sides or banks, is to be obtained where applicable. The greatest coefficient will have to be applied to the surface velocity of the section *d*, a smaller one for *c*, and less again for *b*, and the least for *a* and *e*, where the friction will in every way be more marked in its effects.

We prefer obtaining our mean velocities by direct experiments in the following manner, supposing we are still adopting floats, of which we take a couple, connected together by eyes and a cord long enough to reach from the surface to near the bottom of the water; *one* of these we load sufficiently to sink nearly to the bottom, and to draw down the other, left quite empty, so that it is just perceptible on the surface; the velocities thus obtained we take for mean velocities. The advantage of having one float loaded and the other unloaded, is, that the slightest obstruction on the lower float, or on the line, becomes quite perceptible on the upper one, in which case we reject the trial thus made; after taking half a dozen satisfactory experiments, we take the mean for the mean velocity.

Elliott's Current Meter, Fig. 129A, is a most useful instrument for ascertaining velocities, either a few inches below the surface, at the bottom, or at any depth between. A is a rod, by means of which the instrument is sunk to any required depth; C, a vane consisting of four spiral wings, called the *rotator* or *fly*, the axle of which carries an endless screw which turns a toothed and graduated wheel, B, by which the velocity is indicated; F is a portion of a plate by which the instrument is steadied in the course of the current.* The *constant* due to each instrument must be ascertained by direct experiment, by drawing it through still water a certain number of feet per minute, or submitting it to the action of a current, the velocity of which has been carefully ascertained. Massey's Patent Log, for measuring velocities at sea, is constructed on a similar principle; the whole is contained within a brass wedge-shaped box; there are three cogged wheels instead of one, acting on each other in such proportion that a total revolution of one completes a division of the next, or $\frac{1}{20}$, a revolution of the next $\frac{1}{8}$, registering thus from 160 miles to tenths and decimal parts. It is towed astern by a stout lead line of 60 fathoms. Mr. Beardmore says that he has had a current meter constructed with two wheels, the second having one tooth less than the first, and showing in its revolution about 1128 turns of the first; this gives the power of leaving the instrument a considerable time under water, which is very desirable to obtain a good mean velocity. There are two or three other instruments made for measuring velocities, but the current meter above described is now in general use in preference to any other.

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* The "Fly" is brought into action on the wheel by tightening the line which acts on the spiral spring, and the action ceases as soon as it is loosened.

In connexion with stream gauging, it is generally of the greatest importance to ascertain the effect of rain on the discharge of streams, both as regards depth of rain and time. Where there are many overfalls and rain-gauges, and consequently a great number of observations to make during the day, the operation of gauging the depth of rain fallen by means of the glass we have described becomes troublesome, and takes up a good deal of time. At Fig. 129 is shown an addition to the rain-gauge, by which the depths of rain falling during a wet day may be ascertained with great facility; it will be perceived that leading from the funnel to the reservoir R is a tube containing an inner one, the space between the two conveying the water from the funnel to the reservoir; to the upper part of the inner one is fixed a glass tube T; the dimensions of the reservoir are made according to what we have already said about the gauge glass, so that any defined rise in the reservoir gives a corresponding depth of rain; at the bottom of the reservoir is a float, which rises as the rain falls, and carries with it a very slender rod rising up through the inner pipe into the glass tube, which is graduated; the depth of rain falling is thus ascertained at a glance on the graduations above zero, with which the top of the rod coincides when the reservoir is empty. When the float is not in use, the inner pipe may be stopped up by means of a cork, so that there is but little room for the action of evaporation.

It is generally also very desirable to distinguish the average flow of a stream from the flood streams, which is often difficult to do. Mr. Leslie has proposed the following method:—

Tabulate the gaugings in the order of their quantities, and divide the *number* of observations into four equal parts, and take the least fourth for droughts and the highest for floods, the *average* of the middle half is then obtained, and all quantities in the whole table above that average are set down as flood water; next, all the gaugings not exceeding the average of the middle half are tabulated at their actual quantity; but all that are above the average are put down as equal to that average quantity; the mean of the whole of the new table is to be considered as a fair average of the water flowing in the stream, exclusive of floods. Mr. Leslie gives a table of a stream thus treated, which varied in its run from 1902 to 59,861 cubic feet per minute. Average of the whole was 10,231 cubic feet per minute; the average of the middle half was 7,234 cubic feet per minute; and the average quantity, exclusive of floods, was 5,830 cubic feet per minute.

Having the depth of Rainfall in inches, to ascertain the number of cubic feet per acre.—Multiply the constant, 363, by the depth of rain in tenths of inches; the product gives the cubic feet per square acre.

Examples.—Depth of rain 4 inches, required the number of cubic feet per acre.

363 × 40 = 14520 cubic feet per acre.

Depth of rain 5½ inches, required the number of cubic feet per acre.

363 × 55·0 = 19965 cubic feet per acre.

Having the depth of Rainfall in inches, to find the number of cubic feet per square mile.—Multiply the constant, 23232, by the depth of rain in tenths of inches; the product is the number of cubic feet per square mile.

With the depth of Rainfall in inches, to find the weight in tons per square acre.—Multiply the constant, 10128, by the depth in tenths of inches; the product is the weight required nearly, with three places of decimals.

To find the weight in tons per square mile from the depth of Rain in inches.—Multiply the constant, 64821, by the depth of rain in tenths of inches; the product is the weight required nearly, with two places of decimals.

To find the power of a Fall in Mill-work.—Multiply the discharge in cubic feet per minute by the height of the fall in feet, and by 62·5, and divide the product by 44000; the quotient is the amount of horse power. In practice, a discharge of twelve cubic feet per second, with a fall of one foot, is taken as equal to one-horse power; therefore the number of cubic feet discharge per second, divided by 12, and multiplied by the fall in feet, is equal to one-horse power.

Example.—130 cubic feet discharge per second, with a fall of 48·75 feet = 528 horse power. In the ordinary state of works the wheels are about filled to two-thirds, that is, two-thirds of the bucket are considered as filled to produce a useful effect; and in the above, two-thirds of 528 = 352 horse power is about the useful effect.

CHAPTER XV.

LEVELLING.

Curvature.—The Dumpy Level.—Adjustments.—Levelling Staves.—The Dumpy Level.—Levelling Staves.

If the earth were a perfect sphere covered with water, unaffected by wind or tide, the surface of the water would trace a surface parallel to the horizon in every direction, which surface would consist of an infinite number of level lines, all of which would intersect the radii drawn from the earth's centre at equal distances, and at right angles to them. The tracing of such a line of any length on the earth's surface, or at any given distance from it, would be setting out level lines.

The practice of levelling, in the engineering or surveying application of the term, consists of the ascertaining of the difference of level between any two places, or the differences of level between any number of places, whether in a straight line, or situated in any other manner on the earth's surface.

Let A E B, Fig. 130, be a portion of the surface of the earth, with its centre at C; let an observer be supposed standing at E, with a spirit-level in adjustment; *t t′* will be the line coinciding with the axis of the telescope; this line would be parallel to T E T, at right angles to E C, a line supposed to be dropped from the station E′ to C, the centre of the earth; the true level line would be the curve *a* E′*b*, parallel to A E B; *t* E′*t′* is the apparent level, and *a* E *b* the true level line; and the difference equal to *a t*, *b t′*, E′*t* or E′*t′* is the sensible horizon, *a* E or E *b*, the real horizon; but the visual line along the axis of the telescope produced is distorted by the effect of refraction, and both these circumstances have been alluded to under the head of "Surveying Instruments." If the level between E′ and *b*, instead of being set out at once from E′, had been set out, say at 10,000 stations, each time gradually nearing *b*, then the apparent levels would have so nearly coincided with the true levels, that the differences would not have been worthy of consideration.

This is exactly what obtains in the practice of levelling; in ten chains, for instance, the combined allowance for curvature and refraction is only nine-thousandths of a foot, and ten chains is a tolerably long sight. As, however, instances may occur when it would be useful to apply it, we will give one example for doing so:—Let E in the last figure be a spot where a level is set up and adjusted; on making an observation on a house seven miles off, the centre of the diaphragm intersects a window on the first floor of the house; from the table of differences between the apparent and true levels for miles, the tabular number is found to be 28·01 equal to the difference of level, and the window which appeared on the same level as the eye of the observer at the telescope, is in fact 28·01 feet higher; but to find the difference of level between the ground on which the observer is standing and the window, the height of the instrument above the ground must be added to the above correction; let this be four feet, then $28{\cdot}01 + 4 = 32{\cdot}01$ equal to the difference of level between the ground at the observer's station and the window in question. In the absence of tables, a convenient rule to remember is that two-thirds of the square of the distance in *miles*, is equal to the correction in feet.

The practice of levelling is a very simple operation, but it is very certain that it is one of the most delicate the engineer has to perform, and demands the greatest caution and attention.

We will first describe the levelling instrument, that is the "level," and shall confine ourselves to that so well known as Gravatt's "Dumpy Level," being the one used by at least ninety-nine persons out of every hundred. It is shown at Fig. 130A. As much of what has been said on the subject of the theodolite is applicable to this instrument also, our remarks will be shorter than they otherwise would be. As generally constructed for packing in its case, the "Dumpy Level" consists of the telescope, the tube above carrying the spirit-bubble, or rather, the air-bubble in the spirit, with a small mirror and cross level attached, and of the compass-box under the telescope; this is screwed on to the axis of the instrument with the parallel plates P P P P, and these again on to the tripod staff, the same as the theodolite. The spirit-level carries a diaphragm consisting of two vertical wires and one horizontal wire. The object of the small mirror, placed on the telescope and mounted on a hinge-joint when it is in use, is for the observer to be able to ascertain, at the same time that he is reading the levelling-staff through the telescope, that the instrument has remained steady in its position, by noticing in the mirror any alteration in the position of the bubble with regard to the divisions engraved on the glass tube; the small cross level is to assist in setting up the instrument approximatively by means of the legs only, by which a great deal of time is saved, as well as wear and tear of the parallel plate screws. This cross level is always made much too short, the one we have attached to our level is six inches long, and we find it very useful in quickly setting up the instrument; a steady motion, when adjusting to distinct vision, is secured by the mill-headed screw in the side of the telescope. The little mirror we have mentioned is seldom used, but is nevertheless very useful in going over soft springy ground or in windy weather; in boggy ground it is particularly so, for here the slightest motion of the observer may disturb the instrument, which, by keeping the mirror in sight whilst observing, will detect the disturbance. The instrument is so constructed that the diaphragm is adjusted by means of the capstan-headed screw near the eye end of the telescope. The little lens attached to the compass-box is for reading off the graduations on the silver ring.

As we suppose the reader has already studied the construction of the theodolite in former pages of this work, and will therefore have no difficulty in fully understanding that of the level, we shall now give Mr. Gravatt's method of adjusting his level.

"*To Examine and Correct the Collimation.*

"On a tolerably level piece of ground drive in three stakes at intervals of about four or five chains, calling the first stake *a*, the second *b*, and the third *c*.

"Place the instrument half way between the stakes *a* and *b*, and read the staff A, placed on the stake *a*, and also the staff B, placed on the stake *b*; call the two readings A′ and B′; then, although the instrument be out of adjustment, yet the points read off will be equidistant from the earth's centre, and consequently level.

"Now remove the instrument to a point half way between *b* and *c*. Again read off the staff B, and read also a staff placed on the stake *c*, which call staff C (the one before, called A, being removed into that situation). Now by adding the difference of the readings on B (with its proper sign) to the reading on C, we get three points, say A′, B′, and C′, equidistant from the earth's centre, or in the same true level.

"Place the instrument at any short distance, say half a chain, beyond A, and, using the bubble merely to see that you do not disturb the instrument, read all three staffs, or, to speak more correctly, get a reading from each of the stakes *a*, *b*, *c*: call these three readings A'' B'' C''. Now, if the stake *b* be half way between *a* and *c*, then ought $C''-C'-(A''-A')$ be equal $2\,[B''-B'-(A''-A')]$; but if not, alter the screw which adjusts the diaphragm, and consequently the horizontal spider-line, or wire, until such be the case; and then the instrument will be adjusted for collimation.

"To adjust the spirit-bubble, without removing the instrument, read the staff A; say it reads A''', then adding $(A'''-A')$ with its proper sign B' we get a value, say B'''.

"Adjust the instrument by means of the parallel plate-screws, to read B''' on the staff B.

"Now, by the screw attached to the bubble-tube, bring the bubble into the centre of its run.

"The instrument will now be in complete practical adjustment, for level, curvature, and horizontal refraction, for any distance not exceeding ten chains, the maximum error being only $\frac{1}{1000}$th of a foot.

"Example.—The instrument being placed half way between two stakes, *a* and *b*, (at one chain from each,) the staff on *a* or A' read 6·53, and staff on *b* or B' read 3·34, placing the instrument half way between the stakes *b* and *c* (three chains from each), the staff on *b* read 4·01, and the staff on *c* read 5·31.

"Hence, taking stake *a* as the datum, we have

Stake.		*Above Datum.
a or A'	=	0·00
b or B'	=	3·19
c or C'	=	1·89

"The instrument being now placed at *d* (say five feet from *a*, but the closer the better), the staff on *a* or A'' read 4·01, on *b* or B'', 1·03, and on *c* or C'', 3·07. Now, had the instrument been in complete adjustment (under which term curvature and refraction are included) when the reading on staff *a* was 4·01, the readings on *b* and *c* should have been respectively 0·82 and 2·12.

"The instrument therefore points upwards, the error at *b* being 0·21, and the error at *c*, 0·95: now, were the bubble only in error (as is supposed in all other methods of adjustment), the error at *c* ought to be four times as great as at *b*, but 4 × 0·21 = 0·84 only, there is an error, therefore, of 0·95 − 0·84 = 0·11 not due to the bubble.

"For the purpose of correcting this error (and be it remembered contrary to former practice, for this purpose only), we must use the capstan-headed screw at the eye end of the telescope, and neglecting the actual error of level, we are only to make the error at *b* one-fourth that of *c*.

"After a few trials, whilst the reading at *a* continued 4·01, the reading on *b* became 0·75, and that on *c*, 1·84.

"Now $0·82-0·75=0·07$, and $2·12-1·84=0·28$.

"And as $4\times 0·07=0·28$, the telescope is now adjusted for collimation.

"All that remains to be done, is to raise the object end of the telescope by means of the parallel plate-screws, until the staff at *c* reads 2·12, and then to bring the bubble into the centre of its run.

"The operation of collimating, when once performed upon levels on Mr. Gravatt's construction, will scarcely ever need being repeated."

The instrument may also be collimated by means of a pool of still water, if large enough to have a couple of stakes, three or four chains apart, driven exactly level with the surface of the water; the instrument should be set up about half a chain beyond one of the stakes; if the line of collimation is correct, the readings of the staff placed alternately on each of the stakes will be exactly the same; if different, the instrument must be collimated in the manner already described.

We must not omit to state that, in observing through the Dumpy Level objects are inverted, that is, everything is seen upside down. *At first* this has a disturbing effect on many persons; but after a few hours it passes entirely away, and we should now feel at least equally disturbed on looking through a level to perceive any other result; it is merely to read the staff downwards instead of upwards.

In almost all instruments it is very difficult, and sometimes, practically speaking, almost impossible to make the bubble maintain the centre of its run, whilst the instrument is turned completely round on its axis; that is, to use the technical expression, to make the telescope reverse; a tighter or slacker connexion between the telescope and the parallel plate portions of the instrument, will often be the cause of it. For this reason, therefore, and to prevent accidents, we must *particularly impress* on the mind of the student the necessity of always turning the telescope *to the right* when reversing it; if it were often turned to the left, not only will it reverse all the more incorrectly, but the telescope may get screwed off and fall to the ground. When the instrument will not reverse correctly, a mere touch on one of the parallel plate screws will bring the bubble back to the centre of its run.

Parallax is also to be carefully attended to when levelling; it exists when on looking through the telescope at a distant object, and moving the eye sideways, or up or down, the wires appear to move. Generally a motion of the mill-headed screw in the side of the telescope, one way or the other, rectifies this, at the same time that it produces distinct vision; at the same time it brings out the wires clear and distinct against the divisions on the levelling staff.

In looking at our drawing of the Dumpy Level, it will be perceived that one of the parallel plate screws rests in a notch or groove on the lower plate.

When setting up the level for use, we prefer to have this particular screw over one of the legs of the tripod, and this leg we keep nearest to us in planting the instrument on the ground, at the same time we bring the eye end of the telescope towards us; so that in fact, it is at right-angles to the position shown in the drawing. We obtain with the legs an approximation of the bubbles to the centres of their runs; we then fix the two legs furthest from us firm in the ground, and by means of the third leg moved backward or forwards, or sideways, as the bubbles may appear to require, we get them so nearly in the centres of the tubes, that a touch or two on the parallel plate screws brings the whole into complete adjustment, as far as the bubbles are concerned. The next adjustment is for distinct vision, and to prevent parallax, which we need not repeat here.

The tripod we always prefer to have made of fir, instead of mahogany; stout, and six feet in length when closed for carriage; this length enables us to get a firm wide spread when set up for use, and with such a stand there is less to apprehend from the wind causing vibration of the instrument when observing; when high, however, its effects will be felt more or less; therefore in windy weather we avoid taking long sights as much as possible, ten or twelve chains for instance, or else watch for a lull in the wind to do so.

Levelling Staves.—The different descriptions and modifications of levelling staves introduced at different times since the old vane-staff has become obsolete, and which we shall therefore not describe, have been almost innumerable; that in use now was introduced by Mr. William Gravatt, and a description of its principle of construction will enable the reader to make any modification of it he may please. This staff varies in length from 14to 17 feet; it consists of three joints, and when stowed away for carriage, they are thrust down into each other, but drawn out when the staff is in use; the bottom joint is about four inches wide, the whole length is divided into feet, tenths, and hundredths; the two first are numbered, but not the latter, the first in large, and the tenths in smaller figures; the figures of the feet are often painted red, those for the tenths being always black. Figure 131 will save a lengthened description, and make the whole plainer to the reader than words can possibly do; Fig. 131 shows the whole of one foot reduced to one-fourth of the actual size; and 131A shows a little above and below one-tenth of a foot full size; the figures for tenths are, it will be seen, entered alternately, when a staff thus graduated is observed through the telescope of the level, the divisions are read off by the observer himself; but when it happens to be at a distance of ten or twelve chains, every possible assistance is desirable to make the divisions more distinctly visible, and observe that here the figures are made of such lengths that at top and bottom they coincide with a full tenth of a foot, as 9·10, 0·20, 0·30, &c., which is in every case at the *top* of a black division, and 0·15, 0·25, 0·35 at the *bottom* of a black line longer than all the other hundredths between the tenths; and so it is with all the odd hundredths, the even numbers are at the *top* of a black line or band, and the odd numbers at bottom. It will also be observed that with the figures 3 and the 9, portions of them coincide with some particular hundredths; when the instrument is not too far from the level, and the wires of the diaphragm are clear and sharp and free from parallax, it is very easy to subdivide the hundredths of a foot, and obtain such a reading as, for instance, 1·325. Such readings are, however, by no means desirable for ordinary levelling, as they greatly increase the labour of *casting out and reducing*, which will be hereafter explained; but occasionally for some hydraulic observations, as for instance, obtaining the fall of the surface of a river, it is extremely useful to be able to read to such a nicety, and with short sights and the instrument in thorough adjustment in all respects, it is quite possible, as the reader will see, to obtain even closer readings of the staff than the above.

Attempts have been made to introduce more minute divisions on the staff, but they have not been successful, the result being that even at short distances the divisions became confused instead of being distinct.

The greatest fault that can be found with a staff, the joints of which, as the above, slide down into each other, is that the topmost joint becomes inconveniently narrow to receive the figures for the decimals, as well as those for the feet; as, for instance, 12, 13, &c.; after a little use, however, one gets accustomed to this; but when stowing away for carriage is not necessary, a 15 and even 20 feet staff may be made of a piece of clean, well-seasoned wood, about four inches wide from top to bottom, and about two inches thick at the lower end, and tapering to about half an inch thick at the top. A staff of these proportions, carefully graduated, is a valuable instrument for nice work, though when longer than 15 feet, it is heavy to carry any long distance. For some hydraulic observations, such as taking levels across a deep river, a 20, and even 30 feet staff is used; to obtain a hold at the bottom of the water, the foot of the staff is armed with three short prongs or points, which stick into the bed. A 16 or 18 feet staff, of the width of the above, may also be made in two or three pieces, which are fixed one on top of the other by means of sockets, like the ferrules of a fishing-rod; for ordinary use, however, the staff made as first described will be found the most useful and convenient. The manner in which the graduations and the figures either for the feet or the tenths are shown on the staff, is of too much importance to be in any way overlooked, for whilst at short distances everything may be very clear even on a staff graduated in an inferior manner, it becomes quite indistinct at some of those longer distances at which the value of time demands that observations should be made; in the sliding staff we have mentioned, from the top joint being so narrow, it is in fact of the highest importance. It is a very good plan, on the left hand of the column of graduations, to paint the whole length of the feet alternately black and white, leaving, however, a vertical band of white three-eighths of an inch wide between the column of graduations and the alternately white and black border. This is certainly a considerable assistance in distinguishing the feet. On some staves the figures instead of being shown upright, appear sideways, which is a very good plan also where the width of the staff will admit of it.

We have already observed that the telescope of the level inverts every object seen through it, and if the instrument is so situated that when adjusted for vision, the axis bears on a point near the ground where the staff-holder is standing, he appears turned upside down. This has a strange appearance *at first*, but as said elsewhere, one becomes immediately accustomed to it, as it resolves itself simply into reading downwards instead of upwards; to remedy this, which some appear to consider so great an inconvenience, the figures have been painted *inverted* on some staves, so that they appear upright when seen through the inverting telescope; but we submit that this is neither one thing nor the other, the figures *seen inverted* naturally lead one to read from the top downwards, which is the proper way, whilst the *figures upright* would lead one to read from the bottom upwards, which would be wrong.

Having now given a description of the "level" and of the levelling-staff, we will enter on the subject of levelling. In Fig. 132, let all the distances between the horizontal lines represent depths of three feet each, and let all the vertical lines be supposed as one chain apart. Let A be the level of the surface of still water in the pond A W, and let it be required to ascertain the difference of level between this point and that at B. Let the levelling-staff be held upright at A, and let the observer proceed with his level up the inclined ground, until he judges by the eye that the level when set up and levelled at 1, or rather the axis of the telescope, will intersect the levelling-staff at A and B; set up the instrument, and level it roughly at first, to see that it will do so, and then perfectly, and do this in the manner we have mentioned when describing the level. When the instrument has been levelled, turn the telescope round to A, always turning round to the right instead of the left, and adjust for distinct vision, that is, so that the wires of the diaphragm come out sharp and clear on the staff, and that they appear steady and motionless when the eye is moved up and down before the eye-piece.* This done, look to the bubble to see that it has not moved, and if it has a little, then just touch either of the parallel plate screws to bring it back to the centre of its run; now read off the staff, observing whether the horizontal wire of the diaphragm intersects the staff at the edge of one of the black bands or stripes, for if in the middle of one of these, then half a hundredth will be a quantity in the reading. In our present case the drawing has been so arranged that the wire intersects exactly the top edge † of the staff, which is 18 feet long; the reading will therefore be 18 feet; had the wire bisected, say 1½ hundredths from the top of the staff, then the *correct* reading would have been 17·985. Our reading, however, is 18 feet, which is to be entered in a note-book; having made the entry, again look to the bubble to see that it has not moved from its central position. Signal the staff-holder, who will now carry his staff on to B. Now reverse the telescope so as to bear on B, adjust for distinct vision, which will be required on account of the distances B 1 and A 1 being very unequal; examine the level as to any slight alteration in the position of the bubble, correct it as before, if required, and read off the staff at B; the horizontal wire now intersects exactly the foot of the staff as it did before the top, and the reading will therefore be zero or nothing; then B is evidently 18 feet higher than A, correction for curvature, &c., being at present left unconsidered.

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* See *ante* what we have said on the subject of "Parallax."

† In the staff we have described, this edge would be a brass plate one hundredth of a foot thick, always fastened on to the top of the staff as a means of drawing out the top joint from the one below it.

Now let it be required to ascertain the difference of level between the point C and the same point at A as before. The difference of level is too great for us to be able to ascertain this at one setting up of the level as last done, but it may be done at twice by setting up the level at 1 and 2, and the staff at A, B, and lastly at C; the difference of level between A and B having been already ascertained, we have now to find the difference between C and B. With the level set up at the point 2, read off the staff at B, which is supposed not to have been moved from its exact position, but merely turned round, so that the figures may face the level at 2; let this reading be 15; signal the staff on to C, reverse the telescope as before, and read off the staff at C, and let this be 0·75. Had this reading been zero from 2 to C, as it was from 1 to B, then the difference would have been the total of the first reading from 2 on B, and we should have 15 feet difference of level, and $15+18$ would be the total difference of level from A to C. But instead of zero we have 0·75, and exactly as the reading B subtracted from the reading A gave 18 for difference of level, so now the reading C=0·75 subtracted from the reading B=15, leaves 14·25 for the difference of level between C and B, and 14·25+18=32·25, the difference of level between A and C. Let us now put *d* for the difference of reading between any two staves, and pursue our course on to E; at 3 we shall have 15·75 minus 0·75, equal to 15·00; at 4 we shall have 15·75−1·00=14·75; at 5 we shall have the reading on D=4·00 and on E=4·00; the two readings are equal to each other; then there is no difference, and from E to D is a dead level. Let us now tabulate the above readings, calling B the first reading at each setting up of the instrument, F the second reading, and *d* the difference between any two readings, which, as we have seen, gives the difference of level, and we shall have

	B	F	*d*
(1)	18·00	0·00 =	18·00
(2)	15·00	0·25 =	14·75
(3)	15·75	0·75 =	15·00
(4)	15·75	1·00 =	14·75
(5)	4·00	4·00 =	0·00
	68·50	6·00	62·50 = difference of level between A and E.
	6·00		
	62·50 = *d*, or diff. of level between A and E.		

Observe that by casting up the column *d*, we obtain the total difference of level between the points A and E; also, that by adding up the columns B and F, and subtracting the lesser from the greater, we obtain the same result, the total of B minus the total of F being equal to *d;* we might therefore have obtained the difference of level without ascertaining the differences of readings between B and F each time.

Observe also that in the above, the readings for which we have called B are *Backsights*, and F are called *Foresights;* also that we have been *ascending* all the way from A to D, getting a rise at each set of readings from the instrument, and that the column for which we have put *d* is, as far as D, called *Rise;* and in the columns B and F, the backsight, is each time greater than the foresight, which *is always the case with ascending levels*, and that the difference between the two gives us the entries for column *d* or *Rise;* where there is no difference in the readings, as between D and E, there is also no difference of level, and neither rise nor fall, and therefore zero or a blank appears in column *d*, which in future we shall call by the technical term *Rise*, as also B and F, the *Backsights* and the *Foresights*.

This much being thoroughly understood, let us pursue the levelling operation from E to F; at the setting up at 6, the backsight reads 1·00, and the foresight 18·00; the difference equal to 17·00, but instead of being a rise, it is a fall; at 7 we find a difference of level equal to 12·00, also a fall, and at 8 another fall of 12·00. Now if we were to enter all these in the column of rise, and add up all the readings, we should be adding rises and falls, and making no difference between the two; and inasmuch as in ascending levels, we add all the rises together to find the total rise, so in descending levels, the fall must be subtracted from the rise, to get the fall; and if the total of falls is subtracted from the total of rises, we shall obtain the difference of levels between two points on which there may be any number of falls or rises, or which is the same thing—any number of variations in the inclinations, up or down.

An additional column is required in the field-book for the differences of the readings in descending levels, and which column is called "Fall;" *the difference of the readings will have to be entered in this column of Fall whenever the foresight is greater than the backsight.* In the same manner with regard to the totals of the backsights and of the foresights, whenever the total of the first is greater than the total of the second, the difference of level will be a rise; and when the total of foresights is greater than that of the backsights, the difference of level will be a fall.

The operation of taking out the differences of level at each reading and entering them in their respective columns, is termed *casting out*, in doing which the principal thing to remember each time is, that when the backsight is greater than the foresight, the entry is made in the column of *Rise;* and when the foresight is greatest, then the difference is entered in the column of *Fall.*

At all times the differences of the totals of backsights and foresights is equal to the differences of the totals of the columns of *Rise* and *Fall.*

By referring to the above readings between A and F, it will be found that all the

Backsights equal to	72·50	Total Rise equal to	62·50
Foresights „	51·00	Total Fall „	41·00
Difference of level	21·50	=	21·50

The above "tottings up" are checks on the correctness of the *casting out.*

If we now continue our levelling operations on to G, we shall find that G is six feet below A, the descending preponderating over the ascending levels.

As yet we have only ascertained the differences of level, and not measured any of the distances as from A to B, to C, &c.; we are not therefore in a condition to make a drawing of our work, that is, "plot the section;" but if at the same time that the levels are taken the distances from staff to staff are measured, we can then lay down the distances at which the differences of level occur; from A to B the distance is 33 chains, and B is 18 feet higher than A; on to C the distance is 33 + 26 = 59 chains, and the difference of level 32·75; the chaining is carried on simultaneously with the levelling operations, and we get on to D and E; the distance being measured up to every staff, and the difference of level ascertained, we can lay down the distance and corresponding difference of level on paper; up to D we add to the heights all the way; at E there is no difference of level from that at D; but from E we begin to descend, and the differences of level to be deducted all the way to G; or, instead of rising we drop all the way from E to G when plotting the section as well as in the levelling. With the ruled paper in our example, and the figures above given, the reader will have no difficulty in plotting this section; this method, however, is tedious and liable to error in the drawing, and in practice we arrange our figures so as to obtain such results that we are enabled to lay out all the differences of level as heights above a certain point. In our example we have assumed the point A as the point of departure for distances and relative heights of level, and we may assume a line level with this point throughout the section from which to set off the heights, *b* B, *c* D, &c.; but on reaching G we shall have to set off a depth instead of a height: a negative instead of a positive quantity; to avoid this, it is the practice to assume this imaginary horizontal line at such a depth below the starting-point that no such inconvenience arises, and so that all the horizontal distances may be laid down on this line, and from the distances all the levels plotted as heights above this line, which is called *Datum*, or the *given* line to the level of which all the levels throughout the section are relative. It may be assumed at 10, 100, 200, or any other convenient depth below a point from which the levels commence, which point should always be of a permanent character, and thoroughly well-defined, so as to admit of no mistake at any time of future reference. The arrangement of the figures which we have mentioned, so that the levels may all refer to this line, is termed reduction of levels, and in our example the whole will be tabulated in the following form, assuming our *Datum* to be 18 feet below our starting-point.

Datum 18 *feet below Low-water mark.*

Distance.	Backsight.	Foresight.	Rise.	Fall.	Reduced Levels.
					18·00
33·00	18·00	0·00	18·00	...	36·00
59·00	15·00	0·25	14·75	...	50·75
68·00	15·75	0·75	15·00	...	65·75
77·00	15·75	1·00	14·75	...	80·50
91·00	4·00	4·00	...	...	80·50
98·00	0·50	18·00	...	17·50	63·00
103·00	3·00	15·00	...	12·00	51·00
108·00	0·00	12·00	...	12·00	39·50
118·00	3·00	16·50	...	13·50	26·00
128·00	2·50	16·50	...	14·00	12·00

In *plotting the section*, only the first column of distance and the last of reduced levels are referred to—the first for obtaining the horizontal lengths, and the last the vertical heights.

The *datum* line might be assumed at any height above instead of below the starting-point, but we are not aware that it is ever made so. If it so happens that levelling operations fall in a hilly country, no information about the levels of which is to be obtained, and where perhaps several lines of level are contemplated to ascertain the relative merits of different lines, some at a high and others at a low level, it is often as well to assume *datum* as 1000 feet below the starting-point; it is easy to take off 500 feet from the reduced levels if afterwards desired.

We must now make a few observations on the above method of keeping a field-book and the corresponding section.

The distances apart are all a certain number of whole chains, which is not likely to be the case on the ground, where not only differences of level between any given number of points are to be ascertained, but for the preparation of sections, all the undulations along the surface, so that this latter may be truly represented by the line of surface on the section; and in order to make any small differences of level the more apparent on the paper, it is customary to plot the vertical heights to a much larger scale than the horizontal distances; thus a scale of 80 feet to the inch is often used for plotting the vertical lines, whilst one of 6 chains to the inch is used for the horizontals, and again scales of 30 or 40 feet to the inch vertical against scales of 3 and 4 chains horizontal.

Whilst on this subject, we will refer to the ruled paper on which Fig. 132 is drawn; it is called section-paper, and was introduced by Mr. Brunel some years ago; it is not in general use except in his office; but it certainly has many advantages over plain paper, inasmuch as there is no necessity of drawing a vertical line at right angles every time to the datum on which to plot the reduced levels, because it is already done; neither is there any plotting of distances, as these are also marked on the section-paper according to the scale employed. The principal difficulty there is to contend with in using it is when joining the sheets end to end, to make the horizontal lines coincide, and to get them to lay in straight lines; of course scales are not required with this paper, as it is already done to hand. When instead of the section-paper, the plain ordinary paper mounted on linen is used, the datum line is first drawn in, and from the "cockling" of the paper, often occasions some trouble to get straight; on this line the horizontal lengths are first marked off in miles and furlongs, and the same written in; after this the chain distances where the levelling staves were planted, from which verticals are drawn up from which to plot the reduced levels.

Still referring to Fig. 132, it will be seen that the distances A B and B C are respectively 33 and 26 chains in length; and that the backsight 1 A 25 chains long, almost too great a distance to read with accuracy, and requiring the instrument to be in a very perfect state of adjustment; besides this, however, where correct levelling is required, it will be necessary to take the correction for curvature and refraction into consideration.

In the tables for corrections of curvature and refraction, 20·00 chains is the highest figure in the column of distances for chains, the correction being as the square of the distance; take the correction due to 5 chains, which is ·00224, and multiply by $5^2 = 25$; the product will be ·056, which will be the correction to apply to the reading 18·00 from 1 to B, which will reduce it to 17·94; the same method of proceeding may be applied to the backsight 2 B.

But by always setting up the instrument about midway between the staves sufficient compensation for practical purposes is made, without any other reference to the subject; by this midway setting up, any little mechanical defects in the level, either from construction or from its being out of order, is remedied so far that we need not be troubled by them; by this, however, it is not to be understood, that by midway we mean exactly half the distance between the staves, because any such close approximation would in practice be out of the question; what is really meant is, that when the distance between two staves is very great, it is not desirable to set up the instrument so that there shall be an extreme disproportion in the distances between it and the staves.

Intermediate Sights.—Instead of taking the levels along the previous surface A, B, C, D, ... to G, let us consider it as required to take the levels for a section along the line A, B, *c*′, *d*, *e*, *f*, *g*, *h*, *i*, F. The level of B we have already found to be 36 feet above *datum*; we *set up* the level at 11, and the staff is planted at B, and we get for backsight 1·50; the staff is moved on to *c*, and we read for foresight 7·50, which gives a fall of 6 feet, and distance 61·00. Now to obtain the difference of level between *c* and *d*, we might set up the level between, and so consecutively along the broken ground between *c*′ and *i*; but we may save much time under such circumstances by following another course. Now let it be carefully observed, that what is called *backsight* may just as well be termed first sight, and that *foresight* may equally well be termed second sight, for they mean one as well as the other; therefore, instead of moving the level from 11, after reading the foresight *c*, let the staff be moved on to *d*, and the reading taken from 11, which will be 13·50; the difference between the readings at *c*′ and *d* is equal to 6·00, which is the difference in the levels as may be seen on the section; then the reading, 7·50, which was formerly fore or first sight, has become back or second sight; let it be entered as such in the column *foresight* of the level-book, and the difference is a fall, for the foresight is greater than the backsight. Let the staff be moved on to *e*, and read off still from 11; this reading will be 4·50; but 13·50, the last foresight, now in its turn becomes backsight, and the difference, which is a rise—for the foresight reading is less than the backsight—is equal to 9 feet. We may proceed exactly in the same manner on as far as *i*, after which the level may be fresh set up where required, between *i* and F, to continue the levels onwards; from A the level-book will read in the following manner.

Datum 18 *feet below Low-water mark.*

Distance.	Backsight.	Foresight.	Rise.	Fall.	Reduced Levels.
					18·00
33·00	18·00	0·00	18·00	...	36·00
67·00	1·50	7·50	...	6·00	30·00
62·75	7·50	13·50	...	6·00	24·00
64·00	13·50	4·50	9·00	...	33·00
65·00	4·50	7·50	...	3·00	30·00
67·00	7·50	1·50	6·00	...	36·00
69·00	1·50	4·50	...	3·00	33·00
71·00	4·50	1·50	3·00	...	36·00—(*i*)

The above is the theory and practice of "Intermediate sights;" it is merely necessary to remember that the figures last read as foresight become the backsight to the next reading, as *c* the foresight to B becomes backsight to *d*, and *d* the foresight to *c′* becomes the backsight to *e*, and so on as far as *i*, or as long as the levelling *instrument* is not fresh planted. In practice it is of eminent service, as in taking levels over broken ground, across brooks, canals, rivers, or to obtain the details of the surface of a road, or indeed any other details as to levels when they are not far removed from each other; by following it out, the time saved in fresh setting up of the instrument is often very considerable, and the student will do well first to impress the theory on his mind and then practise it.

Bench Marks.—This is another subject which must be well understood before any levelling operations are undertaken. In Fig. 132, and according to what we have said before, we have commenced our levels from *Low-water mark*, and if in doing so we had no other defined mark than the surface of the water, we might never probably be able to find it again for future reference, as for instance, to check the accuracy of our levels, or to level from the same point for another section, for low-water may and does alter from day to day; but if we have started from a point permanently fixed, as for instance, a particular landing, or any other permanently established level, *known* as low-water mark, then we can refer to it at any time; with reference to levelling operations, this would be a "Bench mark." Similarly, in going over some miles of levels for a section, or any other object, we are certain to require at some future period some permanently fixed points, the levels of which may at any time be referred to without any matter of doubt; they may be on or off the actual line of section, so that they are not too far, and provided they are to be readily found. Without such marks we should have to look for exact spots on the surface of the ground where the staff had been held, and no reliance could be placed on anything of the kind, in such an important matter as that of a level, where a few inches are certain to be more or less of great importance; but if at the same time that the levels have been taken over a line of country, the levels have been taken at the same time to the sill of a particular door or window, the level coping of a wall, the lower or upper* hook of a gate, the top of a milestone, or horsing-block, the root of a tree where a *bench* is cut for the levelling-staff to be held, the up or down rail under the soffit of a bridge, at the north or south, east or west side, then we have so many defined points over which the levels may be again taken, and by such means the accuracy of former levels may be and are constantly checked. Therefore it should be remembered, that in taking a line of levels, it is of the utmost importance to take levels to some such bench marks, not at greater distances than a mile apart along the line, and nearer if convenient; if not directly on the line, it is of no consequence; no distance is measured to them, and they are merely noted as B. M. in the column of distances, and described in the page devoted to remarks. This page is always the right-hand page of the level-book, the left-hand one only being reserved for the various columns of entries such as we have already shown. Bench marks are not to be plotted on the section, as they may be at any time referred to in the level-book. It does not follow from what we have said that such marks are always to be found exactly at the end or beginning of a line of levels; they may be found some greater or less distance off, and either at a higher or lower level than the commencement of the line of section; suffice it to say, that they must be found somewhere, or else such permanent marks must in some manner be firmly constructed or sunk in the ground. For reasons to be explained hereafter, bench marks should be taken at every road crossed by the line of levels.

Staff-holder and Levelling-staff.—We have already described the construction of the levelling-staff; we must now make a few observations as to the method of using it, for as much care is required in doing this as the level itself. In setting up the staff for observation, with its face towards the observer, it should be held *firmly* on the ground, and perfectly upright; the staff-holder should stand facing the observer; he should hold the staff close to him, with the foot near his toes, and his hands on the *sides* of the staff, and not with his fingers across the face of the graduations. On level ground there is no difficulty in holding the staff perfectly upright, but on sloping ground it is quite different; if the staff leans at all *sideways*, it will not be parallel with the *vertical* wires of the diaphragm, and the observer signals right or left to the staff-holder accordingly; but if it leans from or towards the observer, he will not be able to discern this by the appearance of the staff; but the telescope will give the least reading upon it when it is quite vertical, therefore on sloping ground, when the staff is drawn out to two joints or to its full length of the three, the staff-holder should, as soon as he sees that the observation is being made, lean the staff backwards and forwards towards the observer, when the least reading will be the correct one, for the staff will then be perfectly upright. The observation being recorded, the staff-holder, at a signal made, shoulders his staff, and goes forward to the spot where he is next to set up; the same operation is repeated, and, unless in the case of intermediate levels, the observer moves forward next with his level, and passes the staff-holder to get to the next spot where the level is to be planted. The staff-holder will now have to turn the staff round with its face towards the observer, and it is here that considerable care is required, for a careless man, or one who knows nothing about it, will very likely lift the staff with a clod of earth to it, and set it down again two or three inches from the right place; an inch or two difference of level may thus be made; this repeated occasionally throughout a line of levels, will render the whole of the observations worthless. It will therefore be seen that it is of the utmost importance that the staff be replaced exactly at the same spot, and without any additions in the shape of earth attached to its foot. This deserves far greater consideration than may at first sight appear, in going through ploughed ground for instance. We have said at the beginning that the staff should be pressed firmly on the ground, and if in addition to this the staff-holder gives the staff a side or screwy pressure on the ground, it will prevent the earth from adhering to the brass-plate at the foot. It will now be observed that if the observer and his staff-holder are some eight or ten chains apart, a kind of freemasonry must be established between them, for it is only by signals that the observer can make himself understood by the staff-holder, and therefore a set of intelligible signals must be established between them, about the meaning of which there can be no mistake, and which as the work proceeds the observer will have an opportunity of repeatedly explaining as he and the staff-holder pass by each other.

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* The lower hook is generally preferred.

When taking a set of levels to which more than ordinary care is being given, it constantly occurs that the horizontal wire intersects the staff more or less half way between the different small divisions on. the staff, when either thousandths must be recorded, or the staff-holder must be instructed to give the staff a smart rap on the ground, which will set this right by bringing the staff a little lower down.

There is one particular precaution which, being carefully enforced, secures the accuracy of position of the staff for fore and backsights; this is a thin substance of some kind placed on the ground for the staff to rest upon, as then when the staff-holder turns round for the next backsight, it indicates where the staff is to be held. A triangular piece of sheet-iron, with the corners turned down and a stud in the middle, with a bit of chain and ring attached to carry the whole, has been used; but it is rather awkward for the man to carry when dragging through hedge and ditch. We always use a large penny-piece, or a bit of slate, and satisfy ourselves that it is used by occasionally looking under the staff as we pass along; if neglected, a fine generally sets the matter right.

Enough has been said to show the importance of a staff-holder's duties; and although men of humble position are generally employed for this work, when they are intelligent, it is surprising what a little trouble taken with them at the onset will do in fitting them for their work. A good staff-holder will not only give but little trouble as to the manner he holds his staff, but he will know exactly where to set it up,—he will understand signals readily, and often be useful in looking out for bench-marks. The amount of time saved by this means is often surprising.

When taking levels along a line of country having but little inclination, it is not necessary that all three joints of the staff be kept drawn out, two will be sufficient. This considerably diminishes the staff-holder's labour, which must be fast, by decreasing the leverage, from the lesser length of staff resting on his shoulder.

Along. a line of country where long sights may be taken, a very considerable amount of time may be saved by using two levelling staves instead of one; but care is required in doing this, and it should not be resorted to until the engineer or surveyor has got accustomed to his work; indeed it is only when he has got into the habit of levelling that he can do so with safety. In the first place he must ascertain not only that his staves are of equal length, but also that they are equally divided. In practice it is carried out in the following manner:—At Fig. 132, let one staff-holder set up a staff at A, and let the other staff-holder set up the other at B; as soon as backsight A is taken, a signal is made, and A runs on C, whilst the foresight B is read; the observer moves on to 2, looks to see that C is in the right place, plants the level, and reads off backsight B; he then signals B, who moves on towards D, whilst the foresight C is being read, and so on; but for intermediate levels the two staff-holders would only lead to confusion, therefore as soon as we had read off B, we should send him on towards F, and only make use of A to hold the staff consecutively at *c′*, *d*, *e*, &c., until we had reached *i*, where we should leave him, and again make use of B for the next foresight between *i* and F, wherever that might happen to be.

Chaining; setting out a Line of Levels.—The duties of the principal chainman, or follower, are as important as those of the staff-holder, for we must depend on him for distances, and therefore as to the correct position of the levels. An error in chaining may vitiate a section as much as an error in levelling; it may cause the total length to be wrong, or a reduced level to be shown in the wrong place. The chainman's duty is to give the distance from staff to staff, and also the distances to the different fences, when the staff is not set up on either side of them, which is occasionally the case; the distances to these fences should then be entered on the blank page of the levelling-book, which, as already observed, is devoted to remarks. It is of importance to do this, as then by means of the plan it is a check on the chainage, as given by the follower. Unless a *very safe man*, he should never be allowed to proceed beyond the foresight staff, as this gives the observer an opportunity of reading the chainage himself as he passes onwards; the chainman should be particularly careful to call out every ten chains, or change of arrows; neglect of this precaution is sure to lead to fatal errors. In our remarks under the head of surveying, so much has been said about chaining that further remarks are unnecessary; the importance of the subject is evident enough, and where we cannot secure a good chainman we always chain ourselves, employing a man to carry our level as we go along. We may as well mention here that when pressed for time, it may be economised by employing a man to carry the level, and casting out and reducing in the field; it is only, however, where the surface of the ground admits of long sights that much time is saved, and it is only after having had experience, and with good assistants, that where time is very precious, this course is advisable. We prefer ourselves setting up the staff at every mile, and stop the chainage there to recommence for the next mile, and in the column of distances we enter the distance as so many miles instead of so many eighty chains.

The subject of setting out a line of country along which the staff is to be set up, in order to take the levels, now demands a few observations. As a guide over the district, a set of tracings from the plan of the lands through which the works proposed are to be carried, and on which the centre line is laid down, is taken into the field; by means of these tracings it will be evident that the line along which the levels are to be taken may be easily set out; to facilitate this, and to avoid the necessity of scaling in the field, it is customary to mark on the tracings the distances from neighbouring fences, buildings, corners of roads or fields near which the line crosses, always selecting on the plan or tracings the objects nearest to the centre line, as it then gives less trouble to measure off the distances in the field. If at the spots thus measured off "whites" (that is cleft sticks with pieces of paper stuck in them) are set up, the line of levels is set out, except through very large enclosures proposed to be crossed by curves; for these allowances must be made according to the deflection or bend of the curve; and in sidelong ground this must be done with caution in proportion to the steepness of inclination. Of course we are not now alluding to a working section, the centre line for which would be accurately set out before the levels are taken.

We hope that the above observations have sufficiently explained the subject of chaining and setting out a line of levels.

Longsights and Shortsights.—This is a subject of some importance if extreme distances are repeatedly taken. If along a line of country having a very gentle rise, many backsights of some twenty chains in length were taken with corresponding foresights of only three or four chains, the correction for curvature would in the long run give a very appreciable difference of level; but, in the first place, these very long sights are very difficult to read off to the hundredth of a foot, even with a very good telescope, and with every pains being given to the adjustment of the eye-piece, and by the milled head to obtain distinct vision and destroy parallax. The reader may satisfy himself of this at any time; a gloomy day, or a very bright sunshiny day make the difficulty all the more palpable; in the second place, the instrument requires for such long sights to be in a perfect state of adjustment; in the third, there must not be sufficient wind to communicate the slightest motion to the instrument. After being set up it requires to be levelled with the nicest care, and the staff must be held very steady and upright; now the state of the weather *may* be very favourable, and all the rest may be obtained if sufficiently cared for; but when the end is gained it is scarcely worth the trouble taken, for after all we shall only have saved one planting of the level. If the levels had been taken at twice instead of at once planting and setting up, the same or better results would have been obtained, and not three minutes gained or lost either way; perhaps, after all, the time saved will lay in setting up twice, for it is very likely to be found that after having walked the longer distance, a little wind causing almost an invisible motion to the level or even the staff, a very cloudy day, or the sun being in the same direction as the staff may prevent distinct vision, and it is found advisable by the observer to retrace his steps. It is true that it may so happen that it is not a matter of choice, but one of necessity, as for instance, in taking the levels for a section either across or along the bed of a river, or for a pier extending perhaps half a mile out to sea, or through marshy ground where it is difficult to find a spot sufficiently firm to plant the level; but then in such cases, instead of the sliding staff we have described, and which is narrow towards the top, we always use one four inches wide from top to bottom, all in one piece, admitting of the graduations being much more distinctly marked, with the foot armed with three short spikes, which being pressed into the ground give the staff greater steadiness; the level instead of being ten or fourteen inches long in the barrel of the telescope, measures eighteen inches, with a proportionately large object glass to obtain greater clearness; the conditions and means are thus totally different, and according to their comparative relations, other results are obtained. We are alluding now to an ordinary line of levels, as for a railway, road, or canal; and unless pressed by circumstances, we seldom care for taking sights longer than about ten chains, and then, with few exceptions, correction for curvature and refraction may be disregarded.

There is another subject connected with long and short sights which is of greater importance, and that is a fault that some persons have in being over anxious to obtain the greatest range of level from staff to staff; that is, in ascending or descending levels, to plant the instrument so far from the staff as to try to catch the very top of it in ascending, or its foot in descending levels; the consequence is, that after setting up, and partly levelling the instrument, they find the horizontal wire of the diaphragm perhaps a foot or a few inches either above the top or below the bottom of the staff accordingly as the levels are rising or falling; they are then obliged to take up the instrument and go back for another plant, or have recourse to the dangerous expedient of setting up the staff on a foot-rule to bring it within range of the level, or what is still worse, guess the difference between the top of the staff and the horizontal wire of the diaphragm; that is in the case of taking a backsight with ascending levels; for in the case of backsight when going down, the observer would, under the circumstances we are mentioning, find the axis of the telescope intersect the ground below the staff, and must of necessity go back, which, after all, would be better than the very "rough-and-ready" remedy we have just mentioned above. It will thus be seen that, taking all things into consideration, there is very little gained by this over anxiety to obtain very long sights, and sometimes it occasions a great deal of loss of time. It is better, safer, and more economical in the long run, to take care as you move along the ground that you do not go on so far as to be obliged to retrace your steps, and to plant the instrument at a rough guess in such a situation that you may be certain that when levelled the horizontal wire will come within a foot or two of the top or the bottom of the staff.

When a set of very careful levels are required, as for instance, for a working section, where the line is stumped out at every chain's length, we never care about taking more than ten chains at a plant where the inclination of the ground admits of it, setting up the level at about the fifth stump, so as to get five chains at furthest each way; the instrument is then easily levelled, perfect vision obtained, and consequently a correct reading, and we distinctly see what the staff-holders and chainmen are about. To those commencing levelling, we recommend to pursue this course of short sights and clear vision until they have acquired sufficient experience to determine for themselves the course to pursue under varying circumstances.

There is another error which inexperienced observers are liable to make, and that is, to plant the level on the highest or lowest spot near them along a line of levels. Now, inasmuch as in taking levels for the preparation of a section, we require to trace all the undulations of the ground, and particularly the highest and lowest spots, it is the staff and not the level which should occupy such highest and lowest spot of ground as we proceed along the line.

In going over ground where the undulations are very numerous, we may often save a good deal of time by a judicious selection of the spot for planting the instrument, so that the telescope shall just overtop the high ground in the manner shown at Fig. 133. The method of planting or setting up the level has already been described, and it will be made more fully intelligible if necessary by reference to what has been said about the theodolite, the principles of which, as far as setting up levels are concerned, are the same. In taking a line of levels, although it is necessary that the levelling-staff be set upon such line, it is not required that the level should be; sometimes indeed it will be found advantageous to have the level to the right or left of the line, as a better range may sometimes then be obtained; another observation on setting up the level is, that it is very desirable to do so in such manner that when taking the sights the telescope shall range in taking backsight and foresight over one pair of the parallel plate screws; any little inaccuracy in the reversing may then be corrected by just touching one or the other of these screws; this can only, however, be done when the level is planted very nearly on the line of levels.

We have already explained the method of plotting a section, and need only observe that the forms in which such documents are to be prepared for public purposes may be gathered from the abstract of *Standing Orders*, published annually by Vacher and Sons, of Parliament Street; with regard to manner, such as writing up and so forth, it will be understood from the section and corresponding plan for a railway at Fig. 134. We must conclude this portion of our work by observing once more, to the inexperienced leveller, that the object of such a section is not only to find differences of level as to high and low ground, but also a faithful delineation of the surface of the ground, in all its undulations, bends, and breaks, from one end of the line to the other; at each of these spots, therefore, the staff must be set up and read off from the level.

In going through a town, it will constantly occur that in consequence of the buildings, it is not possible to carry on a straight line of levels, nor to chain the distances; in such a case we must level round the obstructions until we come to the various points on the line, and instead of entering the chainage, which we cannot obtain, in the column of "distance," the name of the street, lane, road, or whatever it may be, is entered on the page of remarks; if there happen to be raised footpaths or causeways, their levels must be taken, as well as the centre of the road; and a corresponding sketch, that is, corresponding to the lines of backsights and foresights, must always be entered in the page of remarks.

The same thing may occur on a line where the engineer or surveyor is turned off an estate by an opposing landlord; he must leave a bench mark where turned off, and level round the obstruction, until he can return to his line on the next estate. The levels of the ground where the obstruction exists must be obtained afterwards, as best we may; we have done this at night by the means of a lantern held near the staff, and another at the level, taking short sights all the way.

Retrograde Levelling.—In the observations we have made as to casting out and reducing levels, and as regards bench marks, and referring to Fig. 132, it will be remembered that we commenced our levelling operations from the point A, being 18 feet above datum; from A to D we have ascending levels, and therefore the backsights are greater than the foresights, and the differences in the readings of the staff are all inserted under the column "Rise;" from E to G this is all reversed. Let us now suppose that we are going to level from the point G to A, and that we have to reduce our levels to the assumed level of A, which is 18 feet above datum, without having any knowledge of the level of G above this datum line. Let it now be observed that in carrying out the levelling operations from G to A we invert the former relations of the ascending and descending levels; that which was a rise becomes a fall, and *vice versâ;* from G to E every difference in the back and foresight *now* gives a rise, and from D to A it gives a fall; then the levels being *only cast out,* let us invert the order of things in *reducing; opposite* the last level cast out at the bottom of the page, enter the assumed height of A above datum, and treat every fall as a rise and every rise as a fall, and thus go up the page or pages in this order, which a moment's consideration will show to be the right order in which to put the differences of the readings; for though we have now levelled from G to A, and *cast out* accordingly, from the top to the bottom of the page, we are now about to reduce in the inverse order of this, and though the difference of level—as for instance from B to A—*is a fall in casting out,* it must be treated as *a rise in reducing.*

Thus, then, in this retrograde work we cast out the levels in the natural order of things as we find them; on reaching the extremity of the line we introduce in the column of reduced levels the assumed height of this point above datum, opposite or on a line with the last casting out, and in reducing we treat every fall as a rise and every rise as a fall, that is, we subtract the rise and add the fall. Although at first sight this may appear contradictory, a little consideration will show that it is rationally perfectly correct.

But let us now suppose that we have a bench mark at D, the height of which above datum we know; then on reaching this point from G the inverted order of reducing may cease, for if opposite the level cast out at this bench mark we insert its height above datum in the column of reduced levels, we shall only have to invert back from this point, and then from D to A proceed in the reducing on the usual system.

We have plotted the Fig. 132 on "section paper," in order that the reader may satisfy himself—geometrically as well as arithmetically—as to the accuracy of this as well as in elucidation of the remarks we have made on ordinary levelling. As to the delineation of the form of the surface from the levels obtained, it will be seen that this will depend on the number of the plants. For instance, if in levelling over the surface A, B, C, Fig. 135, we only planted the staff A, D, B, E, C, we should obtain from the levels for delineation of the surface the form shown by the dotted lines, but with about four more plants and a little feeling in drawing in the curves, we obtain the correct form of the ground; nothing but experience will teach the extent to which this has to be carried under constantly varying circumstances.

Cross Sections.—The levels for cross sections are taken to obtain the inclination of the ground across the *main line* of levels; they may be required merely for three or four chains on each side of the main line, as for instance, at a road crossing; or the cross sections may be wanted of much greater length. On reaching a road of which a cross section is required, we level along the main line up to the road, and then, as we have said elsewhere, up to a bench mark, as the hook of a gate or the stump of a tree where the bench mark is cut, and opposite the entry of the B.M. in the level-book we insert X.S., No. 1, 2, or 3, or whatever the number of the cross section may be. We then turn over the leaves of the level-book till towards the end, and proceed from the bench mark to take the levels and lengths along the centre of the road, to the extent required, noting the distance where the line of cross levels intersects the main line of levels, and there setting up the staff, and so obtain what we require. The cross levels being thus taken, we return to the main line, the levels of which we start afresh, commencing from the bench mark, which is in fact left there for this purpose, and continue our levelling and chaining as though we had not been interrupted. We cast out and reduce the levels from the datum of the bench mark, and, if convenient for the plotting, lessen this height by any number of feet we please, but we always prefer in our level-book to have one datum throughout for each work. It is scarcely necessary to add that each set of levels for the cross sections is numbered to correspond with the number above mentioned as inserted on the main line of levels. For a cross section of any length, as for instance, for half-a-mile or mile through the fields across the main line, we proceed in the same manner to avoid inverting in reducing, unless indeed the line of cross levels is to be of great extent. Before, however, levelling to the bench mark, we drive in a stump on the main line and level to it, then to the B.M., and proceed next to set out the line of levels for the cross section, *at right angles to the main line,* unless it is shown to be required in any other direction on the plan, for it is necessary that, for such cross sections as we are now mentioning, their direction and lengths should be carefully *shown on the plan,* besides being mentioned on the main section as well as the distance shown at which they are taken; it is also to be observed of such cross sections as these, that they are generally plotted to the same datum as the main line of levels. With regard to the cross sections of roads, it is hardly necessary to mention, that as the levels are taken down the centre of it, they are understood to cross the main line of levels at the same angle as the road itself. Whether or not these cross sections are plotted on the same sheet of paper as the main line, which is generally the case as regards those for roads, they are all carefully numbered according to the reference made to the main section, and sometimes on the plan. Besides the above, other methods are adopted in cross sectioning, but we consider that which we have given as the safest.

Check Levelling, the object of which is to ascertain the correctness of any particular line of levels, requires but very few observations. It has been understood from what has been said above that there is a B.M. at each end of the line, besides intermediate ones; let it be supposed that such a line has been levelled from end to end through fields, gardens, villages, &c.; the work is *check levelled* not necessarily by going over the whole of the line again, but by proceeding along the most convenient roads or paths, *from one end to the other only;* or from end to end, at the same time *taking the levels to any intermediate bench marks* that happen to be near the roads, canals, rivers, or other important positions that are crossed by the main line. Calling A one end of a line, and B the other, if we level from A to B by one route and then by another, and we find in both cases that the difference of level is the same, then we have a right to assume that both sets of levels are correct. But if instead of levelling by any road whatever, merely from the B.M. at A to the B.M. at B, we proceed so as to take up some of the intermediate bench marks, and find all the differences of levels the same as when the work was first done, then there will be still greater reason to be satisfied. In check levelling the chain is not used, differences of levels and not distances being the objects sought; in check levelling, more particularly when pushed for time, much longer sights are taken, but we have generally a better opportunity of making the backsights and foresights approximate to equality. As the observer who takes the check level is not always the same person who had levelled over the line, it is necessary that the first observer should leave some distinguishing sign at the bench marks, or else that he should enter a correct description of them in the level-book, so that the next observer should meet with no uncertainty on the subject. Keeping in mind what has already been said about ascertaining the differences of levels, by merely casting up the backsights and foresights, and subtracting one from the other, it will be perceived that for check levelling it is not necessary to cast out and reduce, as we can obtain the required differences without; but to prevent mistakes in the "tottings up," they should be gone over twice. Although the same description of field-book is used, entries are only made in the two columns of backsights and foresights. Let us suppose we have several pages of check levelling, and that at the bottom of the first page the backsights are greater than the foresights by 92·15, then the difference of level is a rise equal to this amount; to carry it forward to the next page it is entered at the top of the column of backsights, and the casting up again proceeded with as before; had it been a fall it would have been entered at the top of the column of foresights.

Trial and Explore Levels.—The object of trial levels is to try how far it may be preferable to lay down a line in one part of a country instead of another, as by adopting one line instead of another we may avoid not only tunnels and viaducts, but also heavy cuttings and embankments, expensive road crossings, bridges, &c., and all those heavy items which swell up an estimate to such formidable dimensions. For the purpose of taking trial levels, two or three and even more lines may be laid down on a general map of the country, and the salient points in the surface are levelled over, so as to obtain the *general features* of high and low ground, road, river, and stream crossings; the little undulations or bumps along a rise or fall in the surface it is quite unnecessary to notice, because whatever line is adopted, a survey of the lands will have in the next place to be made, and a regular line of levels taken over it; neither is it necessary to lay down the centre line with the same care in the field; indeed it cannot be done if we have only a general map to guide us, as the scale of it will prevent our reading off minute dimensions, and we must be directed by the corners or bends of roads, streams, woods, buildings, &c. Care should be taken to leave bench marks along a line of trial levels for the same purposes as those already mentioned, and more particularly on high and low ground, at roads and river crossings, and near all points where another trial line is shown to join on the general map. Besides, however, taking the levels along the different lines, one after another, the engineer may often do good service by taking levels to points off the line, as to objects to which he can refer on the map by lettering them with letters corresponding to those in his level-book; even to objects at a distance of half a mile or a mile useful observations may thus be made, particularly if a description is added in the column or page of remarks. This may sometimes be done approximatively without sending the staff-holder on to such points, which might involve loss of time. For instance, in taking trial levels along any particular line, we may observe the windings of a road in the distance, and which windings are shown on the map by some feature or other; or we may take the bearing of it by the compass, or by half a dozen ways explained in our remarks on surveying. Now we may turn the telescope of the level in such direction, and see whether we intersect with the horizontal wire the surface of the road, or a man's or horse's head, the top of a hedge row, some particular clump of branches springing out of a remarkable tree, the top or bottom of a door or window, the roof of a house or chimney stack. Of the first of these we know the heights within a few inches, and the heights of the others may be measured at any future time, and we have thus the means of getting an approximation to the relative level of such road at those points as compared with the level of the spot on which we are standing. If there is no road there may be some farm buildings, with labourers or cattle about, or the corner of a wood, and many other objects near marked features on the plan, which a little ingenuity on the part of the observer will enable him to take advantage of. These observations may at times be of the greatest value, more particularly in a country of which the relative levels are not known, or only very imperfectly so. It is to be considered that in trial levels the minutiæ of the surface are quite matters of secondary consideration, and that the objects sought are the levels of the salient points in the formation of the ground. Trial levels are almost always plotted to a small horizontal scale, as three or four inches to a mile, and several lines of trial levels may be plotted from one datum line, as of course they must all be referred to one datum level. The lengths of these lines will all probably differ more or less, and consequently some will be longer on the section than others. If they all begin or terminate at one point, such points of termination or beginning should be the same on the section. The surface lines of trial levels so plotted must be tinted in different colours to distinguish them one from the other. For our own part, we do not like the system of plotting trial levels in this manner, for when the inclinations or gradients come to be shown on them to try comparative merits, the whole often looks very confused. We therefore prefer to plot the whole of them on one sheet of paper, but on different horizontal lines, so as to keep the sections distinct, and we tint each surface line of the same colour as that corresponding to the colour of the line on the general map.

Working or Contract Section.—These documents only differ inasmuch as the second is prepared from the first to be delivered to the contractor under his contract. A perfectly correct idea may be formed of a working section from Fig. 136. The different manner in which it is prepared we will endeavour to explain. The horizontal scale to which it is plotted, instead of being 5 or 6 chains, is generally 3 or 4 chains to the inch, and the vertical instead of being 80 or 100 feet, is generally 30 or 40 feet to 1 inch. At every crossing of road, river, canal, stream, brook, &c., is inserted some remark as to the work to be constructed and a reference to the No. of the working drawing prepared; this may be a special drawing as for a bridge, or a general drawing as for a level crossing and gates, or for a culvert according to the dimensions of the volume of water discharged by the brook. The different borings taken are also shown on the working section.

We have already made some slight allusion to the levels taken for the preparation of a working section, but we will detail them here rather more at large. It is to be understood that the centre line is first stumped out from end to end, that is, a short stump about two feet long and two inches square is driven into the ground at every chain's length, except at junctions of curves where three are driven across the line, one in the centre and one on each side, and the two latter instead of being driven upright are driven in slanting. Some engineers drive these stumps level to the surface, and by the side of each a guide peg is left to point out the direction of the line, in which case where the levelling is carried on the staff is set up on every stump. Others drive the stump to within three or four inches of the top, and the peg is driven behind it down to the surface for the levelling-staff to be held upon it. The advocates of either system are often very zealous for *the one* plan, but we do not believe that the matter is of the slightest importance. The centre line being thus prepared, the levels are not only taken over the surface at every stump or level peg as the case may be, but also over every little undulation or break in the surface of the ground, and to the bed and surface water of every stream, and to the flood height where attainable. Every indication is also noted in the remarks that may determine the diameter of the culvert. Where roads are crossed levels are taken to the centre of the road, to the sides and to the footpath where there is any; to the surface water in canals, to the bottom of the canal and the towing path. With levels thus taken a very faithful document may be prepared in the shape of a working section as giving a true representation of the surface of the ground. Levels are taken to a sufficient number of bench marks, and where none suitable are found ready, large square-headed stakes are driven for the purpose. In preparing the working section vertical lines are drawn in blue at every chain's length up to the surface of the ground, and on each vertical is written the reduced height above datum, not as read off by scale, but from the column of reduced levels in the level-book. The next operation is to lay down the gradients, the one representing the surface of rail and the one below gives the average depth of formation below the level of rail, and is called "formation." This being done, the height of formation above datum is *calculated* from the inclination for every chain's length, and the difference between the reduced level to the natural surface and the level of formation last found is entered as depth of cutting or height of embankment at every chain also on the working, but not necessarily on the contract section. The inclination of the gradient per mile and per chain is also shown, the width of formation and the slopes as 1½ or 2 to 1, or whatever it may happen to be. The nature of the borings are also shown. Fig. 136 will sufficiently explain the above observations.*

Setting out Levels.—After what has now been said there will be but little difficulty in understanding this portion of our subject.

An inclined plane is one inclined to the horizon, and by the term "gradient," numerically expressed, we imply the proportion which the height of rise or fall of the inclined plane bears to the length of the horizontal base of the incline, as 1 in 100, 1 in 200, 1 in 300, are expressions which mean, that in 100 feet the rise or fall is 1 foot, that in 200 feet the rise or fall is 1 foot, &c. &c.; and by keeping 1 as a constant number, the various proportions which inclined planes bear to each other is better expressed and understood than by varying both expressions, or employing fractional quantities. Having to ascend or descend to a given point above or below the starting point, and along a given route, we shall know by levels and measurements the height of ascent or descent, and the distance by which the difference of levels will be regulated into one or more inclined planes; for a short distance, and slightly undulating ground, this may often be done by one incline; for any considerable length, more particularly in a broken country, we shall find that hills and valleys, rivers, canals, and roads will compel us to consider these as guiding circumstances, or we must encounter innumerable tunnels and viaducts, and disproportionate cuttings and embankments, expensive bridges, and other crossings; but a well-regulated series of inclined planes graduated in a measure proportionately to the inclination of hill and valley, skirting along the one and skimming over the other, and reaching from height to height, enables us, by moderate works, and therefore at a moderate expense, to attain the desired end; and generally the reaching from valley to hill, and hill to valley, the crossing of rivers and roads, the equalization of excavations and embankments, will be the existing circumstances in compliance with which certain heights are to be reached, and by which the inclined planes, and therefore the gradients, will be determined. The working section will show a vertical plane passing along these various points, the relative heights of which have been ascertained by levelling, and their distances from each other being known, it remains to calculate the gradients. Let us suppose that at a distance 3 miles 8 chains, there is an incline which is to reach from 161·64 above datum to 14·50 above a road, which is 6·50 links further on; the level of the road is 151·43 above datum, and 151·43 + 14·50 = 165·93; but 165·93 = 161·64 + 4·29, or the difference of level between a point we have reached on the section and one to be attained. But 650 links are equal to 429 feet, and this divided by 4·29, the difference of level will give 1 in 100 for the gradient; for 4·29 : 429 :: 1 : 100, or AB : BC :: DE : EC, Fig. 137; therefore, in all cases, to find the ratio of inclination, or gradient, divide the length in feet by the rise or fall, and the quotient will be the answer; but having the gradient, or ratio of inclination, it is also necessary to know the rise or fall per chain, in order to calculate the depth of cutting or embankment, or rather the height of formation above datum, as this height, deducted from that of the surface above datum, will give the depth of cutting, whilst the deduction of the height of surface from the height of formation will give the depth of embankment, as at 2 miles 10 chains 171·70 − 162·96 = 8·74, or depth of cutting; and at 2 miles 12 chains 164·28 − 150·83 = 13·45, or depth of embankment. To find the rise or fall per chain divide the difference of level by the number of chains in the incline, as 4·29 ÷ 6·50.

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* For further information on this subject the reader is referred to *Railway Construction*, by the author of this volume.

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6·50 ) 4·290 ( ·66
       3900
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        3900
        3900
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Therefore, to find the gradient, or ratio of inclination, divide the length of the incline in feet by the difference of level. And to find the rise or fall per chain, divide the difference of level by the number of chains, as shown above.

The ratio of inclination of a plane, or the gradient, being given to find the inclination per mile, divide the number of feet in a mile by the ratio, as

100) 5280 (52·8 per mile.
500
280
200
800

The gradient being given, to find the inclination in a chain, divide 66 (the number of feet in a chain) by the ratio as for 1 in 100, 66÷100—

100) 660 (·66
600
600
600

Also, the gradient being given, to know the rise or fall per yard, divide 3 (the number of feet in a yard) by the ratio, as in 1 in 100, what is the inclination per yard?

100) 300 (·03
300
000

And at any time, for any length, divide the length in feet by the ratio, as in 1 in 78, how much in 12?

78) 12·0 (·15 (sufficiently near for practice)
78
420
390

In calculating the formation heights for the contract section, from which, as before observed, the depths of cuttings and embankments are afterwards ascertained, the greatest care is required in doing this correctly, for if, in this first step, an error be committed, it will be carried on into the depth of cutting or embankment, and the contractor working from such contract section may cut to a wrong depth, for which, in justice, he is in no way to blame; to men of habits at all careful, this may almost appear impossible, but it is nevertheless, however gross the blunder, one of common occurrence. Suppose that at 2 miles 33 chains, the formation height is marked 182·48 instead of 181·48, which it should be; now the heights for the springers would be set out by reference to bench marks, when the arch might be turned to a height of 21·41, instead of 20·41, making the crown of the arch 1 foot too high, for 181·48 − 161·07 = 20·41; but 182·48 − 161·07 = 21·41. It is hoped that the importance of a careful calculation of the gradients at every chain stake is now distinctly understood; the ratio of inclination being ascertained, and the rise or fall per chain, every stump height will require to be calculated separately, and checked by tens, twenties, &c., and in as many places as possible—the reader may rest assured, that for whatever trouble 20 or 30 miles of such calculations may give, he will be most amply repaid by ease of mind on this important subject during the working of the contract. From formation height 161·64, before referred to, we have an incline of ·66 per chain for 7 chains; and ·66 × 7 + 161·64 = 166·26; but 161·64 + ·66 = 162·30 + ·66 = 162·96 + ·66 = 163·62, and so on until we have 166·26 at the 7th chain, if no error be made; on an incline of a greater length we check these results by multiplying the rise or fall per chain by 10, 20, 30, &c., which is very quickly done, and examining the 10th, 20th, or 30th chain stakes taken at different places. This being gone through, it only remains to get the depth of cutting by deducting formation height from the surface height, or the reverse, when we get the height of embankment.

The section being prepared so far, we should, as soon as the works of construction are determined on, insert notes from the working drawings or otherwise, of the angles of skew at which the line crosses roads, canals, &c., the spans of arches on the square and skew, the rise of the arch, the depth of arch stones, of puddle, if any; also, if the works be on an inclined plane, the rise or fall from centre to centre of piers; memoranda also, of nearly similar nature, should be made of girder bridges, culverts, drains, and other works occurring along the line. These remarks are more than necessary, because, when on the works, the drawings, when required, are often mislaid, or partially defaced or destroyed; it must be added, however reluctantly, that the tracings with which contractors and sub-contractors are supplied, are often wrongly figured, and the site of construction, amidst the moving to and fro of masons, labourers, and 'navvies,' is not the place where such errors may be most readily detected and corrected. The acting engineer always affixes his signature to the working drawing, but not always to the tracings, the correctness of which is often entrusted to a confidential office assistant, but who, from want of field practice, is not always aware of the value of correct figures; it will be found advantageous, and only consistent with duty, to compare the copy of contract works with the office original.

The contractor is supplied with a copy of the contract section of the intended line of railway, or any portion of it, the performance of the works on which is entrusted to his execution, and if the levels are set out by his agents, the engineer has but to check them, a simple matter merely demanding correct levelling; but he may have to set out the bed moulds, which should be done in the following manner:—take Fig. 137 for a cutting to be worked from both ends. At stump, 2 miles 27 chains, we are to have for formation 176·41 above datum, and the height of surface is 174·05; plant the level at a convenient point, and direct the levelling-staff to be held on the level peg at 174·05, and read off, say 7·00; but 176·41 − 174·05 = 2·36 for height of embankment, and 7·00 − 2·36 = 4·64; now if the staff be raised until we read through the telescope 4·64, it is very evident that the foot of the staff will be held at 2·36 above the surface, or at 176·41 above datum, or formation height; next, direct the staff to be moved along the nick or centre line, and towards the cutting, until we read off with the level 4·64 again, we shall then have found along the nick a point exactly level with the intended formation height at 176·41; but our gradient is 1 in 78, or ·846 per chain; the formation therefore at the point sought will be lower than 176·41 by 1 in 78, according to the distance from 176·41; let this distance be half a chain, the formation height will then be 176·41 $-\frac{846}{2}$ = 175·99, or ·42 lower; and if we add this ·42 to 4·64, we shall have 5·06; move the staff nearer until with the level this depth 5·06 be read off, when we shall have the desired point; for if when we first read 4·64 the foot was at formation height, as we hope is evident, by adding to it ·42, and reading off 5·06, we have a point ·42 nearer to datum at half a chain distant, and therefore parallel with the incline; at this point found, drive in a stout stake, until on reading off the staff we have 5·06, and another at 176·41, until we have 4·64, and we shall have two stakes at the required formation height half a chain apart, and by boning from these two heights any tolerable excavator can direct the formation of the bottom of his cutting until fresh levels are given. Boning is performed with boning rods, which exactly resemble T squares, in the following manner:—let A and B, Fig. 138, be two stakes driven to a certain depth, and according to a given inclination; if on both of these stakes boning rods, of exactly equal lengths, be held perfectly upright, it is plain that the tops of these will be parallel to the incline, and if a third rod be carried along the intended slope, the top of it will be in line with the top of the other two, if the incline be correct; if it is above there will be more to cut away, and if it is below the excavation will have been made too deep; this method is certainly but approximate, but quite sufficient to guide the excavators for a time. As another example of setting out the bed moulds, we will take the other end of the cutting, Fig. 137. At formation 165·60 we have the surface height of 161·80, and therefore a bank of 3·80; plant the level, and then set the staff on the level peg, and read off, say 9·50; but 9·50 − 3·80 = 5·70, and if the staff be held up until such depth be read off, then again will the foot of it be at the formation height at this point; move the staff along the centre line until this 5·70 be again read off, and we shall have a point level with the intended height of embankment; but the gradient here being 1 in 100, the new point sought must be higher in that proportion, as we are now rising, and the distance is 52 feet; for this distance the rise will be ·52, and as we are *rising*, be it remembered, we must deduct ·52 from 5·70, when we shall have 4·18; by moving the staff along the nick until from the level we read off 4·18, we shall find the new point for formation, at which drive in a stake. Under no circumstances should this part of the work ever be considered as completed until carefully checked, which should be done by carrying on the levels to the nearest P.P., or permanent post.

The incline for the cutting being given, will be, however, but an indifferently approximate guide for the height of an embankment, particularly if of clay and tipped in dry summer weather, for under such circumstances, such an embankment may settle one-fifth of the height at which it is first formed; let Fig. 139 be a sketch of a cutting and an embankment, in which suppose A B the inclined plane, and the required formation height at B C to be 22 feet; if the embankment be formed at once to this height, and the above mentioned settlement before perfect consolidation takes place, and it will not be much less, the general height will be something like A D, and will require reforming; it is therefore absolutely necessary to keep the bank high in proportion to the expected settlement, whatever may be the material of which the bank is formed, and a near approximation of the depression may be very easily ascertained a few weeks after the tipping has commenced, due allowance being made for extremes of weather. At 1 chain from the cutting we have embankment height of 8 feet; as soon as the tip is formed beyond this point, ascertain the height by levelling; a few weeks after, the depression from the weight of the earth and that of the loaded waggons may be ascertained by levelling again, when an approximate ratio of settlement will be determined, by which the formation of the embankment may be directed.

Viaducts and bridges are works which require the greatest accuracy as regards levels, and an error of this description is unpardonable, as attention and care are all that are required to carry the construction to the proper height; settlement in the masonry will of course take place, but unless the work is very bad, or the foundation vicious, the difference of level occasioned by this settlement will be immaterial: by attending to the following advice the heights of the crowns of the arches will be found to be perfectly accurate. Let A B, Fig. 140, be a gradient of 1 in 100 along the extrados of a set of arches of 13 feet span and 13 feet rise, with voussoirs 2 feet thick; the piers up to springing must all be built up to a height of 13 + 2 below formation, that the line under the springers may be parallel to formation. Let formation at A be 240·00, surface height 245·00, or a cutting of 5 feet; near this a permanent post, or B.M. 243·59, or 3·59 above formation at A; plant the level and let the staff be held on the B.M., read off, say 5·25; send the staff on to A, or to *b*, which will make the work much shorter, and this point may be pretty nearly ascertained by the eye; supposing the staff therefore at or near this latter point, read off, say 8·00; 8·00 − 5·95 = 2·75 fall, and 243·59 − 2·75 = 240·84, or ·84 higher than 240·00, and also, therefore, ·84 higher than a point level with the required formation; now by moving the staff lower down, so that 8·84 be read off, we shall have a point level with formation at A, for 8·84 − 5·25 = 3·59 fall, and 243·59 − 3·59 = 240·00. Now measure the distance from stump 240·00 to the point where the staff was last held, and say it is 15 feet; the inclination for this distance being ·15 rise, the required height for formation will be 240·15; but the height of B.M. is 243·59 − 240·15 = 3·44, and when the staff is held on the B.M., we read from the level 5·25; add to this 3·44, when we get 8·69. Now drive in a stout stake, iron hooped, at the said distance, until the staff being placed upon it we read off 8·69, and it will be at the required height, for 8·69 − 5·25 = 3·44 fall, and 343·59 − 3·44 = 240·15. It has been already observed, that a line drawn along the tops of the piers and abutments under the springers must be 15 feet below formation height; now it will not be sufficient to deduct this 15·00 from 240·15, giving 225·15, and drive in a stump at that height, because we cannot do so vertically under *b*, and at *c* an allowance must be made for the inclination of the gradient corresponding to distance. But by levelling from *b* downwards to *c*, we can find the height 225·15, which will be level with a point 15·00 below *b*; measure the distance from A to *c*, call it 30 feet, the inclination for which being ·30 we shall have 340·30 for formation at this point, and 240·30 − 15·00 = 225·30, to which height drive in a stake as before, and then check by levelling back to B.M. 243·59, which height exactly we shall get if the work has been done correctly. From the appearance of Fig. 140, the reader may be led to think that the above mentioned stakes are driven on the centre line, but as in this position the excavators would dig them up, they are driven in some 15 feet on one side; in rock or sound soil these bed moulds are little liable to disturbance, but they are not so safe in a loose ground, and it is always better occasionally to check their heights. If at the other side B of the viaduct the ground was similarly high, we should have but to repeat a set of levelling operations analogous to the last, to get the lines A B and *c d*, but being an embankment this cannot be done, and we must employ another method; we will suppose that at abutment C, and pier D, the masons have reached the heights marked by the dotted lines, we must first ascertain the formation heights at those points; let these be 241·66 and 242·45, and these minus 15·00 will give 226·66 and 227·45 for the heights at the springers. By levelling from a B.M. we find that the height of the abutment at C is 290·05; 227·45 − 190·05 = 37·40, or the height below springers to which the masons have reached, that is that they have 37·40 feet of masonry to put on to reach springing height, and this height, 37·40 should be entered in feet and inches by the engineer on a memorandum book with the date and afterwards on the section. At pier D the height of masonry is ascertained by levelling to be 177·76; and 226·66 − 177·76 = 48·90, the depth at D below springing, and this depth should be similarly entered; and in both cases a crow's foot mark should be made on the face of the masonry where the levelling-staff has been held, that it may be found at any future period. When the piers have reached within a few feet of their due height, it is well again to take the levels, as by then giving the height to be added, the masons or bricklayers, but of course more particularly the former, can regulate to a nicety the depth of their courses, and some allowance will have been made for settlements.

Contour Levelling.—The foregoing observations on the practice of setting out levels, will be of material service in explaining the subject of contour. If we can imagine a valley with the water steadily rising in it, and leaving a mark of its presence at every five or ten feet rise, we shall have the most perfect idea of setting out contour levels, for the bubble of the spirit-level defines the horizontal line which would be assumed by the water, and if we remove the level higher and higher each time, the axis of the telescope performs, as regards this, the same process as the rising water; but it is necessary to drive in a number of stumps in order that the contour lines may be found on a future occasion. If the lines of contour levels are all set out at any *given number* of feet one above the other, and a survey of these lines is afterwards made, then the plot of such a survey will give a plan and also a section, as far as representing the relative height of levels over the ground, and according to the pains bestowed on the work, a section more or less accurate may be plotted from the plan.

Let A B C D E F G, Fig. 141, be a valley which it is required to contour level with a rise of 5 feet, with the lowermost contour also 5 feet above the point X; remembering that not only the levels have to be set out, but a survey to be obtained by some means in order to lay down the contours on the plan. Range out the lines A G, H B, H C, &c., all of which are carefully fixed by their respective angles; plant the level at *a*, with the staff first set on X, where a permanent bench mark has been established; say that the reading is 12·50; this reading, minus 5 feet, equal to 7·50, and minus 10, equal 2·50; let the staff-holder move up the hill toward G, until the observer from his telescope reads 7·50, then drive in a stake; let the staff-holder then move on higher up, until the observer reads 2·50, and then drive in a stake. Let the staff be now moved towards *a*, until 2·50 is again read off, which will be another point to be staked off, and then lower down until 7·50 is again read, which will give a fourth point to be marked. The stumps for this work may be much lighter than the stumps we have already mentioned; in fact, if no time is lost, mere laths are sufficient. Let the observer thus range out with tolerable longsights up to the line H F, getting the staff held exactly on the line, so as to have the stumps to answer here the purposes of stations; of course, after the first remove, the reading on the staff will be different; for instance, when the level is first planted and the staff set up at the last stump driven in—say on the first 5 feet contour—it may, perhaps, read 11 feet; then 11·00 will be the reading to continue this lower contour, and 11·00 minus 5, equal to 6·00 will be the next. With a long staff and 5 feet contours we may always arrange, with a little judgment, to set out three different heights at each fresh plant of the instrument; but, until experience is acquired, it is advisable to be satisfied with two. On reaching a contour-stump on the line H F, set up a flag-pole, when an assistant, who has already ascertained the starting-point of the contours with reference to the line X G, and the distances of the first and second from the point X, takes up the chaining with a good prismatic compass, and taking the bearings, measures with the offset staff, or better, with the tape or another chain, up to all the stumps driven in, until he reaches the line H F, at the same time he ascertains the distance from H or H F, where the flag-pole has been set up. In the meantime the level has gone on from the line H F to H E; the assistant follows with the chain, and so one set after another is set out with the level, and then chained. If in chaining and offsetting the assistant is careful to obtain *horizontal* distances, more particularly with the offsets, a very correct plan will be produced, which will be very useful in moderately undulating countries. One line of contours being taken all round, the work is recommenced. To effect the same object approximatively, sets of cross sections may be taken, as the undulations may require, and the positions of these cross sections very carefully shown on the plan. The differences in the readings of the level-staff must, as before, be 5 feet each time, or 10 feet if the contours are to be ten feet above each other.

Improved Dumpy Level.—The most experienced observers with the level have remarked the difficulty that exists with every instrument to get it to reverse well; that is, without any alteration in the position of the bubble; and when taking long sights or levels about which there happens to be more than ordinary anxiety, it is often a constant source of discomfort. When after several removes of the instrument over very steep ground, the parallel plate screws have been so worked as to bring the plates into a very oblique direction, this difficulty in getting the telescope to reverse correctly generally increases, so much that it is often necessary to readjust the screws so as to bring back the plates to their parallel positions. To this it may be observed that in planting the instrument even on very inclined ground, we should roughly level with the legs first, as we have pointed out in our observations on the use or rather application of the instrument; but when levelling operations are hurried, as they often are, attention to this very important point as regards this rough levelling of the instrument is often neglected. The improved level, as constructed by Elliott, to which we now call attention, in a great measure removes these defects, both as regards the degree of tightness with which the telescope portion of the instrument is screwed on to that part which consists of the parallel plates and axis, as also with regard to the difficulty arising from the levelling of the instrument on very steep ground. In the improved dumpy level, the telescope and the parallel plates are all in one, and a little circular level is applied, the object of which is to bring the instrument into rough adjustment; for by attending to the bubble of the circular level occupying its proper centre while setting up with the legs only, this rough adjustment is secured; instead of the ordinary ball and socket fitting usually applied to the level, the ball and socket joint of the theodolite has been adapted, by means of which the inconvenience arising from steeply inclined ground is done away with. These improvements are so considerable that we fully intend to adopt this improved level in our own practice. In appearance when set up it does not differ from the ordinary level, or we should have given an illustration of it.

APPENDIX.

THE TRANSIT INSTRUMENT.

As the name of this instrument implies, it is used principally astronomically, for observing the passage of celestial objects across the meridian. We give an illustration of it at Fig. 142, because in long tunnels with deep shafts it may be used to considerable advantage, as giving the means of setting out with minute accuracy the main line of direction and the vertical axis of every shaft.

The telescope is in two parts, connected together at the sphere A, which also receives the ends of the conical axis, CC, at right angles to that of the telescope. The other two ends of the conical axis, consisting of cylindrical pivots, rest in Y's at the summits of the vertical standards, SS, in which the axis of the telescope turns. A level across the conical axis denotes when it is horizontal. The standards are fixed to a circle which rests upon parallel plate-screws, by which the instrument is levelled. The two braces from the top of the standard to the beam across the above-mentioned circle are for the purpose of steadying the motion of the telescope.

The vertical limb, L, is fixed to one of the pivots of the conical axis, and turning with it, denotes the angular elevation of the telescope by means of the two horizontal verniers, which are adjusted in their horizontal position by means of the spirit-level shown above them; the adjustments are effected by means of the arms and screws, *y* and *a;* the wires of the diaphragm, placed near the eye, and of the telescope, are illuminated for night observations by means of a lamp, which throws its light through the pierced cone on to a reflector within the sphere A. The lantern is so constructed that the power of the light may be tempered to suit circumstances.

COPYING AND REDUCING PLANS—THE PENTAGRAPH—THE EIDOGRAPH—SCALING AND THE PLANIMETER.

Copying and Reducing Plans.—To persons unacquainted with the routine business of an office, and to the accuracy that is required to be combined with despatch, the mere copy of a field-plan, or of the plan of a townland survey, must appear a matter of the greatest simplicity, and, but for the necessary correctness, so it would be; notwithstanding, however, that it is a most mechanical operation, it demands great care and system, particularly when the plan is large. A very ordinary method that is adopted is, to lay the original on to the blank paper, and then with a fine needle-point to prick through every corner and bend. These points being thus determined on the copy, the pencil is applied to complete the work, which, after being compared with the original, is completed with the drawing-pen and Indian-ink. This method, however, by no means improves the original plan, which may be of great value, even in respect of finished drawing only, and we consider it in every way inferior to the following mode. The original plan should be laid down on a flat table, of dimensions corresponding to the size of the drawing to be copied; over it should be laid clear tracing-paper, as large as can be obtained, if the size of the original requires it; the whole should then be loaded with any long flat rules that are to be come at, and leaden weights, so that the whole shall lay perfectly flat; the plan is then traced off, in one or half-a-dozen sheets, as may be required. This need not be in ink, a sharp, clean-pointed pencil will do very well; but take care that in tracing sheet after sheet sufficient work is repeated along the length and width of each sheet of tracing paper to determine *accurately* their relative positions if they were all joined together. The correctness of a plan as a whole in a great measure depends on this particular feature, and great care should be taken when first laying down the tracing-paper, not only that it lays flat, but also without any twist in its length or breadth. The blank paper which is to receive the copy is now laid down on the board, and over it thin sheets of tissue-paper that have had the under-side well rubbed with black-lead; two or three of these may be laid down one alongside of the other, if size requires it; over the whole is laid one of the tracings, loaded as before, so as to lay flat and even, and with a tracing-point the plan is thus transferred to the blank sheet of paper underneath, by means of the black-lead paper; this tracer we manufacture ourselves, by driving the point of a fine needle into a pencil, and making use of the head as a tracer. The first sheet being now accurately transferred, the next tracing is laid down so that the above-mentioned *repeated* work along the length and breadth of the tracing shall perfectly coincide with the pencil-work which has just been transferred to the blank paper; it is then fixed in this position, and the black-leaded paper is gently slipped underneath; the whole is then loaded as before and this second sheet thus transferred, and by repetition the whole of the plan to be copied. The whole may then be inked in.

Another very good method, though, as we think, much more tedious than the above, is to triangulate the original, as in surveying; transfer, or rather lay down the triangles on the blank paper, and by a system of offsets copy the whole.

The reduction of a plan is often effected by a system of proportional squares, as one-quarter, or one-fourth, as may be required; these must be all perfectly rectangular, and the perpendiculars should all be carefully drawn in, by means of a large and perfectly correct set square, taking care, both on the original and the copy, to use the same side of the square.

The reduction is, however, much more rapidly effected by means of the two following instruments.

The pentagraph (Fig. 143) consists of four flat rulers, made either of wood or brass; the two outside ones are generally from 15 to 24 inches long, and the others about half that length. The longer ones, A and B, are united together by a pivot, about which they turn like the legs of a pair of compasses, and the two smaller rulers are similarly attached to each other at D, and to the longer rulers. A sliding box is placed on each of the arms, A and B, which may be fixed by a clamp-screw at any part of the ruler; these slides carry a tube, to contain either a blunt tracing-point, a pencil, or pen, or the fulcrum G, which is a heavy weight of lead, having a point on the under side to pierce the drawing-board and remain immovable in its proper position, it being the centre upon which the whole instrument turns. Several ivory castors support the surface of the machine parallel to the paper, as well as facilitate its motions.

The arms, D and B, are graduated and marked with the ratios, ½, ⅛, &c., so that when a copy of a plan is required to be made in any of these proportions, it is only requisite to fix, at the required ratio, the slides carrying the fulcrum, G, and the tube at D, with a pencil or pen, and the instrument will be ready for operation. Thus, suppose it were required to make a copy of a plan exactly one-half the size of the original, the slide carrying the pencil, and that working on the fulcrum are each fixed by their respective clamp-screws at the divisions marked ½, the pentagraph being first spread out so as to give room for the tracing-point to be passed over every line on the plan, whilst the pencil is making corresponding marks on the copy, which it is evident will be equal to one-half the size of the original. A fine string is attached to the pencil-holder, and passed round to the tracing-point, the pulling at which raises the pencil a small quantity above the paper, to prevent false or improper marks upon the copy. It should also be remarked, that the cup represented on the top of the pencil-holder is intended to receive a weight, to keep the pencil down upon the paper, or when a stronger mark is required.

When the instrument is set for work, the tracing-point, the pencil, and the fulcrum must in all cases be in a straight line, which may be proved by stretching a fine string over them.

When it is required to make an enlarged copy of a plan, the setting of the instrument is precisely the same as above-stated, only the tracing-point and pencil must change places. But when a copy is to be made of the same size as the original, the fulcrum must be placed in the middle.

With regard to an *enlarged* copy of a plan, by whichever of the above methods it may be done, it can never be considered a very correct representation of the lands surveyed; and when an enlarged plan is required, a fresh plot should be made from the field-book, according to the scale required.

The eidograph is an improved instrument for copying and reducing plans, and illustrated at Fig. 144. After what has been said about the former instrument, a little experience will fully explain the use of the latter and its advantages.

The Planimeter and Computation of Areas.—Those who have had occasion to perform much scaling and computations of areas are aware of the tedious nature of the task. With irregular figures, such as are constantly met with, it is in the first place often troublesome to give and take correctly, and indeed it is in practice at the best of times a mere guess, however near it may be to accuracy. The figure has then to be divided into triangles, of which one side and one perpendicular, have to be scaled, another constant source of error, however small, or the polygon has to be carefully reduced to one triangle, which requires a great deal of care, and the perpendicular and base again scaled; the computations have then to be made by multiplication, and all of these have to be carefully checked before the work can be depended on. There will therefore be nothing at all surprising when we observe that many inventions have been suggested from time to time either to shorten the *modus operandi*, or to facilitate the calculations. None of them, however, appear to have met with success, and the old fashioned mode has had to be adhered to. The *planimeter*, however, bids fair to make the laborious task much more easy, inasmuch as with it we have only to trace the periphery of the figure with a tracer, and then merely read off the contents. This instrument, illustrated at Fig. 145, and lately introduced by Messrs. Elliott, has already been received with a great deal of favour. E and F are two points at the ends of the moveable arms, A and B; when in use the instrument rests on these two points, and on a third, being that part of the graduated wheel D which is tangent to the paper. G is a graduated disc connected with an endless screw, and H is a vernier to the wheel D. When the instrument is applied to the operation of computing, the point E is firmly fixed at any convenient point on the paper. A very little thought and experience will show that the object sought in fixing this point will be to place the point of the tracer F so that it may be made to run over the greatest possible portion of the periphery of the figure, the contents of the area of which are required. Let it be considered that in the drawing the instrument is so placed, and that the point of the tracer is at a convenient point of the periphery of the irregular figure to be traced. Read off the instrument, trace the contour of the figure, and read off a second time. Subtract the first reading from the second; multiply the remainder by 10, which will give the area required in square inches, with two places of decimals. Let the first reading of the disc G be 2, that of D equal to 80, but with the vernier reading instead of 80 let it be 805; then the reading of the instrument at starting is to be set down as 2·805; let the second reading be 7·553; this minus the first is equal to 4·748, which multiplied by 10 gives 47·48 as the contents in square inches which was required. In carrying the tracer round, care must be taken to observe whether the disc G has performed an entire revolution, for in that case 10 must be added to the unit in the last reading. For instance, in the above example, had the second reading been 7·553, plus an entire circuit, then we should have had 7·553 + 10 = 17·553, from which subtracting 2·805, and multiplying the remainder by 10, we should get 147·48. If the figure is too large for the instrument to contour at once, it may be divided into two or three. The contents of each being taken separately, they are all to be added together.

The instrument is constructed to give areas in other denominations than inches, but we prefer the square inch because it is a convenient constant to work from, whatever the scale of the plan. Suppose, for instance, that in the above example we have been calculating an area from the Ordnance map, having a scale of one inch to the mile; one square inch is one square mile, we should therefore have very nearly 47½ square miles; again, let us suppose that we have been measuring an area from a plan plotted to a scale of five chains to one inch; one inch or five chains square, equal to 250,000 square links; from this we obtain the constant by which to multiply the square inches found by the planimeter to reduce the area to square links for a five chain scale; thus, 47·48 × 2500 = 11,870,000.

Then for a 1 chain scale we shall have . . . 100
" 2 " " . . . 400
" 3 " " . . . 900
" 4 " " . . . 1600
" 5 " " . . . 2500
" 6 " " . . . 3600

by which to multiply the square inches found by the planimetric operations, to reduce them to square links, which will be the more easily remembered from the multipliers being merely *the square of the number of chains to one inch, with two zeros added.*

As before explained, if from square links we strike off five figures to the right, we have the areas on the left, and the little tablet at the end of the volume will give the roods and perches due to the square links first cut off. We are aware that old hands generally object to have their practice of *routine* broken in upon by any novelty. It is natural to all of us; but if they will give this a trial, they will find the labour of scaling and computing very greatly reduced, and, as we think, more accurately performed.

EXPLANATIONS FOR THE USE OF THE TABLES.

TABLE I.—Reference to what has been said as to chaining over inclined ground sufficiently explains the use of this table.

TABLE II. is for facilitating the process of computing areas; one example will explain its use. The area of an enclosure has been found equal to 25·69981; point off five figures to the right, as shown by the decimal point, and in one of the columns marked 1 R, 2 R, 3 R, look for the first three figures of those pointed off. The heading to the column R will give the roods, and in the column headed Pls. and on a line with the figures found, will be the number of perches; the nearest figures to 69981 will be found to be 700 in column 2 R, and on the same line will be found 32, which gives 2 roods, 32 perches, and the total contents are found to be 25A. 2R. 32P.

TABLE III.—*The Lengths of Circular Arcs.*—The application of this table has been explained under the head of length of curve, in the chapter on setting out curves.

TABLE IV.—The use of this table has already been very amply explained. When offsets are required for setting out curves, refer to the particular radius required. In the first horizontal column are the lengths of the arcs in chains; in the second, the lengths of A *c* in chains, from which the right-angled offset is to be set off, and in the third, the lengths of the offsets in feet. Fig. 146 illustrates the whole.

TABLE V.—Corrections for curvature and refraction, when required in levelling operations.

TABLE VI.—Natural sines and tangents to every ten minutes; example illustrating their application:—Required the sine of 9° 47′; from the tables take out the sines of 9° 40′ and 9° 50″; subtract the lesser from the greater; divide the difference by 10, and multiply the quotient by the number of odd minutes, here equal to 7.

Sine of 9° 50′, sine of 9° 40′=·0028669 the difference which divided by 10, and multiplied by 7 is equal to ·0020068
and sine of 9°—40′= ·1679159

·1699227=Natural of 9° 47′ as required.

Let it now be required to find the cosine of 9° 47′. Repeat the above operations on the cosines in tables; but instead of *adding* the quantity found to the cosine of 9° 40′, it is to be subtracted, and the cosine of 9° 47′ will thus be found equal to ·9854565, near enough for all the purposes we require in practice.

TABLE I.

Showing the reduction upon each chain necessary to reduce hypothenusal to horizontal measure.

Angle of assent or descent.		Reduction in links.
°	′	
4	0	¼
5	44	½
7	2	¾
8	7	1
11	28	2
12	50	2½
14	5	3
15	13	3½
16	15	4
17	15	4½
18	12	5
19	6	5½
19	57	6
20	47	6½
21	34	7
22	20	7½
23	5	8
23	48	8½
24	30	9
25	11	9½
25	51	10
26	30	10½
27	8	11
27	45	11½
28	22	12
28	58	12½
29	33	13
30	8	13½
30	41	14
31	15	14½
31	48	15
32	20	15½
32	52	16
33	24	16½
33	55	17
34	25	17½
34	55	18
35	25	18½
35	54	19
36	24	19½
36	53	20
37	21	20½
37	49	21
38	17	21½
38	45	22
39	12	22½
39	39	23
40	6	23½
40	33	24
40	56	24½
41	25	27

TABLE II.

Computation of Areas.

Pls.	Dec.	1 R	2 R	3 R
1	.006	·256	·506	·756
2	·012	·262	·512	·762
3	·019	·269	·519	·769
4	·025	·275	·525	·775
5	·031	·281	·531	·781
6	·037	·287	·537	·787
7	·044	·294	·544	·794
8	·050	·300	·550	·800
9	·056	·306	·556	·806
10	·062	·312	·562	·812
11	·069	·319	·569	·819
12	·075	·325	·575	·825
13	·081	·331	·581	·831
14	·087	·337	·587	·837
15	·094	·344	·594	·844
16	·100	·350	·600	·850
17	·106	·356	·606	·856
18	·112	·362	·612	·862
19	·119	·369	·619	·869
20	·125	·375	·625	·875
21	·131	·381	·631	·881
22	·137	·387	·637	·887
23	·144	·394	·644	·894
24	·150	·400	·650	·900
25	·156	·406	·656	·906
26	·162	·412	·662	·912
27	·169	·419	·669	·919
28	·175	·425	·675	·925
29	·181	·431	·681	·931
30	·187	·437	·687	·937
31	·194	·444	·694	·944
32	·200	·450	·700	·950
33	·206	·456	·706	·956
34	·212	·462	·712	·962
35	·219	·469	·719	·969
36	·225	·475	·725	·975
37	·231	·481	·731	·981
38	·237	·487	·737	·987
39	·244	·494	·744	·994
40	·250	·500	·750	

TABLE III.

Lengths of Circular Arcs.

De.	Arc.	De.	Arc.	De.	Arc.	′	Arc.	″	Arc.	‴	Arc
1	·0174533	61	1·0646508	121	2·1118484	1	2909	1	48	1	1
2	·0349066	62	1·0821041	122	2·1293017	2	5818	2	97	2	2
3	·0523599	63	1·0995574	123	2·1467550	3	8727	3	145	3	2
4	·0698132	64	1·1170107	124	2·1642083	4	11636	4	194	4	3
5	·0872665	65	1·1344640	125	2·1816616	5	14544	5	242	5	4
6	·1047198	66	1·1519173	126	2·1991149	6	17453	6	291	6	5
7	·1221730	67	1·1693706	127	2·2165682	7	20362	7	339	7	6
8	·1396263	68	1·1868239	128	2·2340214	8	23271	8	388	8	6
9	·1570796	69	1·2042772	129	2·2514747	9	26180	9	436	9	7
10	·1745329	70	1·2217305	130	2·2689280	10	29089	10	485	10	8
11	·1919862	71	1·2391838	131	2·2863813	11	31998	11	533	11	9
12	·2094395	72	1·2566371	132	2·3038346	12	34907	12	582	12	10
13	·2268928	73	1·2740904	133	2·3212879	13	37815	13	630	13	11
14	·2443461	74	1·2915436	134	2·3387412	14	40724	14	679	14	11
15	·2617994	75	1·3089969	135	2·3561945	15	43633	15	727	15	12
16	·2792527	76	1·3264502	136	2·3736478	16	46542	16	776	16	13
17	·2967060	77	1·3439035	137	2·3911011	17	49451	17	824	17	14
18	·3141593	78	1·3613568	138	2·4085544	18	52360	18	873	18	15
19	·3316126	79	1·3788101	139	2·4260077	19	55269	19	921	19	15
20	·3490659	80	1·3962634	140	2·4434610	20	58178	20	970	20	16
21	·3665191	81	1·4137167	141	2·4609142	21	61087	21	1018	21	17
22	·3839724	82	1·4311700	142	2·4783675	22	63995	22	1067	22	18
23	·4014257	83	1·4486233	143	2·4958208	23	66904	23	1115	23	19
24	·4188790	84	1·4660766	144	2·5132741	24	69813	24	1164	24	19
25	·4363323	85	1·4835299	145	2·5307274	25	72722	25	1212	25	20
26	·4537856	86	1·5009832	146	2·5481807	26	75631	26	1261	26	21
27	·4712389	87	1·5184364	147	2·5656340	27	78540	27	1309	27	22
28	·4886922	88	1·5358897	148	2·5830873	28	81449	28	1357	28	23
29	·5061455	89	1·5533430	149	2·6005406	29	84358	29	1406	29	23
30	·5235988	90	1·5707963	150	2·6179939	30	87266	30	1454	30	24
31	·5410521	91	1·5882496	151	2·6354472	31	90175	31	1503	31	25
32	·5585054	92	1·6057029	152	2·6529005	32	93084	32	1551	32	26
33	·5759587	93	1·6231562	153	2·6703538	33	95993	33	1599	33	27
34	·5934119	94	1·6406095	154	2·6878070	34	98902	34	1648	34	27
35	·6108652	95	1·6580628	155	2·7052603	35	101811	35	1697	35	28
36	·6283185	96	1·6755161	156	2·7227136	36	104720	36	1745	36	29
37	·6457718	97	1·6929694	157	2·7401669	37	107629	37	1794	37	30
38	·6632251	98	1·7104227	158	2·7576202	38	110538	38	1842	38	31
39	·6806784	99	1·7278760	159	2·7750735	39	113446	39	1891	39	32
40	·6981317	100	1·7453293	160	2·7925268	40	116355	40	1939	40	32
41	·7155850	101	1·7627825	161	2·8099801	41	119264	41	1988	41	33
42	·7330383	102	1·7802358	162	2·8274334	42	122173	42	2036	42	34
43	·7504916	103	1·7976891	163	2·8448867	43	125082	43	2085	43	35
44	·7679449	104	1·8151424	164	2·8623400	44	127991	44	2133	44	36
45	·7853982	105	1·8325957	165	2·8797933	45	130900	45	2182	45	36
46	·8028515	106	1·8500490	166	2·8972466	46	133809	46	2230	46	37
47	·8203047	107	1·8675023	167	2·9146999	47	136717	47	2279	47	38
48	·8377580	108	1·8849556	168	2·9321531	48	139626	48	2327	48	39
49	·8552113	109	1·9024089	169	2·9496064	49	142535	49	2376	49	40
50	·8726646	110	1·9198622	170	2·9670597	50	145444	50	2424	50	40
51	·8901179	111	1·9373155	171	2·9845130	51	148353	51	2473	51	41
52	·9075712	112	1·9547688	172	3·0919663	52	151262	52	2521	52	42
53	·9250245	113	1·9722220	173	3·0194196	53	154171	53	2570	53	43
54	·9424778	114	1·9896753	174	3·0368729	54	157080	54	2618	54	44
55	·9599311	115	2·0071286	175	3·0543262	55	159989	55	2666	55	44
56	·9773844	116	2·0245819	176	3·0717795	56	162597	56	2715	56	45
57	·9948377	117	2·0420352	177	3·0892328	57	165806	57	2763	57	46
58	1·0122910	118	2·0594885	178	3·1066861	58	168715	58	2812	58	47
59	1·0297443	119	2·0769418	179	3·1241394	59	171624	59	2860	59	48
60	1·0471976	120	2·0943951	180	3·1415927	60	274533	60	2909	60	48
De.	Arc.	De.	Arc.	De.	Arc.	′	Arc.	″	Arc.	‴	Arc

20 Chains Radius.

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	1·336	161·64
1	0·999	1·64	11	2·226	131·40
2	1·996	6·60	12	3·137	104·15
3	2·984	14·78	13	4·068	79·99
4	3·975	26·40	14	5·017	58·96
5	4·945	40·98	15	5·980	40·98
6	5·908	58·96	16	6·949	26·40
7	6·856	79·99	17	7·940	14·78
8	7·787	104·15	18	8·929	6·60
9	8·699	131·40	19	9·926	1·64
10	9·589	161·64	20	10·925	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage:

Chains.	Angles.	Chains.	Angles.
1	1° 25·94′	11	15° 45·34′
2	2 51·88	12	17 11·28
3	4 17·82	13	18 37·22
4	5 43·76	14	20 3·16
5	7 9·70	15	21 29·10
6	8 35·64	16	22 55·04
7	10 1·58	17	24 20·98
8	11 27·52	18	25 46·92
9	12 53·46	19	27 12·86
10	14 19·40	20	28 38·80

25 Chains Radius.

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·834	130·22
1	0·999	1·32	11	1·759	105·83
2	1·997	5·28	12	2·705	83·74
3	2·989	11·83	13	3·657	64·30
4	3·982	20·95	14	4·626	47·27
5	4·969	32·91	15	5·599	32·91
6	5·942	47·27	16	6·586	20·95
7	6·911	64·30	17	7·579	11·83
8	7·863	83·74	18	8·571	5·28
9	8·809	105·83	19	9·569	1·32
10	9·734	130·22	20	10·568	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage:

Chains.	Angles	Chains.	Angles.
1	1° 8·75′	11	12° 36·25′
2	2 17·50	12	13 45·00
3	3 26·25	13	14 53·75
4	4 35·00	14	16 2·50
5	5 43·75	15	17 11·25
6	6 52·50	16	18 20·00
7	8 1·25	17	19 28·75
8	9 10·00	18	20 37·50
9	10 18·75	19	21 46·25
10	11 27·50	20	22 55·00

30 *Chains Radius.*

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·572	109·00
1	0·999	1·10	11	1·522	88·39
2	1·998	4·39	12	2·482	69·98
3	2·994	9·88	13	3·452	53·66
4	3·990	17·60	14	4·429	39·46
5	4·977	27·44	15	5·411	27·44
6	5·959	39·46	16	6·397	17·60
7	6·936	53·66	17	7·393	9·88
8	7·905	69·98	18	8·389	4·39
9	8·865	88·39	19	9·389	1·10
10	9·816	109·00	20	10·388	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage:

Chains.	Angles.	Chains.	Angles.
1	0° 57·30′	11	10° 30·26′
2	1 54·59	12	11 27·56
3	2 51·89	13	12 24·86
4	3 49·19	14	13 22·16
5	4 46·48	15	14 19·45
6	5 43·78	16	15 16·75
7	6 41·08	17	16 14·05
8	7 38·37	18	17 10·34
9	8 35·67	19	18 8·64
10	9 32·97	20	19 5·94

35 *Chains Radius.*

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·416	93·64
1	0·999	0·94	11	1·379	75·95
2	1·999	3·76	12	2·350	60·06
3	2·996	8·47	13	3·330	46·03
4	3·992	15·07	14	4·310	33·86
5	4·982	23·53	15	5·298	23·53
6	5·970	33·86	16	6·288	15·07
7	6·950	46·03	17	7·284	8·47
8	7·930	60·06	18	8·281	3·76
9	8·901	75·95	19	9·281	0·94
10	9·864	93·64	20	10·280	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage:

Chains.	Angles.	Chains.	Angles.
1	0° 49·11′	11	9° 0·21′
2	1 38·22	12	9 49·33
3	2 27·33	13	10 38·44
4	3 16·44	14	11 27·55
5	4 5·55	15	12 16·66
6	4 54·66	16	13 5·77
7	5 43·77	17	13 54·88
8	6 32·88	18	14 43·99
9	7 21·99	19	15 33·11
10	8 11·11	20	16 22·22

40 *Chains Radius.*

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·318	82·07
1	0·999	0·82	11	1·289	66·54
2	1·999	3·30	12	2·268	52·61
3	2·997	7·42	13	3·250	40·31
4	3·993	13·18	14	4·235	29·65
5	4·986	20·59	15	5·227	20·59
6	5·978	29·65	16	6·221	13·18
7	6·963	40·31	17	7·216	7·42
8	7·946	52·61	18	8·214	3·30
9	8·924	66·54	19	9·214	0·82
10	9·896	82·07	20	10·214	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage:

Chains.	Angles.	Chains.	Angles.
1	0° 42·97′	11	7° 52·69′
2	1 25·94	12	8 35·66
3	2 8·91	13	9 18·63
4	2 51·88	14	10 1·60
5	3 34·86	15	10 44·58
6	4 17·83	16	11 27·55
7	5 0·80	17	12 10·52
8	5 43·77	18	12 53·49
9	6 26·74	19	13 36·46
10	7 9·72	20	14 19·44

45 *Chains Radius.*

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·250	73·03
1	0·999	0·73	11	1·228	59·36
2	1·999	2·93	12	2·211	46·81
3	2·997	6·59	13	3·196	35·85
4	3·993	11·72	14	4·186	26·35
5	4·990	18·31	15	5·178	18·31
6	5·981	26·35	16	6·174	11·72
7	6·971	35·85	17	7·171	6·59
8	7·957	46·81	18	8·169	2·93
9	8·939	59·36	19	9·168	0·73
10	9·918	73·03	20	10·168	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage:

Chains.	Angles.	Chains.	Angles.
1	0° 38·19′	11	7° 0·16′
2	1 16·39	12	7 38·36
3	1 54·59	13	8 16·56
4	2 32·78	14	8 54·75
5	3 10·98	15	9 32·95
6	3 49·18	16	10 11·15
7	4 27·37	17	10 49·34
8	5 5·57	18	11 27·54
9	5 43·77	19	12 5·74
10	6 21·97	20	12 43·94

50 *Chains Radius.*

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·202	65·95
1	0·999	0·65	11	1·183	53·19
2	1·999	2·62	12	2·169	42·14
3	2·997	5·91	13	3·158	32·17
4	3·995	10·51	14	4·149	23·64
5	4·991	16·58	15	5·143	16·58
6	5·985	23·64	16	6·139	10·51
7	6·976	32·17	17	7·137	5·91
8	7·965	42·14	18	8·135	2·62
9	8·950	53·19	19	9·135	0·65
10	9·932	65·95	20	10·134	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage:

Chains.	Angles.	Chains.	Angles.
1	0° 34·38′	11	6° 18·16′
2	1 8·75	12	6 52·53
3	1 43·13	13	7 26·91
4	2 17·51	14	8 1·29
5	2 51·89	15	8 35·67
6	3 26·26	16	9 10·05
7	4 0·64	17	9 44·42
8	4 35·02	18	10 18·80
9	5 9·40	19	10 53·18
10	5 43·78	20	11 27·56

55 *Chains Radius.*

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·166	59·82
1	0·999	0·59	11	1·151	48·48
2	1·999	2·38	12	2·139	38·32
3	2·997	5·35	13	3·130	29·35
4	3·996	9·52	14	4·123	21·57
5	4·992	14·87	15	5·118	14·87
6	5·987	21·57	16	6·114	9·52
7	6·980	29·35	17	7·113	5·35
8	7·971	38·32	18	8·111	2·38
9	8·959	48·48	19	9·111	0·59
10	9·944	59·82	20	10·110	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage:

Chains.	Angles.	Chains.	Angles.
1	0° 31·27′	11	5° 43·98′
2	1 2·54	12	6 15·25
3	1 33·81	13	6 46·52
4	2 5·08	14	7 17·79
5	2 36·35	15	7 49·06
6	3 7·62	16	8 20·33
7	3 38·89	17	8 51·60
8	4 10·16	18	9 22·87
9	4 41·44	19	9 54·15
10	5 12·70	20	10 25·42

60 Chains Radius.

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·139	54·86
1	0·999	0·54	11	1·128	44·46
2	1·999	2·18	12	2·119	35·13
3	2·998	4·92	13	3·110	26·91
4	3·996	8·74	14	4·103	19·78
5	4·994	13·66	15	5·099	13·66
6	5·989	19·78	16	6·096	8·74
7	6·982	26·91	17	7·095	4·92
8	7·974	35·13	18	8·094	2·18
9	8·965	44·46	19	9·093	0·54
10	9·954	54·86	20	10·093	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage:

Chains.	Angles.	Chains.	Angles.
1	0° 28·64′	11	5° 15·13′
2	0 57·29	12	5 43·78
3	1 25·94	13	6 12·43
4	1 54·59	14	6 41·07
5	2 23·24	15	7 9·72
6	2 51·89	16	7 38·37
7	3 20·54	17	8 7·02
8	3 49·18	18	8 35·67
9	4 17·83	19	9 4·31
10	4 46·48	20	9 32·96

70 Chains Radius.

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·102	47·05
1	0·999	0·46	11	1·093	38·03
2	1·999	1·86	12	2·085	30·16
3	2·998	4·20	13	3·080	23·07
4	3·997	7·60	14	4·075	16·94
5	4·994	11·77	15	5·073	11·77
6	5·992	16·94	16	6·071	7·60
7	6·987	23·07	17	7·069	4·20
8	7·983	30·16	18	8·069	1·86
9	8·975	38·03	19	9·068	0·46
10	9·965	47·05	20	10·068	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage:

Chains.	Angles.	Chains.	Angles.
1	0° 24·55′	11	4° 30·11′
2	0 49·11	12	4 54·66
3	1 13·66	13	5 19·22
4	1 38·22	14	5 43·77
5	2 2·77	15	6 8·33
6	2 27·33	16	6 32·88
7	2 51·88	17	6 57·42
8	3 16·44	18	7 21·98
9	3 40·99	19	7 46·53
10	4 5·55	20	8 11·09

80 *Chains Radius.*

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·078	41·19
1	0·999	0·41	11	1·072	33·37
2	1·999	1·65	12	2·067	26·37
3	2·998	3·70	13	3·063	20·18
4	3·997	6·60	14	4·058	14·84
5	4·994	10·53	15	5·058	10·53
6	5·994	14·84	16	6·055	6·60
7	6·989	20·18	17	7·054	3·70
8	7·986	26·37	18	8·053	1·65
9	8·981	33·37	19	9·053	0·41
10	9·974	41·19	20	10·053	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage :

Chains.	Angles.	Chains.	Angles.
1	0° 21·48′	11	3° 56·35′
2	0 42·97	12	4 17·83
3	1 4·46	13	4 39·32
4	1 25·94	14	5 0·80
5	1 47·43	15	5 22·29
6	2 8·92	16	5 43·78
7	2 30·40	17	6 5·26
8	2 51·89	18	6 26·75
9	3 13·37	19	6 48·23
10	3 34·86	20	7 9·72

90 *Chains Radius.*

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·061	36·63
1	0·999	0·36	11	1·057	29·67
2	1·999	1·45	12	2·054	23·43
3	2·999	3·26	13	3·050	17·95
4	3·998	5·86	14	4·046	13·28
5	4·997	9·16	15	5·044	9·16
6	5·995	13·28	16	6·043	5·86
7	6·991	17·95	17	7·042	3·26
8	7·987	23·43	18	8·042	1·45
9	8·984	29·67	19	9·042	0·36
10	9·980	36·63	20	10·042	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage :

Chains.	Angles.	Chains.	Angles.
1	0° 19·09′	11	3° 30·08′
2	0 38·19	12	3 49·18
3	0 57·29	13	4 8·28
4	1 16·34	14	4 27·38
5	1 35·49	15	4 46·48
6	1 54·59	16	5 5·58
7	2 13·69	17	5 24·68
8	2 32·79	18	5 43·77
9	2 51·89	19	6 2·88
10	3 10·98	20	6 21·97

100 *Chains Radius.*

Length of Arc. Chains.	Ac. Chains.	Offset. Feet.	Length of Arc. Chains.	Tk. Chains.	Offset. Feet.
0	0·000	0·00	10	0·053	32·96
1	0·999	0·32	11	1·050	26·69
2	1·999	1·29	12	2·046	21·10
3	2·999	2·91	13	3·043	16·13
4	3·998	5·27	14	4·040	11·86
5	4·997	8·25	15	5·038	8·25
6	5·995	11·86	16	6·037	5·27
7	6·992	16·13	17	7·036	2·91
8	7·990	21·10	18	8·036	1·29
9	8·986	26·69	19	10·036	0·32
10	9·982	32·96	20	10·036	0·00

For setting out by Tangential Angles.

Angles formed at starting point by the tangent and the chords at the following chainage:

Chains.	Angles.	Chains.	Angles.
1	0° 17·19′	11	3° 9·08′
2	0 34·38	12	3 26·27
3	0 51·56	13	3 43·45
4	1 8·75	14	4 0·64
5	1 25·94	15	4 17·83
6	1 43·13	16	4 35·02
7	2 0·32	17	4 52·21
8	2 17·51	18	5 9·40
9	2 34·70	19	5 26·59
10	2 51·89	20	5 43·78

TABLE V.

Difference between the Apparent and True Level for distances in Feet.		Difference between the Apparent and True Level for distances in Links.		Difference between the Apparent and True Level for distances in Miles.	
Correction in Decimals of Feet.		Correction in Decimals of Feet.		Correction in Decimals of Feet.	
Distances in Feet.	For Curvature and Refraction.	Distances in Links.	For Curvature and Refraction.	Distances in Miles.	For Curvature and Refraction.
100	·00020	100	·00009	¼	·0357
150	·00046	150	·00021	½	·1430
200	·00083	200	·00036	¾	·3216
250	·00128	250	·00056	1	·5717
300	·00184	300	·00081	1½	1·2864
350	·00251	350	·00110	2	2·2869
400	·00328	400	·00143	2½	3·5733
450	·00415	450	·00181	3	5·1469
500	·00513	500	·00224	3½	7·0035
550	·00621	550	·00270	4	9·1474
600	·00738	600	·00321	4½	11·5773
650	·00866	650	·00377	5	14·2929
700	·01005	700	·00438	5½	17·2945
750	·01153	750	·00562	6	20·5817
800	·01312	800	·00572	6½	24·1551
850	·01481	850	·00645	7	28·0143
900	·01661	900	·00723	7½	32·1591
950	·01851	950	·00806	8	36·5883
1000	·02059	1000	·00893	8½	41·3066
1050	·02261	1050	·00985	9	46·3089
1100	·02481	1100	·01081	9½	51·5975
1150	·02712	1150	·01181	10	57·1714
1200	·02953	1200	·01287	11	69·1774
1250	·03204	1250	·01395	12	82·3269
1300	·03465	1300	·01509	13	96·6197
1350	·03738	1350	·01628	14	112·0560
1400	·04019	1400	·01751	15	128·6357
1450	·04311	1450	·01878	16	146·3589
1500	·04614	1500	·02010	17	165·2254
1550	·04927	1550	·02146	18	185·2359
1600	·05250	1600	·02287	19	206·3889
1650	·05583	1650	·02432	20	28·6857
1700	·05926	1700	·02582		
1750	·06280	1750	·02736		
1800	·06645	1800	·02895		
1850	·07161	1850	·03058		
1900	·07403	1900	·03225		
1950	·07798	1950	·03397		
2000	·08203	2000	·03578		

NATURAL SINES.

Degrees.	0′	10′	20′	30′	40′	50′	60′	
0	0000000	0029089	0058177	0087265	0116353	0145439	0174524	89
1	0174524	0203608	0232690	0261769	0290847	0319922	0348995	88
2	0348995	0378065	0407131	0436194	0465253	0494308	0523360	87
3	0523360	0552406	0581448	0610485	0639517	0668544	0697565	86
4	0697565	0726580	0755589	0784591	0813587	0842576	0871557	85
5	0871557	0900532	0929499	0958458	0987408	1016351	1045285	84
6	1045285	1074210	1103126	1132032	1160929	1189816	1218693	83
7	1218693	1247560	1276416	1305262	1334096	1362919	1391731	82
8	1391731	1420531	1449319	1478094	1506857	1535607	1564345	81
9	1564345	1593069	1621779	1650476	1679159	1707828	1736482	80
10	1736482	1765121	1793746	1822355	1850949	1879528	1908090	79
11	1908090	1936636	1965166	1993679	2022176	2050655	2079117	78
12	2079117	2107561	2135988	2164396	2192786	2221158	2249511	77
13	2249511	2277844	2306159	2334454	2362729	2390984	2419219	76
14	2419219	2447433	2475627	2503800	2531952	2560082	2588190	75
15	2588190	2616277	2644342	2672384	2700403	2728400	2756374	74
16	2757364	2784824	2812251	2840153	2868032	2895887	2923717	73
17	2923717	2951522	2979303	3007058	3034788	3062492	3090170	72
18	3090170	3117822	3145448	3173047	3200619	3228164	3255682	71
19	3255682	3283172	3310634	3338069	3365475	3392852	3420201	70
20	3420201	3447521	3474812	3502074	3529306	3556508	3583679	69
21	3583679	3610821	3637932	3665012	3692061	3719079	3746066	68
22	3746066	3773021	3799944	3826834	3853693	3880518	3907311	67
23	3907311	3934071	3960798	3987491	4014150	4040775	4067366	66
24	4067366	4093923	4120445	4146932	4173385	4199801	4226183	65
25	4226183	4252528	4278838	4305111	4331348	4357548	4383711	64
26	4383711	4409838	4435927	4461978	4487992	4513967	4539905	63
27	4539905	4565804	4591665	4617486	4643269	4669012	4694716	62
28	4694716	4720380	4746004	4771588	4797131	4822634	4848096	61
29	4848096	4873517	4898897	4924236	4949532	4974787	5000000	60
30	5000000	5025170	5050298	5075384	5100426	5125425	5150381	59
31	5150381	5175293	5200161	5224986	5249766	5274502	5299193	58
32	5299193	5323839	5348440	5372996	5397507	5421971	5446390	57
33	5446390	5470763	5495090	5519370	5543603	5567790	5591929	56
34	5591929	5616021	5640066	5664062	5688011	5711912	5735764	55
35	5735764	5759568	5783323	5807030	5830687	5854294	5877853	54
36	5877853	5901361	5924819	5948228	5971586	5994893	6018150	53
37	6018150	6041356	6064511	6087614	6110666	6133666	6156615	52
38	6156615	6179511	6202355	6225146	6247885	6270571	6293204	51
39	6293204	6315784	6338310	6360782	6383201	6405566	6427876	50
40	6427876	6450132	6472334	6494480	6516572	6538609	6560590	49
41	6560590	6582516	6604386	6626200	6647959	6669661	6691306	48
42	6691306	6712895	6734427	6755902	6777320	6798681	6819984	47
43	6819984	6841229	6862416	6883546	6904617	6925630	6946584	46
44	6946584	6967479	6988315	7009093	7029811	7050469	7071068	45
	60′	50′	40′	30′	20′	10′	0′	Degrees.

NATURAL COSINES.

NATURAL SINES.

Degrees.	0′	10′	20′	30′	40′	50′	60′	
45	7071068	7091607	7112086	7132504	7152863	7173161	7193398	44
46	7193398	7213574	7233690	7253744	7273736	7293668	7313537	43
47	7313537	7333345	7353090	7372773	7392394	7411953	7431448	42
48	7431448	7450881	7470251	7489557	7508800	7527980	7547096	41
49	7547096	7566148	7585136	7604060	7622919	7641714	7660444	40
50	7660444	7679110	7697710	7716246	7734716	7753121	7771460	39
51	7771460	7789733	7807940	7826082	7844157	7862165	7880108	38
52	7880108	7897983	7915792	7933533	7951208	7968815	7986355	37
53	7986355	8003827	8021232	8040299	8055837	8073038	8090170	36
54	8090170	8107234	8124229	8141155	8158013	8174801	8191520	35
55	8191520	8208170	8224751	8241262	8257703	8274074	8290376	34
56	8290376	8306607	8322768	8338858	8354878	8370827	8386706	33
57	8386706	8402513	8418249	8433914	8449508	8465030	8480481	32
58	8480481	8495860	8511167	8526402	8541564	8556655	8571673	31
59	8571673	8586619	8601491	8616292	8631019	8645673	8660254	30
60	8660254	8674762	8689196	8703557	8717844	8732058	8746197	29
61	8746197	8760263	8774254	8788171	8802014	8815782	8829476	28
62	8829476	8843095	8856639	8870108	8883503	8896822	8910065	27
63	8910065	8923234	8936326	8949344	8962285	8975151	8987940	26
64	8987940	9000654	9013292	9025853	9038338	9050746	9063078	25
65	9063078	9075333	9087511	9099613	9111637	9123584	9135455	24
66	9135455	9147247	9158963	9170601	9182161	9193644	9205049	23
67	9205049	9216375	9227624	9238795	9249888	9260902	9271839	22
68	9271839	9282696	9293475	9304176	9314797	9325340	9335804	21
69	9335804	9346189	9356495	9366722	9376869	9386938	9396926	20
70	9396926	9406835	9416665	9426415	9436085	9445675	9455186	19
71	9455186	9464616	9473966	9483237	9492426	9501536	9510565	18
72	9510565	9519514	9528382	9537170	9545876	9554502	9563048	17
73	9563048	9571512	9579895	9588197	9596418	9604558	9612617	16
74	9612617	9620594	9628490	9636305	9644037	9651689	9659258	15
75	9659258	9666746	9674152	9681476	9688719	9695879	9702957	14
76	9702957	9709953	9716867	9723699	9730449	9737116	9743701	13
77	9743701	9750203	9756623	9762960	9769215	9775387	9781476	12
78	9781476	9787483	9793406	9799247	9805005	9810680	9816272	11
79	9816272	9821781	9827206	9832549	9837808	9842985	9848078	10
80	9848078	9853087	9858013	9862856	9867615	9872291	9876883	9
81	9876883	9881392	9885817	9890159	9894416	9898590	9902681	8
82	9902681	9906687	9910610	9914449	9918204	9921874	9925462	7
83	9925462	9928965	9932384	9935719	9938969	9942136	9945219	6
84	9945219	9948217	9951132	9953962	9956708	9959370	9961947	5
85	9961947	9964440	9966849	9969173	9971413	9973569	9975641	4
86	9975641	9977627	9979530	9981348	9983082	9984731	9986295	3
87	9986295	9987775	9989171	9990482	9991709	9992851	9993908	2
88	9993908	9994881	9995770	9996573	9997292	9997927	9998477	1
89	9998477	9998942	9999323	9999619	9999831	9999958	1000000	0
	60′	50′	40′	30′	20′	10′	0′	Degrees.

NATURAL COSINES.

NATURAL TANGENTS.

Degrees.	0′	10′	20′	30′	40′	50′	60′	
0	0000000	0029089	0058178	0087269	0116361	0145454	0174551	89
1	0174551	0203650	0232753	0261859	0290970	0320086	0349208	88
2	0349208	0378335	0407469	0436609	0465757	0494913	0524078	87
3	0524078	0553251	0582434	0611626	0640829	0670043	0699268	86
4	0699268	0728505	0757755	0787017	0816293	0845583	0874887	85
5	0874887	0904206	0933540	0962890	0992257	1021641	1051042	84
6	1051042	1080462	1109899	1139356	1171781	1198329	1227846	83
7	1227846	1257384	1286943	1316525	1346129	1375757	1405408	82
8	1405408	1435084	1464784	1494510	1524262	1554040	1583844	81
9	1583844	1613677	1643537	1673426	1703344	1733292	1763270	80
10	1763270	1793279	1823319	1853390	1883495	1913632	1943803	79
11	1943803	1974008	2004248	2034523	2064834	2095181	2125566	78
12	2125566	2155988	2186448	2216947	2247485	2278063	2308682	77
13	2308682	2339342	2370044	2400788	2431575	2462405	2493280	76
14	2493280	2524200	2555165	2586176	2617234	2648339	2679492	75
15	2679492	2710694	2741945	2773245	2804597	2835999	2867454	74
16	2867454	2898961	2930521	2962135	2993803	3025527	3057307	73
17	3057307	3089143	3121036	3152988	3184998	3217067	3249197	72
18	3249197	3281387	3313639	3345953	3378330	3410771	3443276	71
19	3443276	3475846	3508483	3541186	3573956	3606795	3639702	70
20	3639702	3672680	3705728	3738847	3772038	4805302	3838640	69
21	3838640	3872053	3905541	3939105	3972746	4006465	4040262	68
22	4040262	4074139	4108097	4142136	4176257	4210460	4244748	67
23	4244748	4279121	4313579	4348124	4382756	4417477	4452287	66
24	4452287	4487187	4522179	4557263	4592439	4627710	4663077	65
25	4663077	4698539	4734098	4769755	4805512	4841368	4877326	64
26	4877326	4913386	4949549	4985816	5022189	5058668	5095254	63
27	5095254	5131950	5168755	5205671	5242698	5279839	5317094	62
28	5317094	5354465	5391952	5429557	5467281	5505125	5543091	61
29	5543091	5581179	5619391	5657728	5696191	5734783	5773503	60
30	5773503	5812353	5851335	5890450	5929699	5969084	6008606	59
31	6008606	6048266	6088067	6128008	6168092	6208320	6248694	58
32	6248694	6289214	6329818	6370703	6411673	6452797	6494076	57
33	6494076	6535511	6577103	6618856	6660769	6702845	6745085	56
34	6745085	6787492	6830066	6872810	6915725	6958813	7002075	55
35	7002075	7045515	7089133	7132931	7176911	7221075	7265425	54
36	7265425	7309963	7354691	7399611	7444724	7490033	7535541	53
37	7535541	7581248	7627157	7673270	7719589	7766118	7812856	52
38	7812856	7859808	7906975	7954359	8001963	8049790	8097840	51
39	8097840	8146113	8194625	8243364	8292337	8341547	8390996	50
40	8390996	8440688	8490624	8540807	8591240	8641926	8692867	49
41	8692867	8744067	8795528	8847253	8899244	8951506	9004040	48
42	9004040	9056851	9109940	9163312	9216969	9270914	9325151	47
43	9325151	9379683	9434513	9489646	9545083	9600829	9656888	46
44	9656888	9713262	9769956	9826973	9884316	9941991	1.0000000	45
	60	50	40	30	20	10	0	Degrees.

NATURAL COTANGENTS.

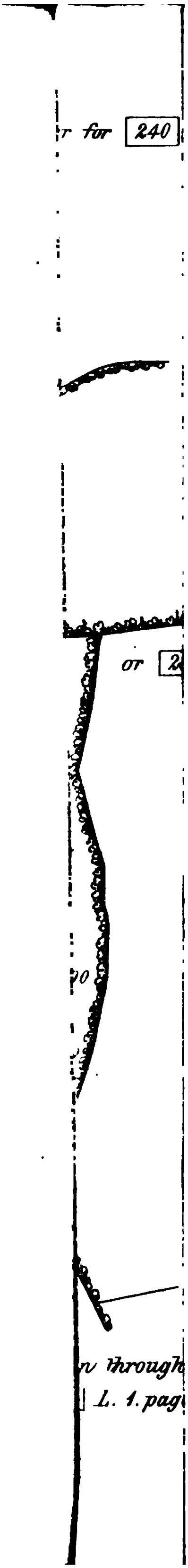
TABLE VI.—*continued.*

NATURAL TANGENTS.

Degrees.	0′	10′	20′	30′	40′	50′	60′	
45	1·0000000	1·0058348	1·0117038	1·0176074	1·0235461	1·0295203	1 0355303	44
46	1·0355303	1·0415767	1·0476598	1·0537801	1·0599381	1·0661341	1·0723687	43
47	1·0723687	1·0786423	1·0849554	1·0913085	1·0977020	1·1041365	1·1106125	42
48	1·1106125	1·1171305	1·1236909	1·1302944	1·1369414	1·1436326	1·1503684	41
49	1·1503684	1·1571495	1·1639763	1·1708496	1·1777698	1·1847376	1·1917536	40
50	1·1917536	1·1988184	1·2059327	1·2130970	1·2203121	1·2275786	1·2348972	39
51	1·2348972	1·2422685	1·2496933	1·2571723	1·2647062	1·2722957	1·2799416	38
52	1·2799416	1·2876447	1·2954057	1·3032254	1·3111046	1·3190441	1·3270448	37
53	1·3270448	1·3351075	1·3432331	1·3514224	1·3596764	1·3679959	1·3763819	36
54	1·3763819	1·3848353	1·3933571	1·4019483	1·4106098	1·4193427	1·4281480	35
55	1·4281480	1·4370268	1·4459801	1·4550090	1·4641147	1·4732983	1·4825610	34
56	1·4825610	1·4919039	1·5013282	1·5108352	1·5204261	1·5301023	1·5398650	33
57	1·5398650	1·5497155	1·5596552	1·5696856	1·5798079	1·5900238	1·6003345	32
58	1·6003345	1·6107417	1·6212469	1·6318517	1·6425576	1·6533663	1·6642795	31
59	1·6642795	1·6752988	1·6864261	1·6976631	1·7090116	1·7204736	1·7320508	30
60	1·7320508	1·7437453	1.7555590	1·7674940	1·7795524	1·7917362	1·8040478	29
61	1·8040478	1·8164892	1·8290628	1·8417709	1·8546159	1·8676003	1·8807265	28
62	1·8807265	1·8939971	1·9074147	1·9209821	1·9347020	1·9485772	1·9626105	27
63	1·9626105	1·9768050	1·9911637	2·0056897	2·0203862	2·0352565	2·0503038	26
64	2·0503038	2·0655318	2·0809438	2·0965436	2·1123348	2·1283213	2·1445069	25
65	2·1445069	2·1608958	2·1774920	2·1942997	2·2113234	2·2285676	2·2460368	24
66	2·2460368	2·2637357	2·2816693	2·2998425	2·3182606	2·3369287	2·3558524	23
67	2·3558524	2·3750372	2·3944889	2·4142136	2·4342172	2·4545061	2·4750869	22
68	2·4750869	2·4959661	2·5171507	2·5386479	2·5604649	2·5826094	2·6050891	21
69	2·6050891	2·6279121	2·6510867	2·6746215	2·6985254	2·7228076	2·7474774	20
70	2·7474774	2·7725448	2·7980198	2·8239129	2·8502349	2·8769970	2·9042109	19
71	2·9042109	2·9318885	2·9600422	2·9886850	3·0178301	3·0474915	3·0776835	18
72	3·0776835	3·1084210	3·1397194	3·1715948	3·2040638	3·2371438	3·2708526	17
73	3·2708526	3·3052091	3·3402326	3·3759434	3·4123626	3·4495120	3·4874144	16
74	3·4874144	3·5260938	3·5655749	3·6058835	3·6470467	3·6890927	3·7320508	15
75	3·7320508	3·7759519	3·8208281	3·8667181	3·9136420	3·9616518	4·0107809	14
76	4·0107809	4·0610700	4·1125614	4·1652998	4·2193318	4·2747066	4·3314759	13
77	4·3814759	4·3896940	4·4494181	4·5107085	4·5736287	4·6382457	4·7046301	12
78	3·7046301	4·7728568	4·8430045	3·9151570	4·9894027	5·0658352	5·1445540	11
79	5·1445540	5·2256647	5·3092793	5·3955172	5·4845052	5·5763786	5·6712818	10
80	5·6712818	5·7693688	5·8708042	5·9757644	6·0844381	6·1970279	6·3137515	9
81	6·3137515	6·4348428	6·5605538	6·6911562	6·8269437	6·9682335	7·1153697	8
82	7·1153697	7·2687255	7·4287064	7·5957541	7·7703506	7·9530224	8·1443464	7
83	8·1443464	8·3449558	8.5555468	8·7768874	9·0098261	9·2553035	9·5143645	6
84	9·5143645	9·7881732	10·078031	10·385397	10·711913	11·059431	11·430052	5
85	11·430052	11·826167	12·250505	12·706205	13·196883	13·726738	14·300666	4
86	14·300666	14·924417	15·604784	16·349855	17·169337	18·074977	19·081137	3
87	19·081137	20·205553	21·470401	22·903766	24·541758	26·431600	28·636253	2
88	28·636253	31·241577	34·367771	38·188459	42·964077	49·103881	57·289962	1
89	57·289962	68·750087	85·939791	114·58865	171·88540	343·77371	Infinite.	0
	60′	50′	40′	30′	20′	10′	0′	Degrees.

NATURAL COTANGENTS.

THE END.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NATURAL TANGENTS.

Degrees.	0′	10′	20′	30′	40′	50′	60′	
45	1·0000000	1·0058348	1·0117088	1·0176074	1·0235461	1·0295203	1 0355303	
46	1·0355303	1·0415767	1·0476598	1·0537801	1·0599381	1·0661341	1·0723687	
47	1·0723687	1·0786423	1·0849554	1·0913085	1·0977020	1·1041365	1·1106125	
48	1·1106125	1·1171305	1·1236909	1·1302944	1·1369414	1·1436326	1·1503684	
49	1·1503684	1·1571495	1·1639763	1·1708496	1·1777698	1·1847376	1·1917536	
50	1·1917536	1·1988184	1·2059327	1·2130970	1·2203121	1·2275786	1·2348972	
51	1·2348972	1·2422685	1·2496933	1·2571723	1·2647062	1·2722957	1·2799416	
52	1·2799416	1·2876447	1·2954057	1·3032254	1·3111046	1·3190441	1·3270448	
53	1·3270448	1·3351075	1·3432331	1·3514224	1·3596764	1·3679959	1·3763819	
54	1·3763819	1·3848353	1·3933571	1·4019483	1·4106098	1·4193427	1·4281480	
55	1·4281480	1·4370268	1·4459801	1·4550090	1·4641147	1·4732983	1·4825610	
56	1·4825610	1·4919039	1·5013282	1·5108352	1·5204261	1·5301023	1·5398650	
57	1·5398650	1·5497155	1·5596552	1·5696856	1·5798079	1·5900238	1·6003345	
58	1·6003345	1·6107417	1·6212469	1·6318517	1·6425576	1·6533663	1·6642795	
59	1·6642795	1·6752988	1·6864261	1·6976631	1·7090116	1·7204736	1·7320508	
60	1·7320508	1·7437453	1.7555590	1·7674940	1·7795524	1·7917362	1·8040478	
61	1·8040478	1·8164892	1·8290628	1·8417709	1·8546159	1·8676003	1·8807265	
62	1·8807265	1·8939971	1·9074147	1·9209821	1·9347020	1·9485772	1·9626105	
63	1·9626105	1·9768050	1·9911637	2·0056897	2·0208862	2·0352565	2·0503038	
64	2·0503038	2·0655318	2·0809438	2·0965436	2·1123348	2·1283213	2·1445069	
65	2·1445069	2·1608958	2·1774920	2·1942997	2·2113234	2·2285676	2·2460368	
66	2·2460368	2·2637357	2·2816693	2·2998425	2·3182606	2·3369287	2·3558524	
67	2·3558524	2·3750372	2·3944889	2·4142136	2·4342172	2·4545061	2·4750869	
68	2·4750869	2·4959661	2·5171507	2·5386479	2·5604649	2·5826094	2·6050891	
69	2·6050891	2·6279121	2·6510867	2·6746215	2·6985254	2·7228076	2·7474774	
70	2·7474774	2·7725448	2·7980198	2·8239129	2·8502349	2·8769970	2·9042109	
71	2·9042109	2·9318885	2·9600422	2·9886850	3·0178301	3·0474915	3·0776835	
72	3·0776835	3·1084210	3·1397194	3·1715948	3·2040638	3·2371438	3·2708526	17
73	3·2708526	3·3052091	3·3402326	3·3759434	3·4123626	3·4495120	3·4874144	16
74	3·4874144	3·5260938	3·5655749	3·6058835	3·6470467	3·6890927	3·7320508	15
75	3·7320508	3·7759519	3·8208281	3·8667131	3·9136420	3·9616518	4·0107809	14
76	4·0107809	4·0610700	4·1125614	4·1652998	4·2193318	4·2747066	4·3314759	13
77	4·3314759	4·3896940	4·4494181	4·5107085	4·5736287	4·6382457	4·7046301	12
78	3·7046301	4·7728568	4·8430045	3·9151570	4·9894027	5·0658352	5·1445540	11
79	5·1445540	5·2256647	5·3092793	5·3955172	5·4845052	5·5763786	5·6712818	10
80	5·6712818	5·7693688	5·8708042	5·9757644	6·0844381	6·1970279	6·3137515	9
81	6·3137515	6·4348428	6·5605538	6·6911562	6·8269437	6·9682335	7·1153697	8
82	7·1153697	7·2687255	7·4287064	7·5957541	7·7703506	7·9530224	8·1443464	7
83	8·1443464	8·3449558	8.5555468	8·7768874	9·0098261	9·2553035	9·5143645	6
84	9·5143645	9·7881732	10·078031	10·385397	10·711913	11·059431	11·430052	5
85	11·430052	11·826167	12·250505	12·706205	13·196883	13·726738	14·300666	4
86	14·300666	14·924417	15·604784	16·349855	17·169337	18·074977	19·081137	3
87	19·081137	20·205553	21·470401	22·903766	24·541758	26·431600	28·636253	2
88	28·636253	31·241577	34·367771	38·188459	42·964077	49·103881	57·289962	1
89	57·289962	68·750087	85·939791	114·58865	171·88540	343·77371	Infinite.	0
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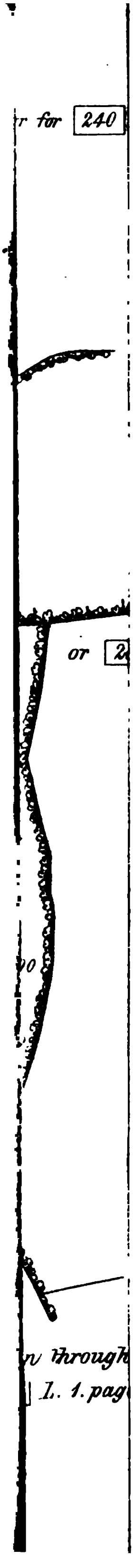
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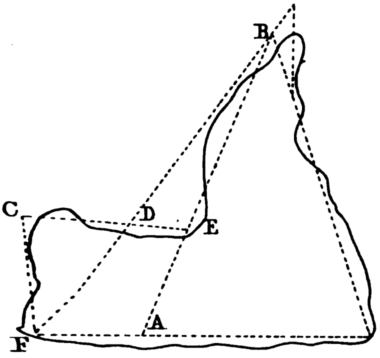
Fig. 2.



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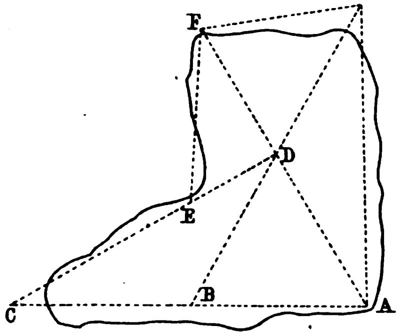

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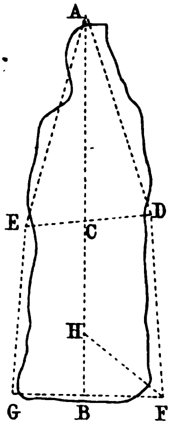

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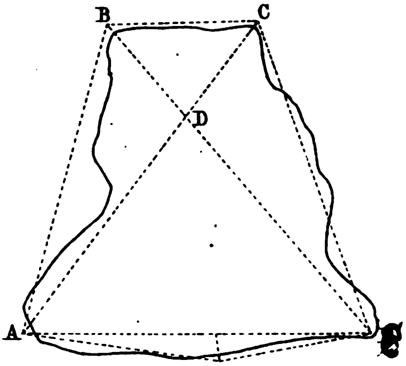

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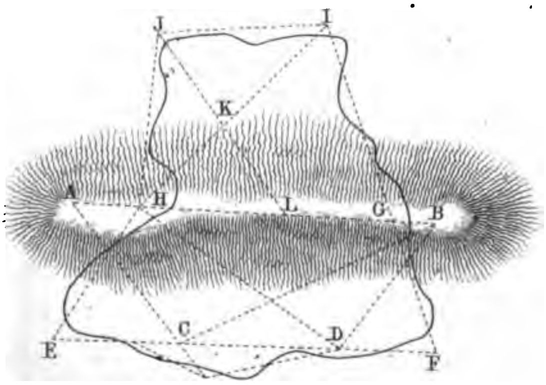


J.R.Jobbi

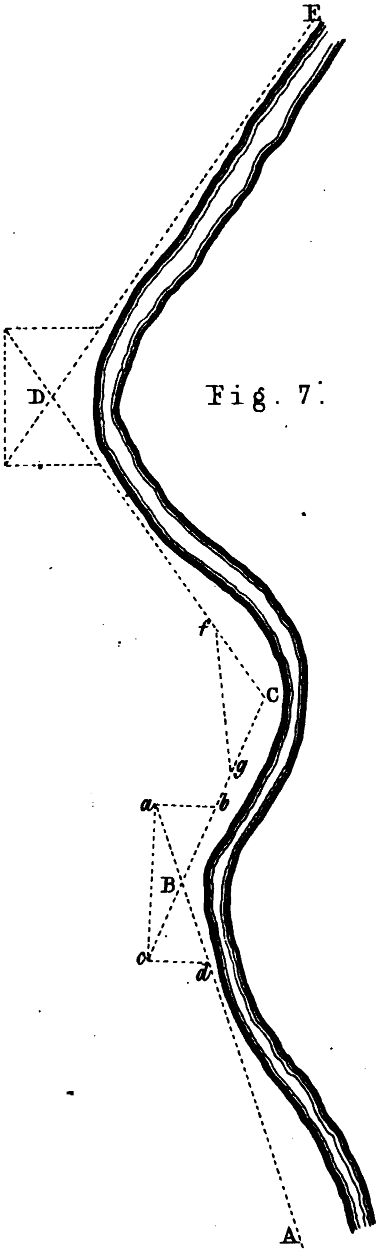
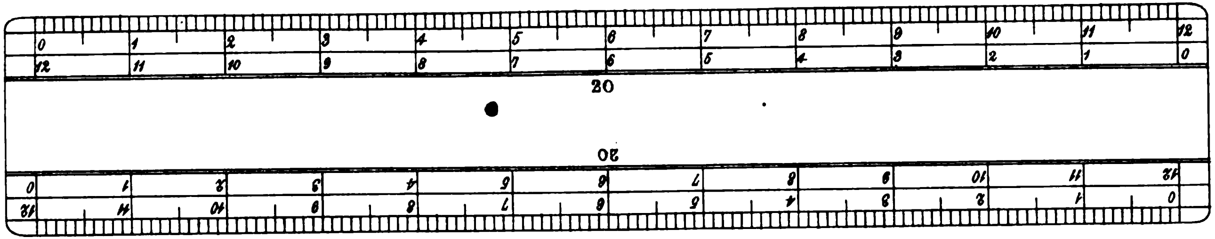


Fig. 7.

Fig. 8.



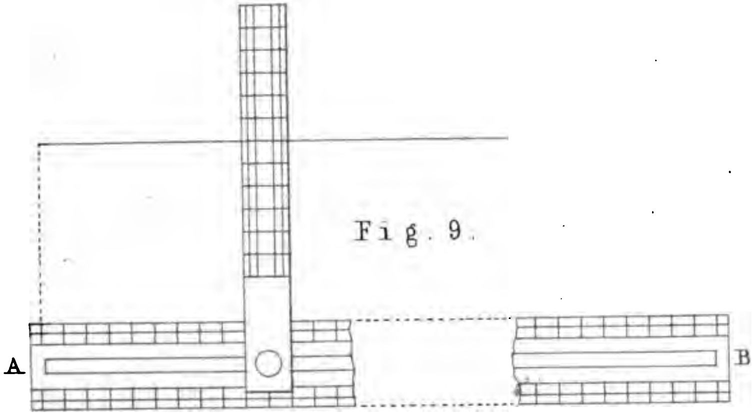


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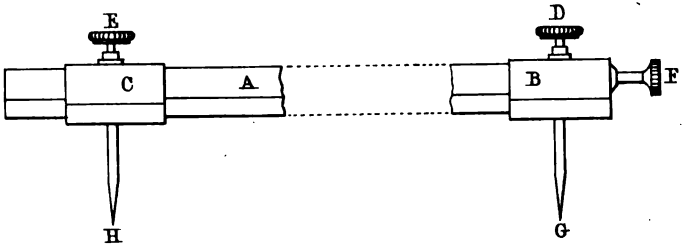
Fig. 10.



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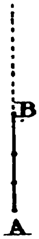

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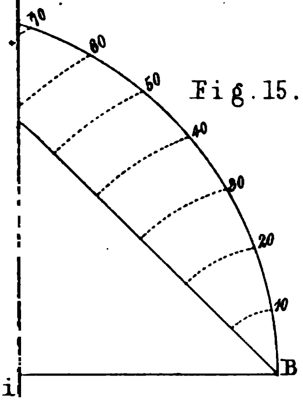

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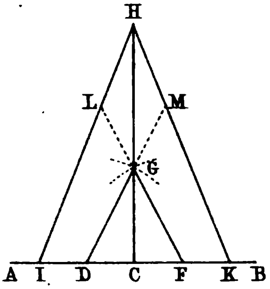

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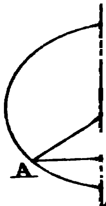

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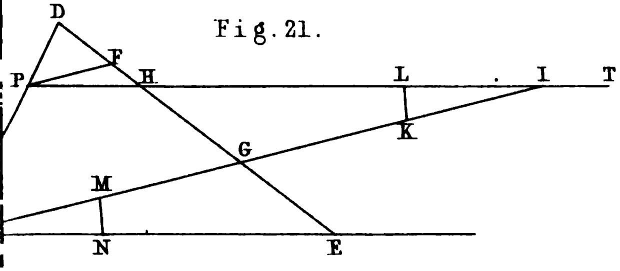

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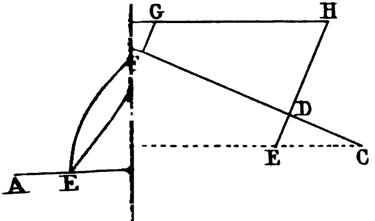

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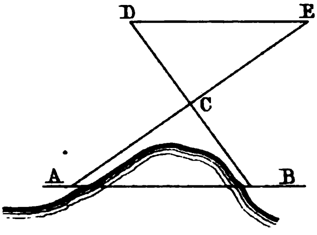

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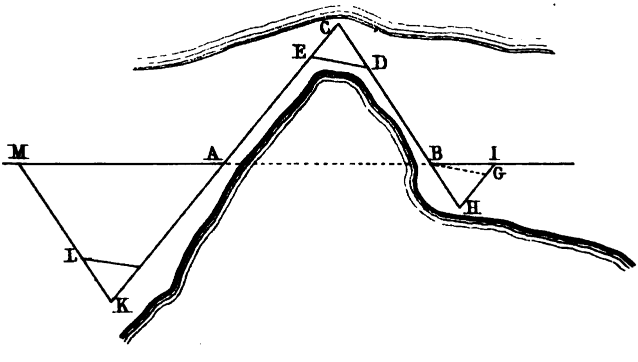

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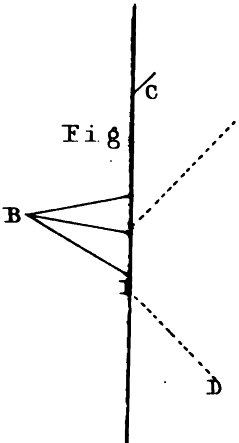

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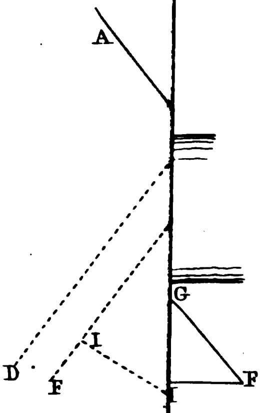

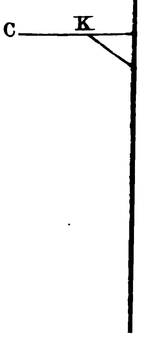

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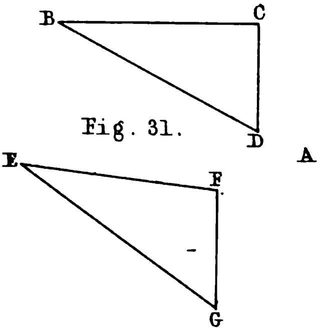

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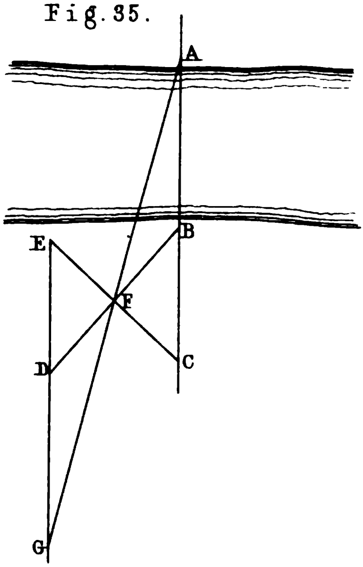

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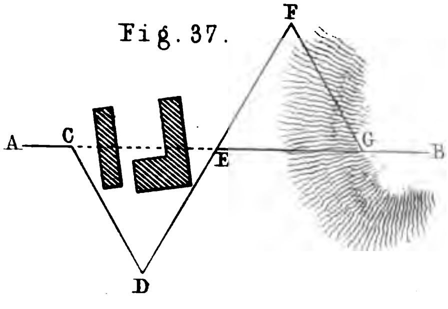

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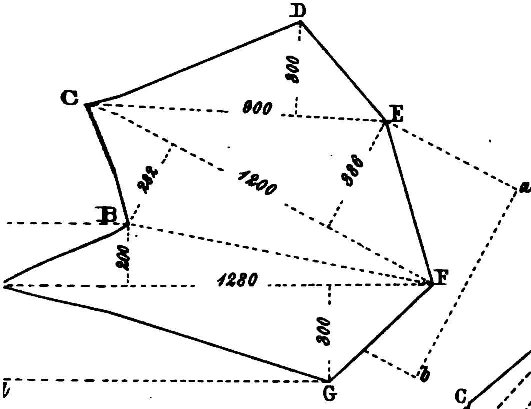

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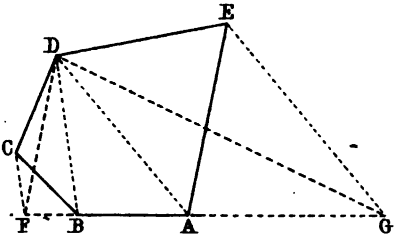

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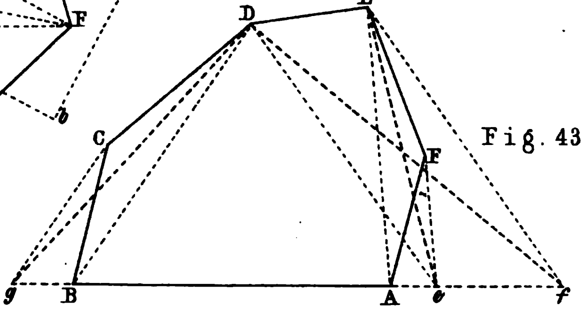

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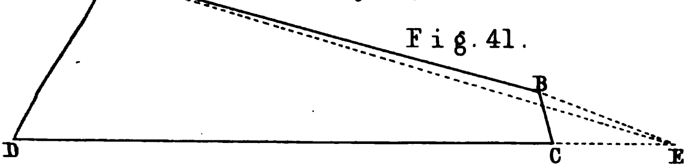

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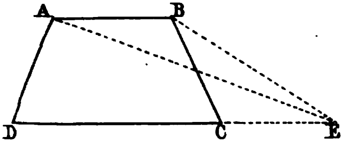

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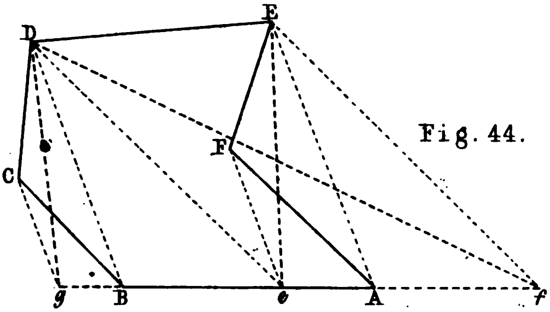

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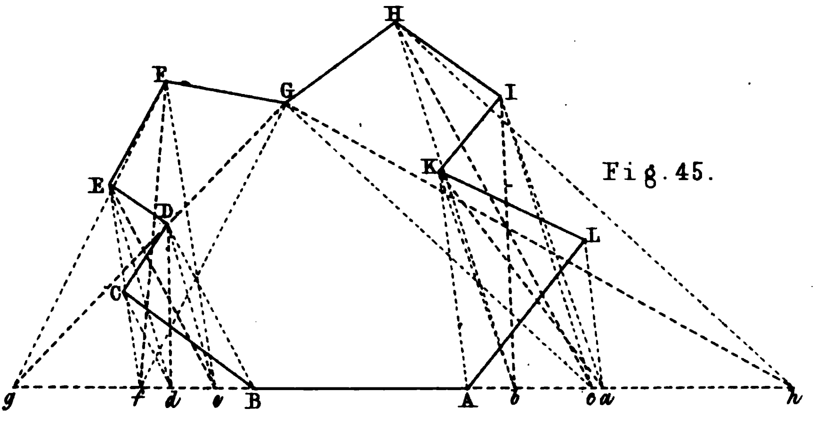

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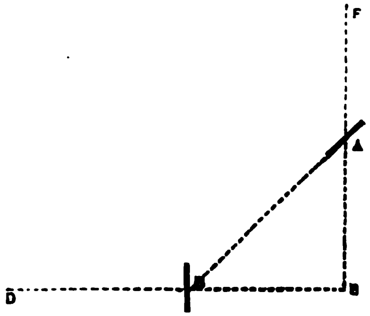
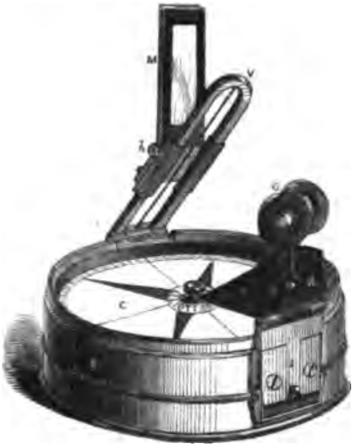


Fig. 47.

THE PRISMATIC COMPASS.



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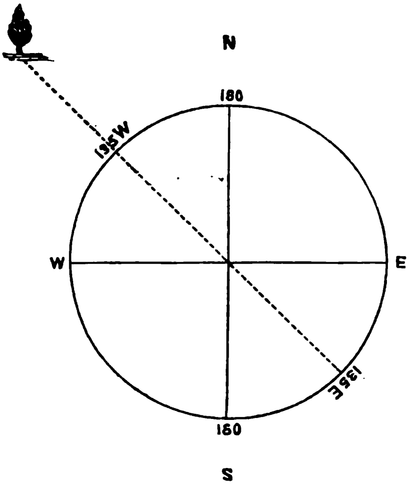

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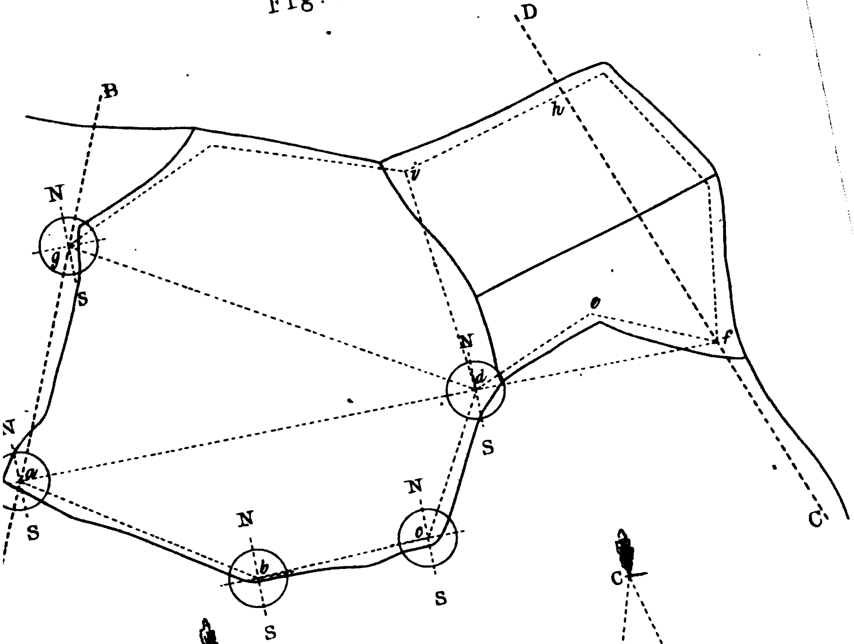

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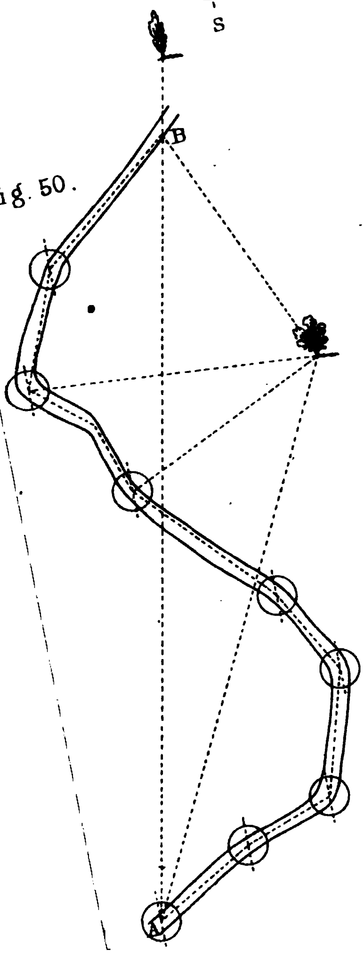

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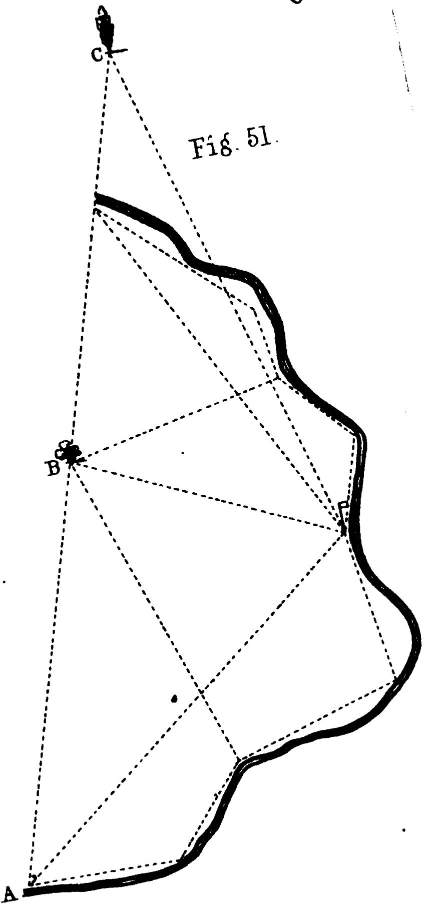

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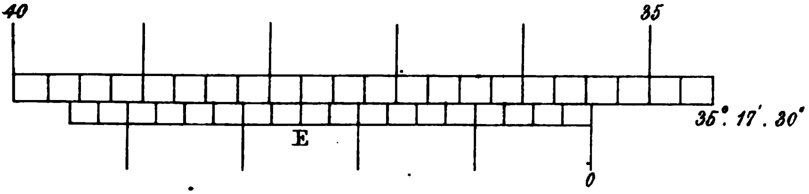

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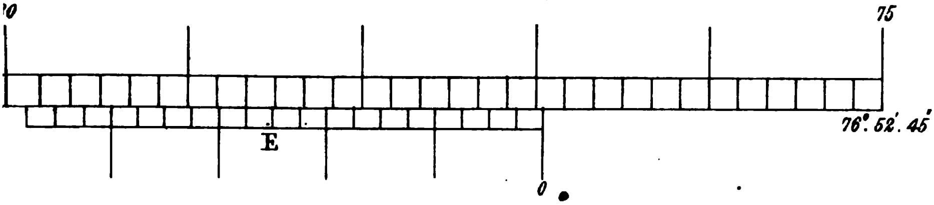

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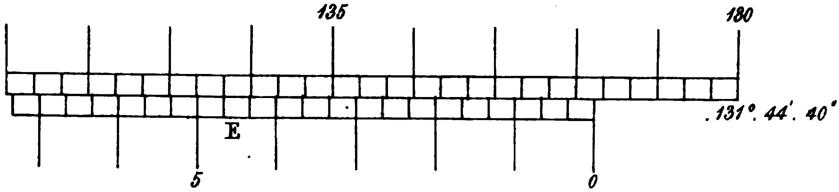

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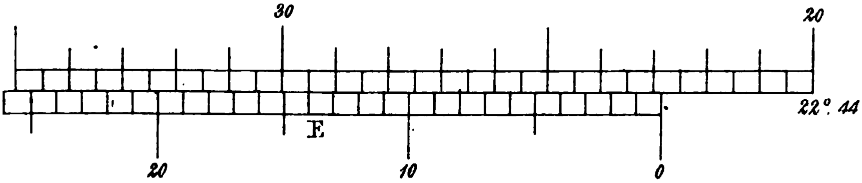

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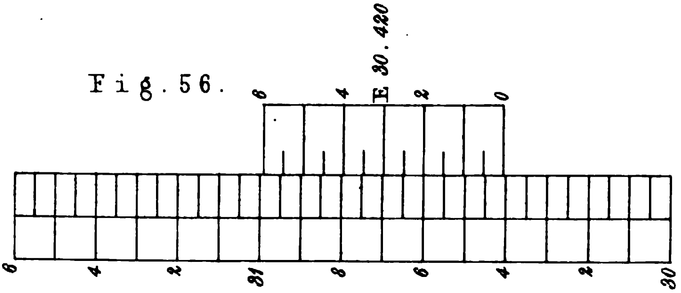

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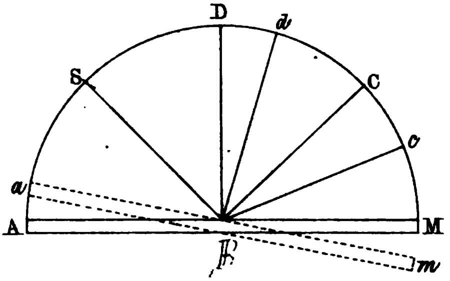
Fig. 57.



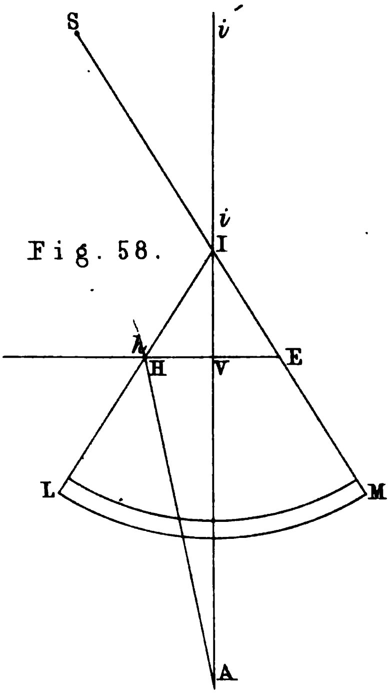


Fig. 58.

Fig. 60.

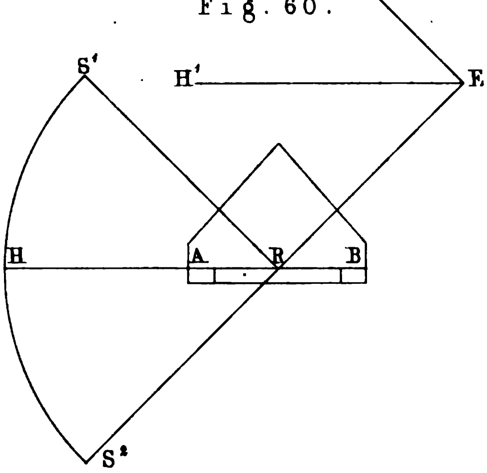


Fig. 59.
The Sextant.

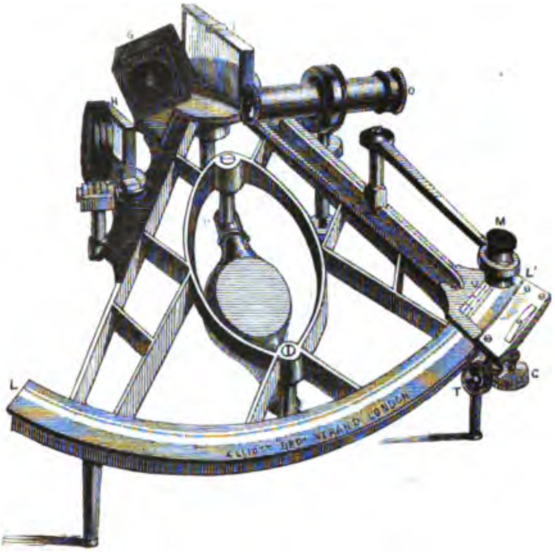


Fig. 61.
The Box Sextant.

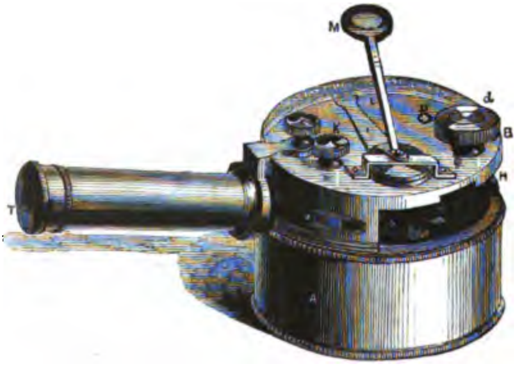


Fig. 62.

The Transit Theodolite.

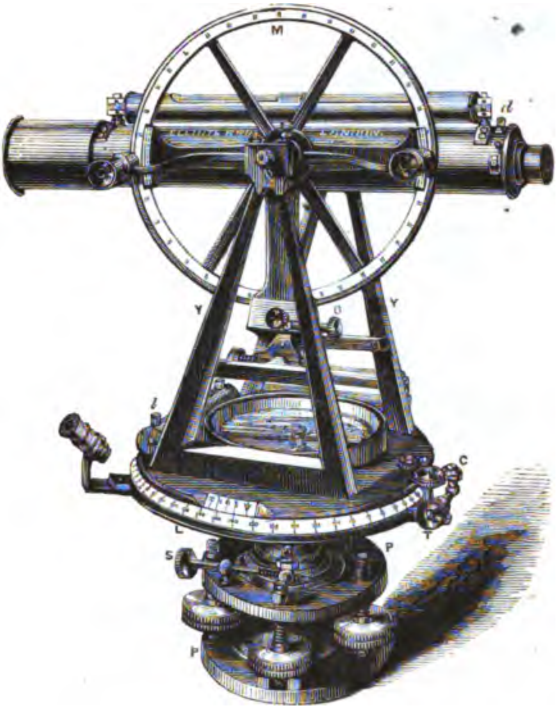


Fig. 63.

The Diaphragm.

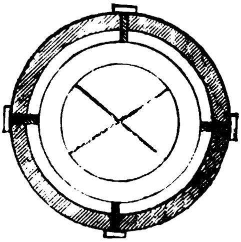


Fig. 64.

THE ORDINARY THEODOLITE.

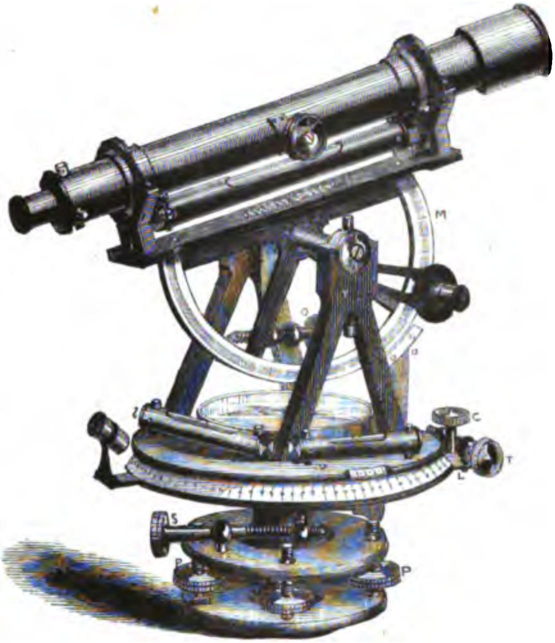
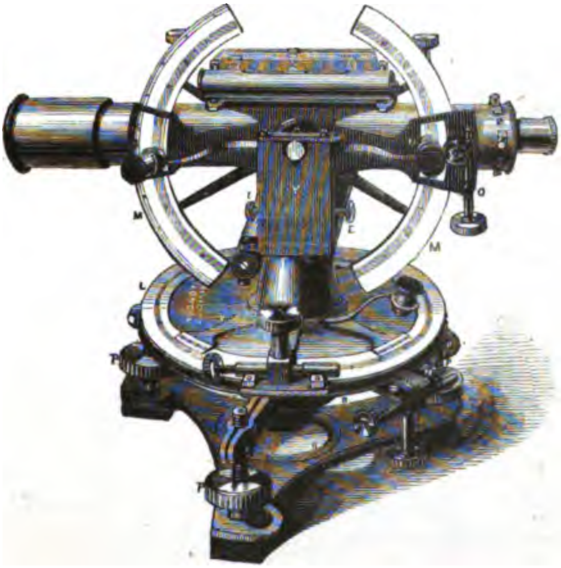


Fig. 65.

COLONEL EVEREST'S THEODOLITE.



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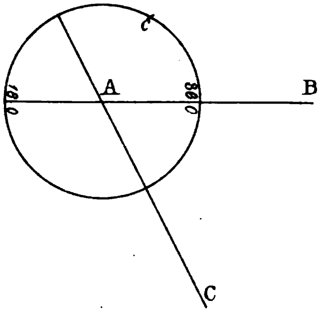

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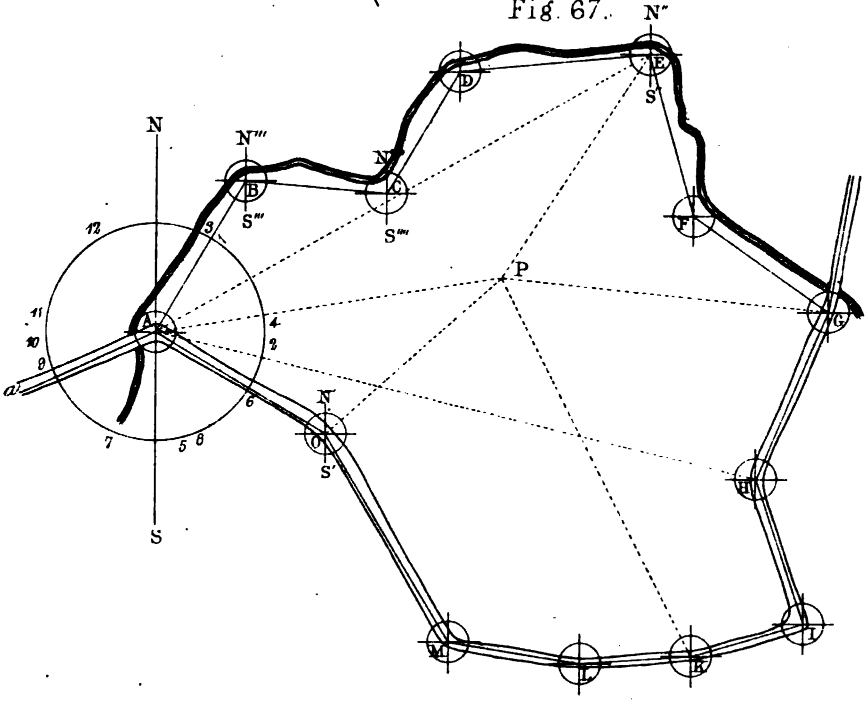

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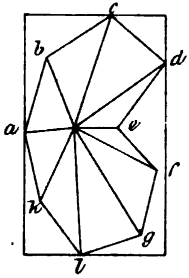

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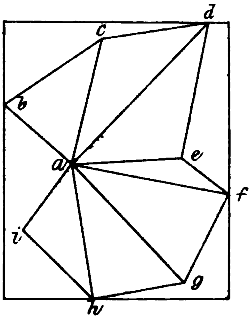


Fig. 70.

The Circular Protractor.

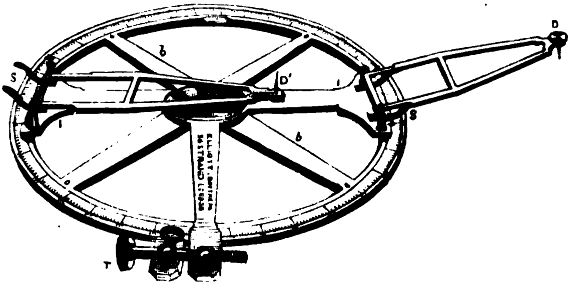
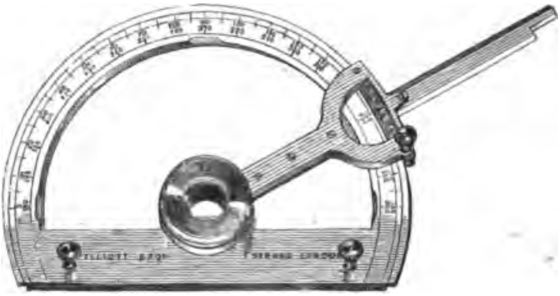


Fig. 71.

Howlett's Protractor.



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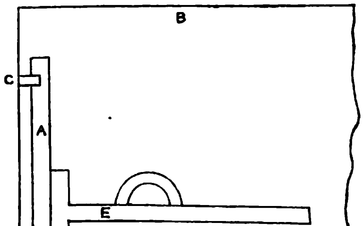
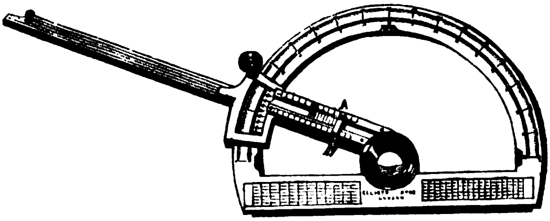


Fig. 73.

Metcalfe's Protractor.



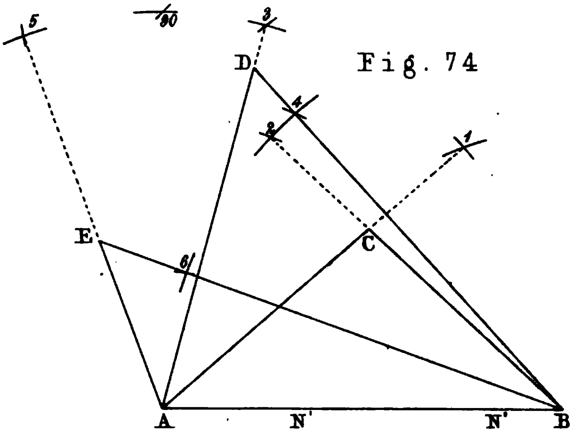


Fig. 74

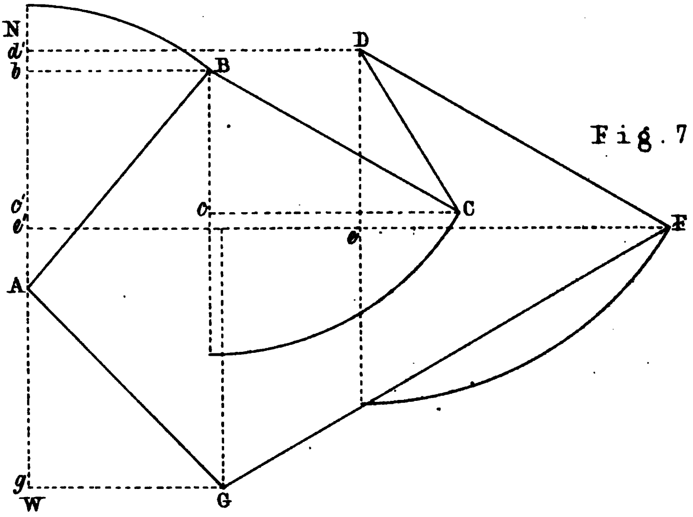
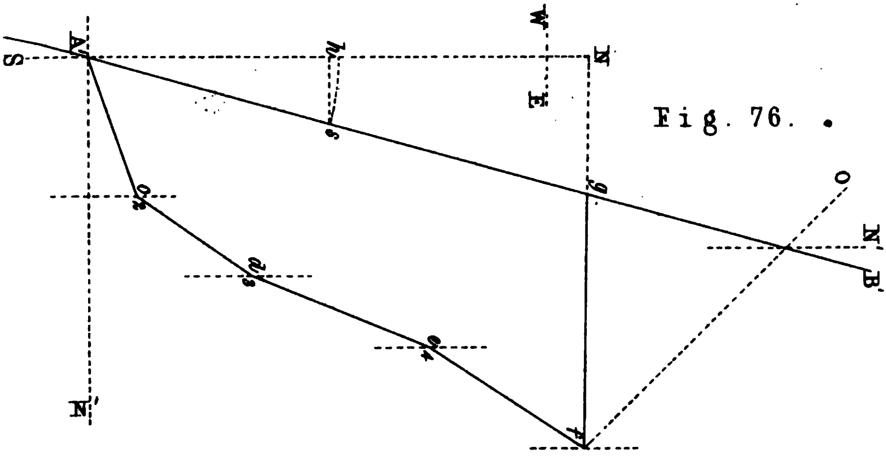


Fig. 75



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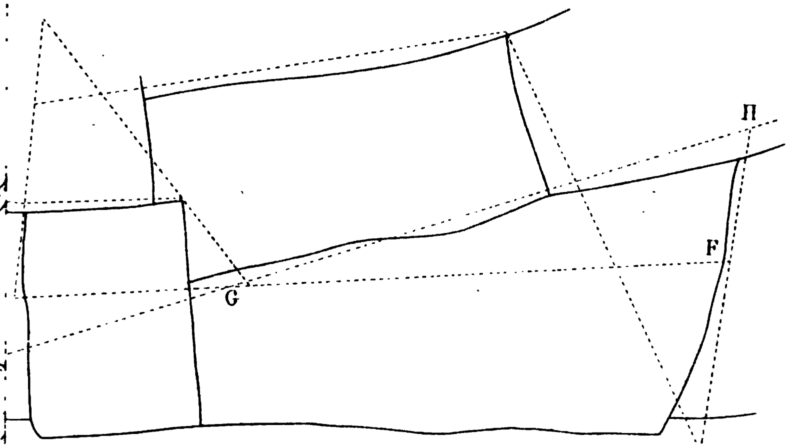

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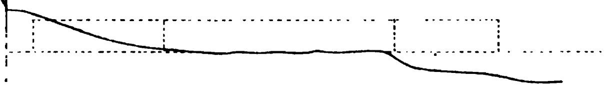
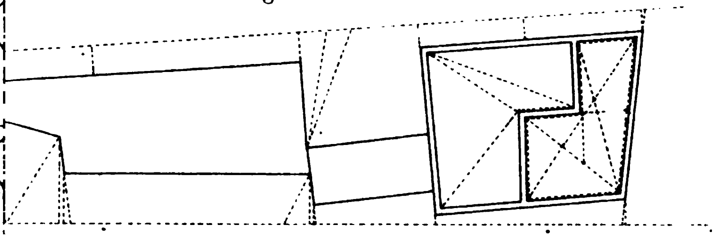


Fig 79



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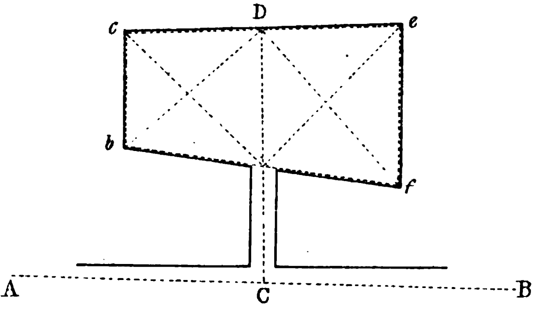
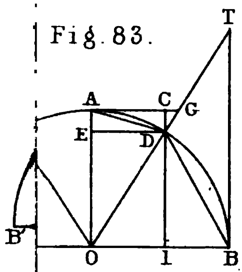


J. R. Jobbins

Fig. 83.



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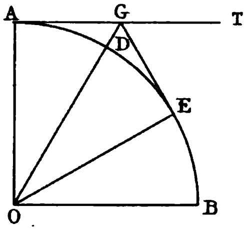


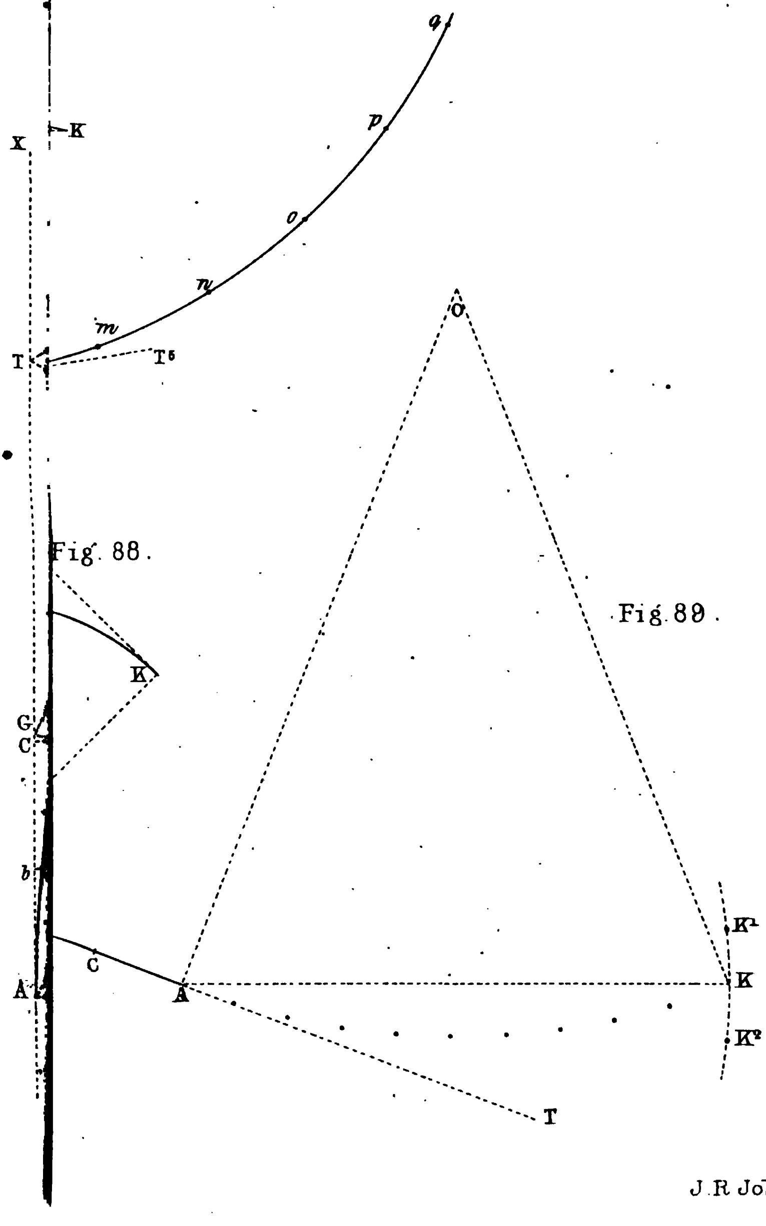
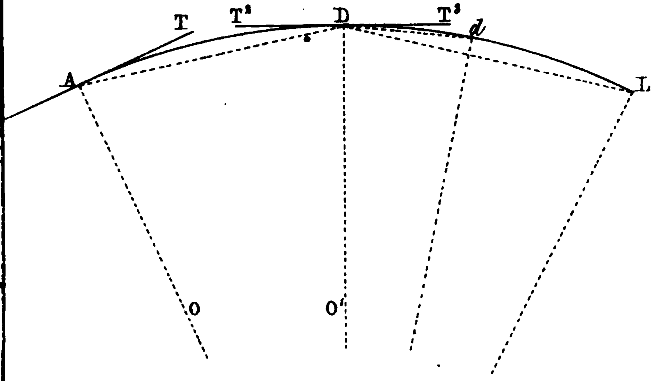


Fig. 88.

Fig. 89.

J. R. Jobbi

Fig. 93.



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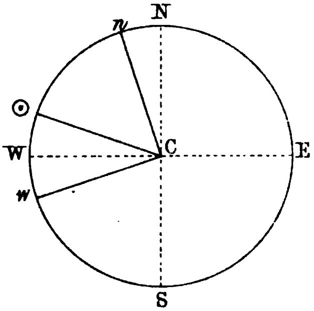

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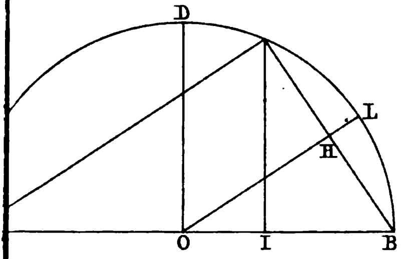

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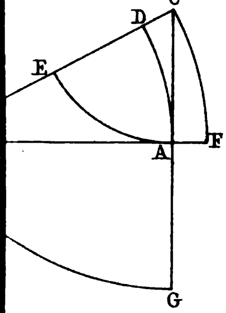

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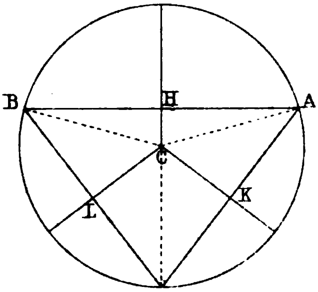
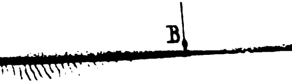


Fig. 98.

Fig. 99.



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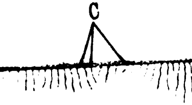

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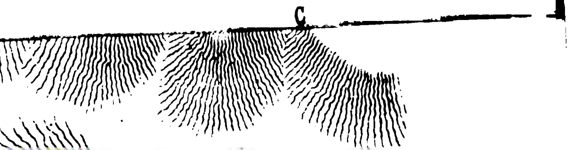

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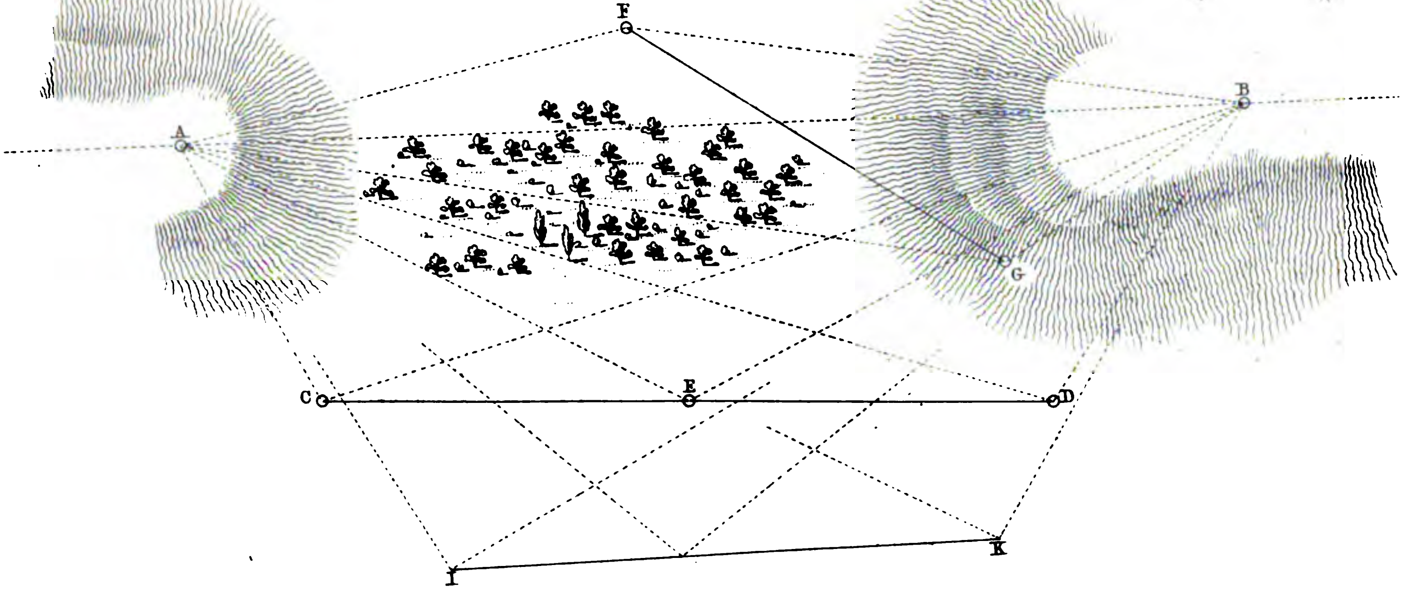
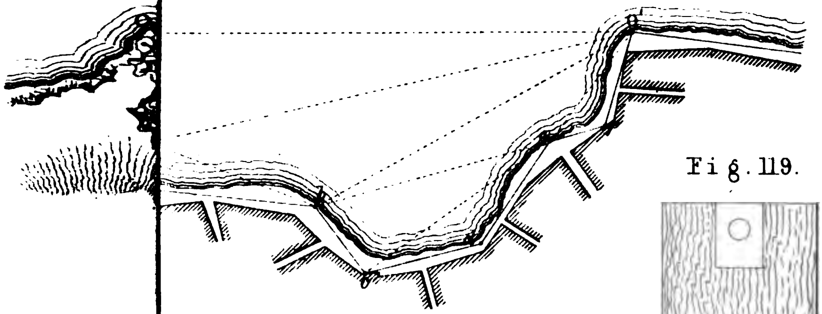


J.R.Jobbins

Pl. 17.

Fig. 109



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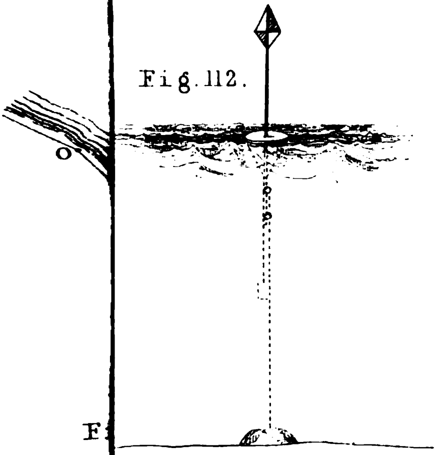

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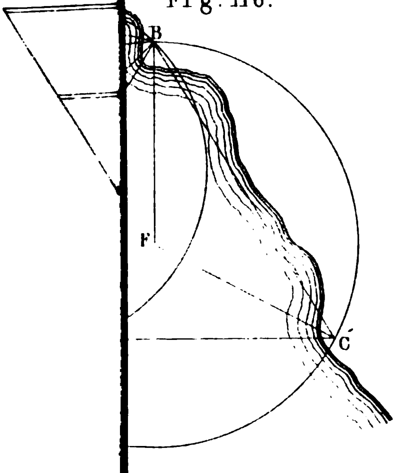

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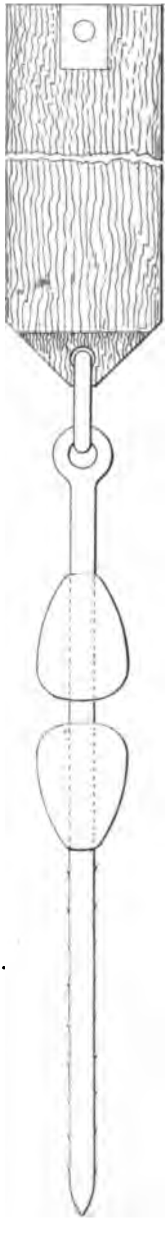

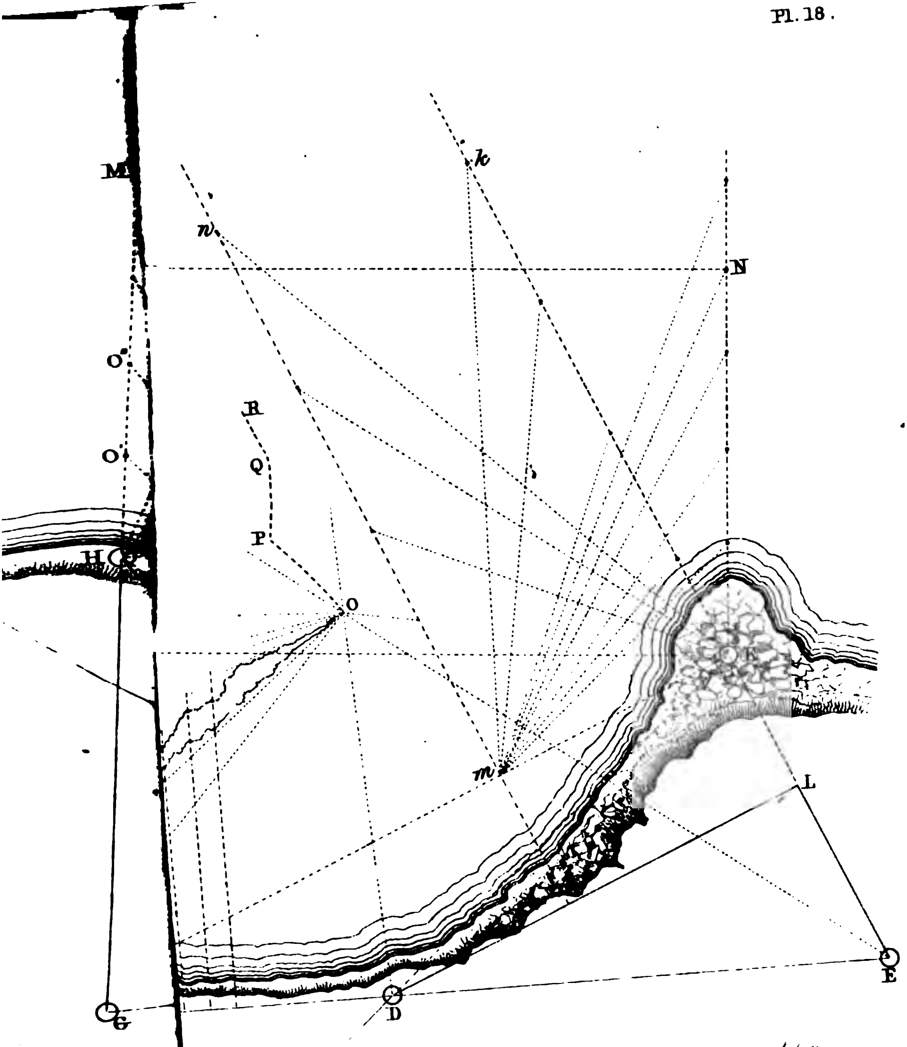

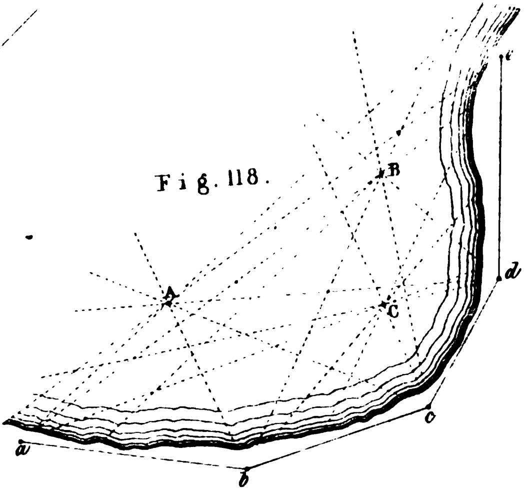

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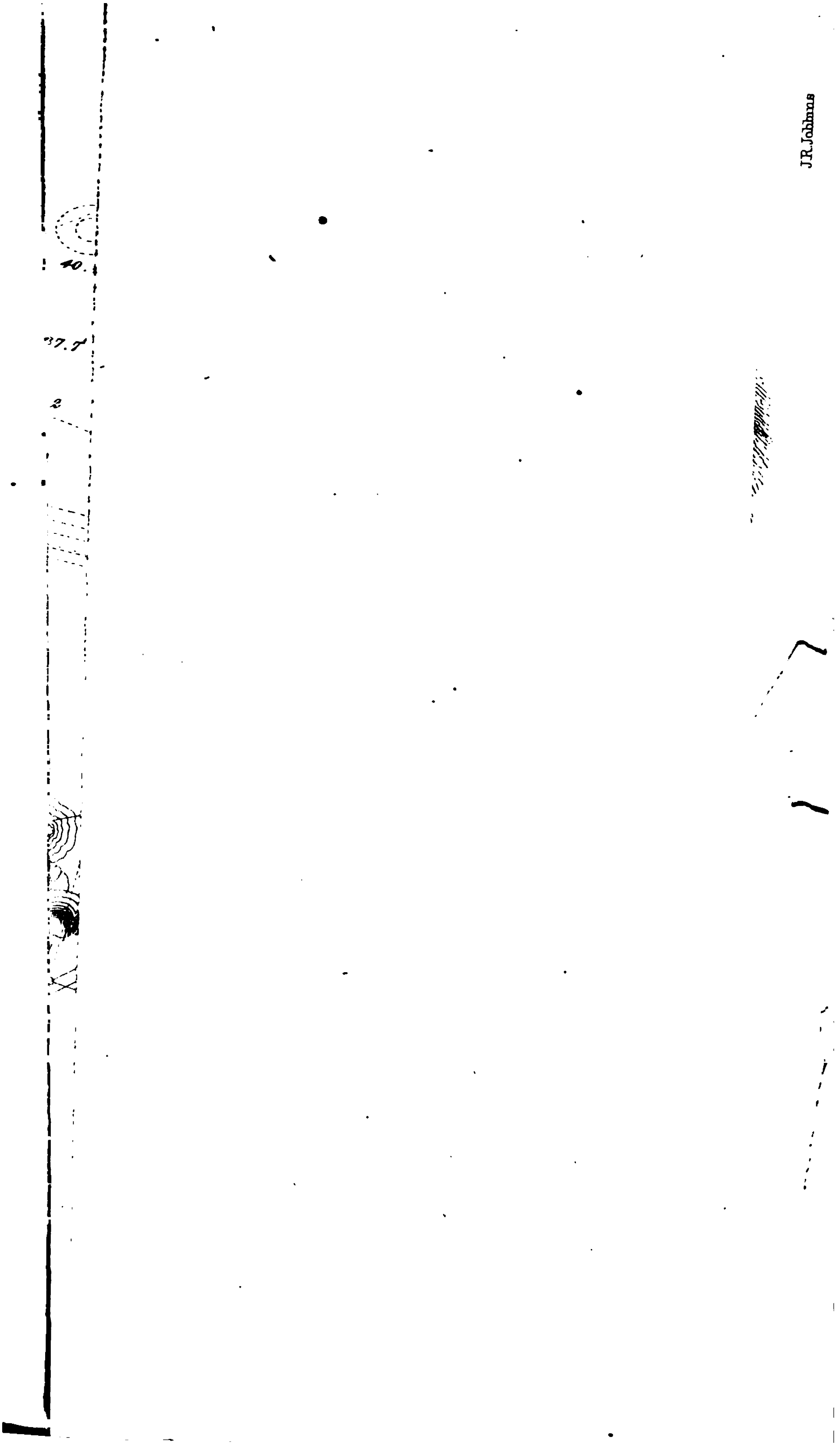
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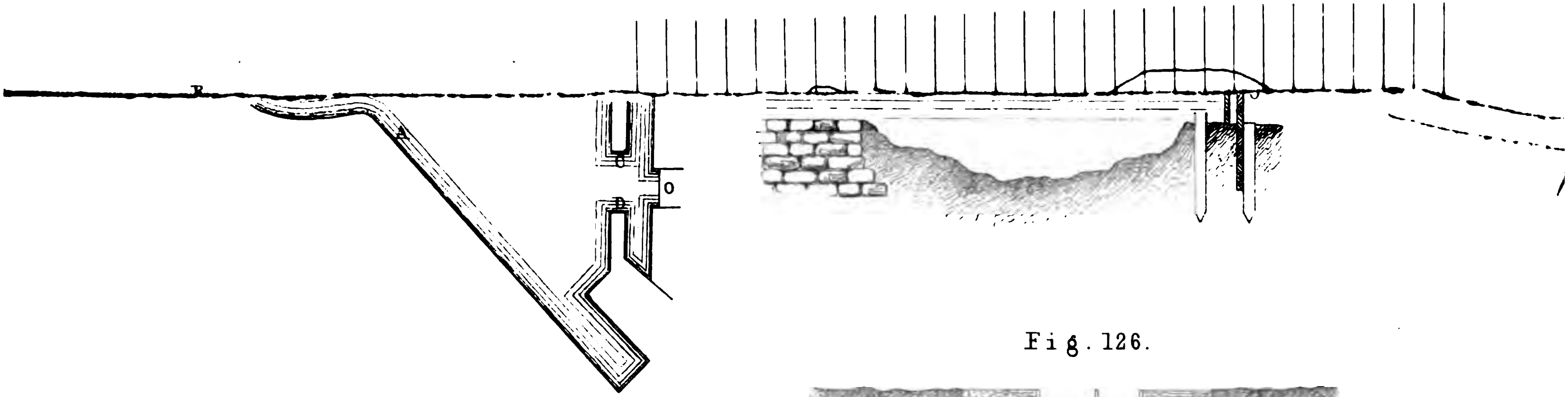
JR.Jobbins

Fig. 122.



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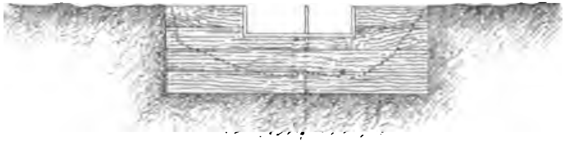

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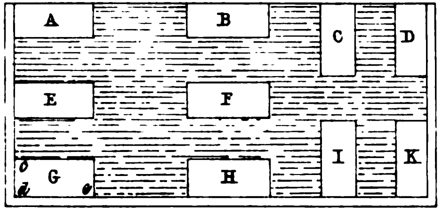

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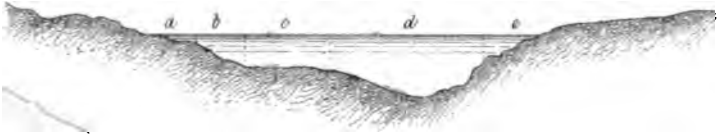


J.R. Jobbins

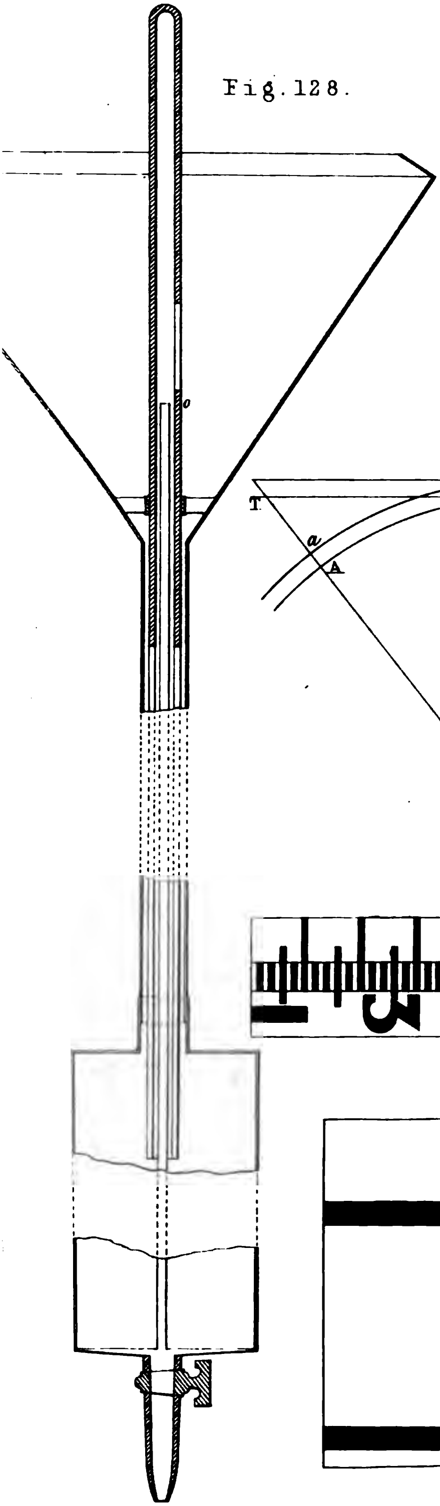
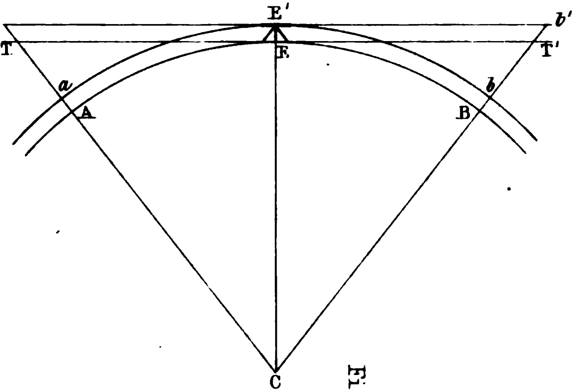


Fig. 128.



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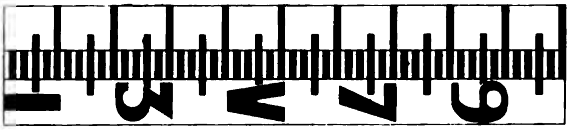

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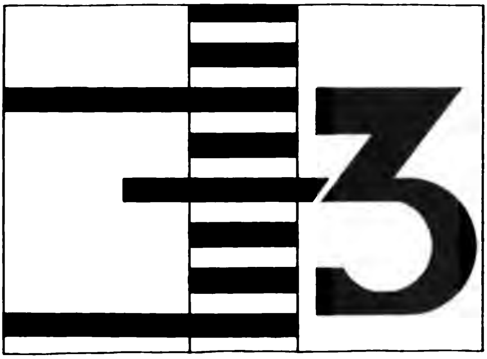


Fig. 131ª

Fig. 129A.

THE CURRENT METER.

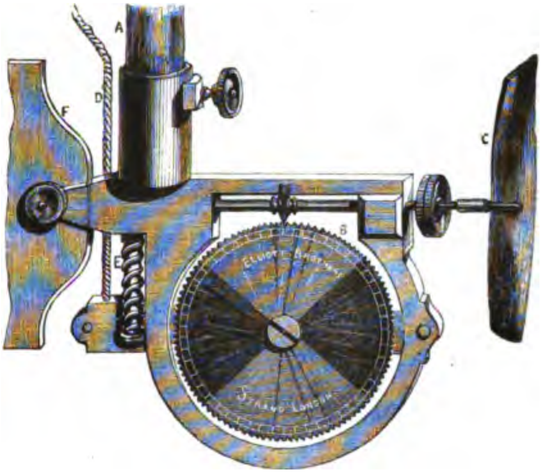
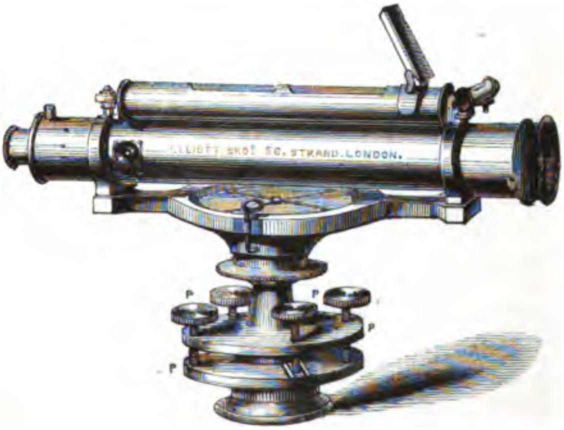
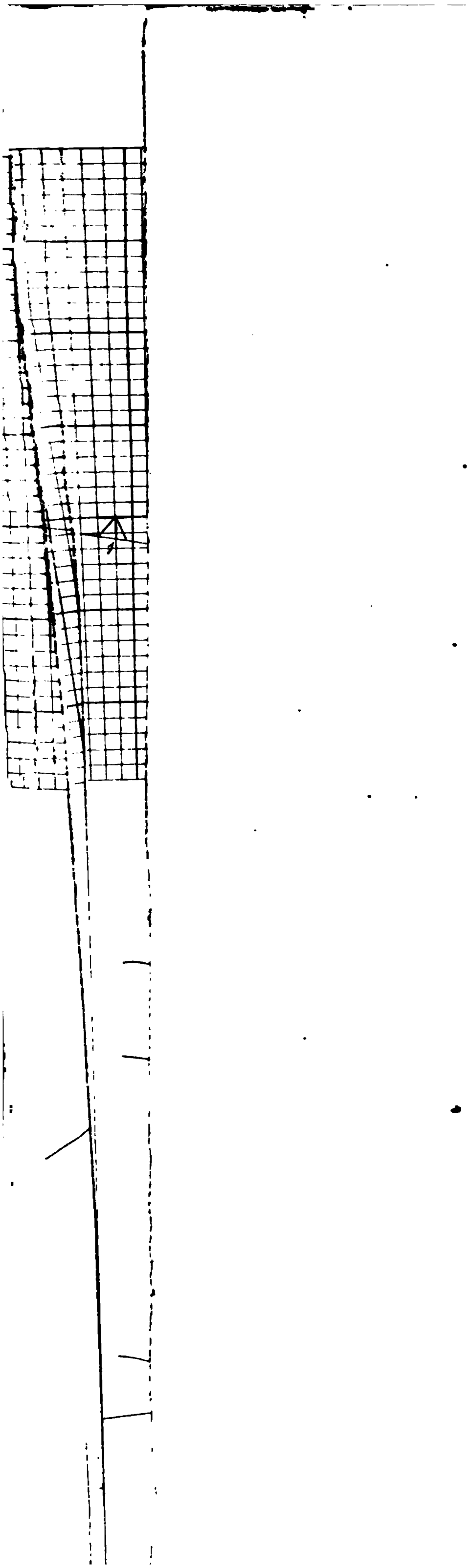
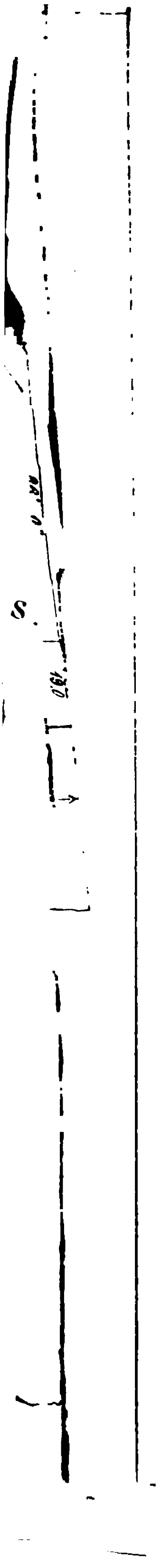


Fig. 130A.

THE DUMPY LEVEL.



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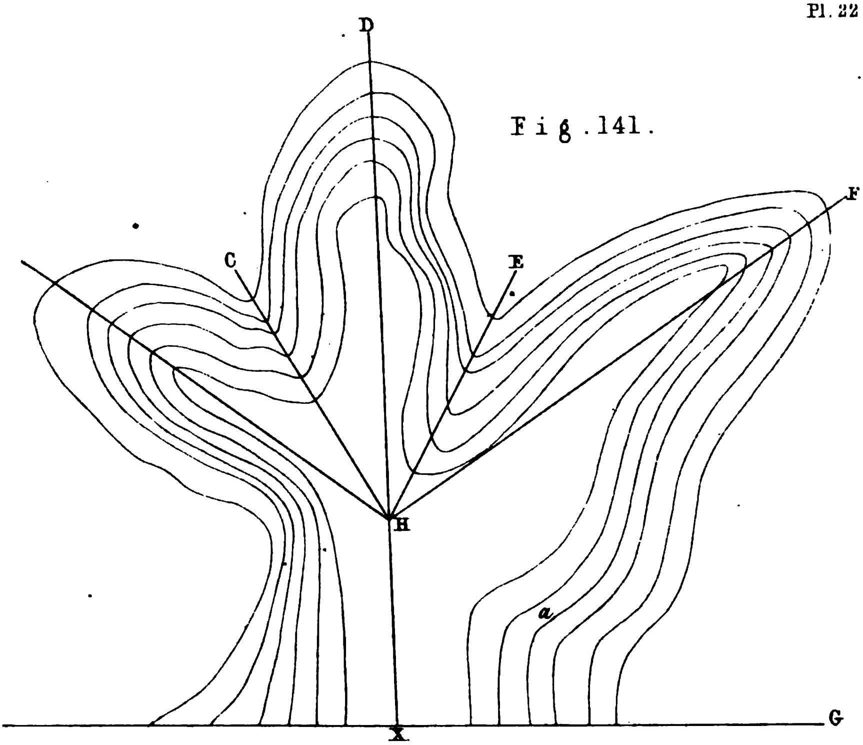


Fig. 141.

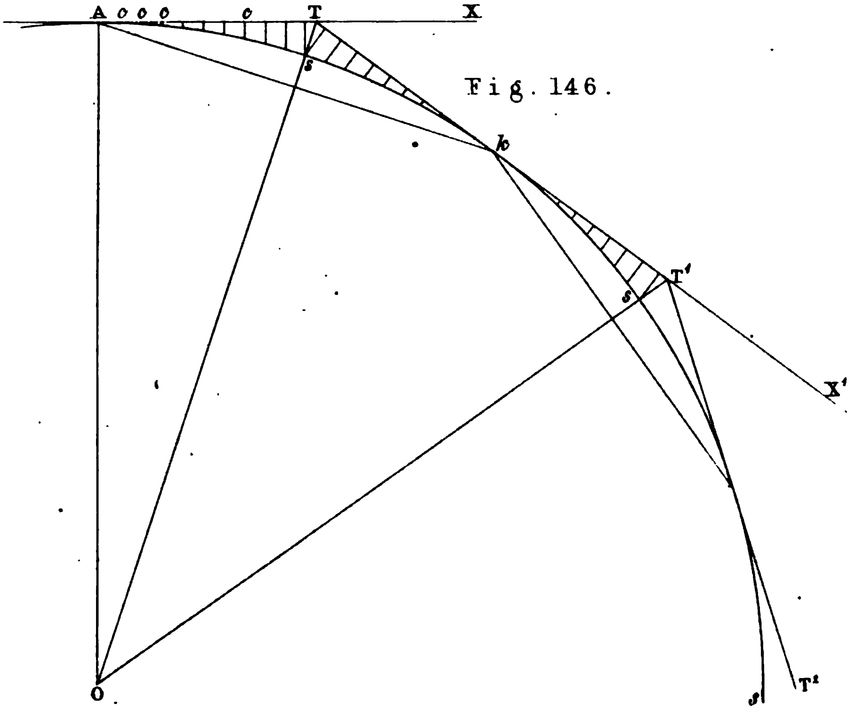
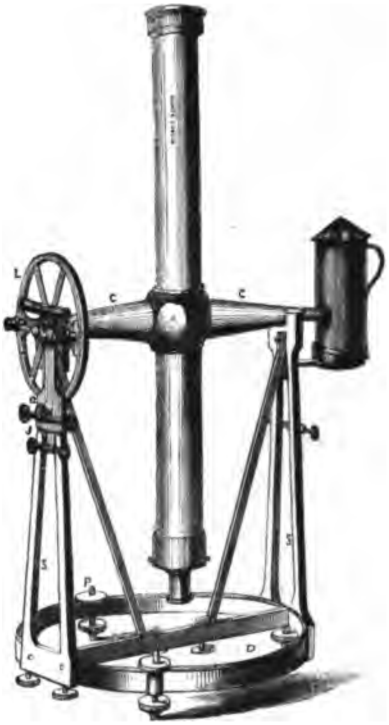


Fig. 146.

Fig. 142.

The Transit Instrument.



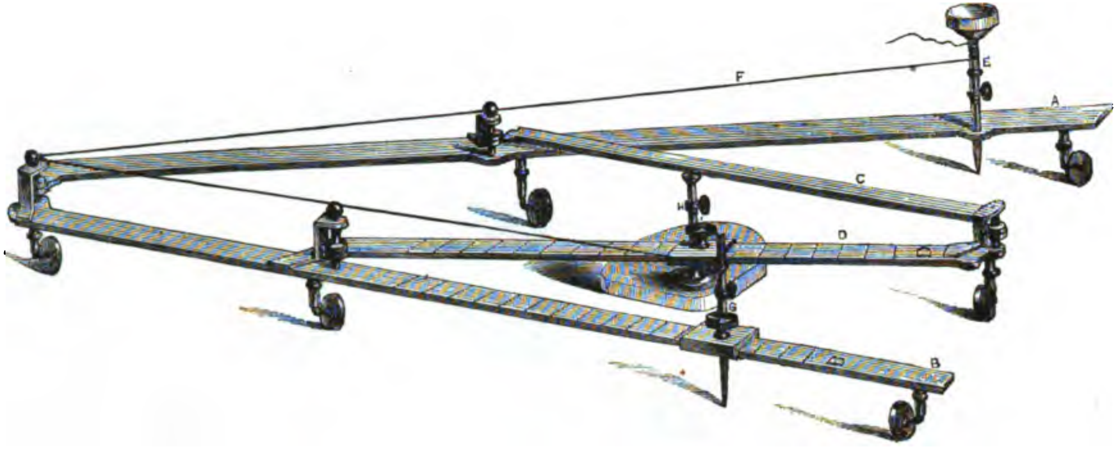


Fig. 143.
THE PENTAGRAPH.

Fig. 144.

THE EIDOGRAPH.

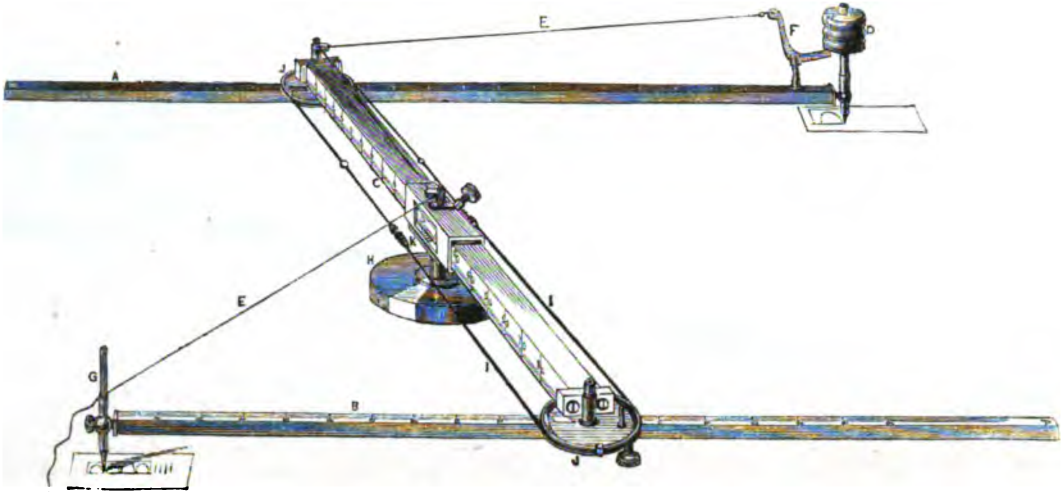
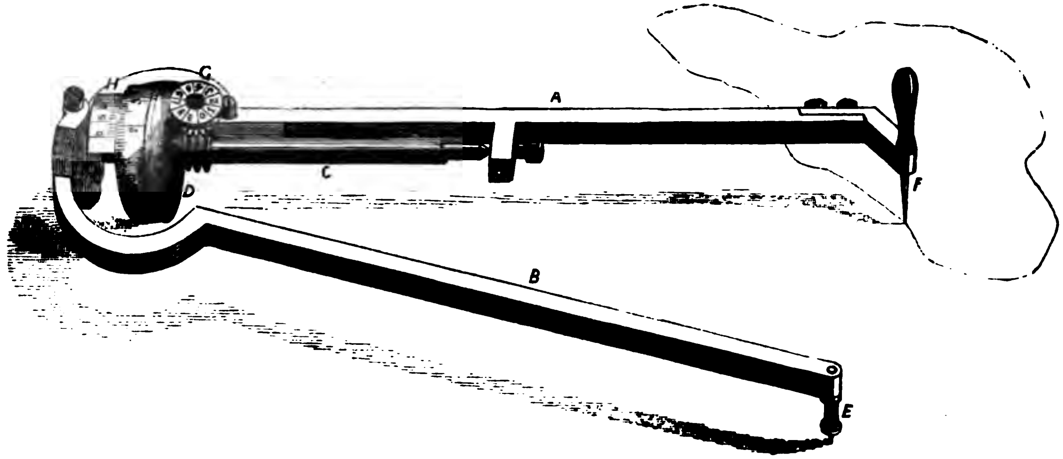
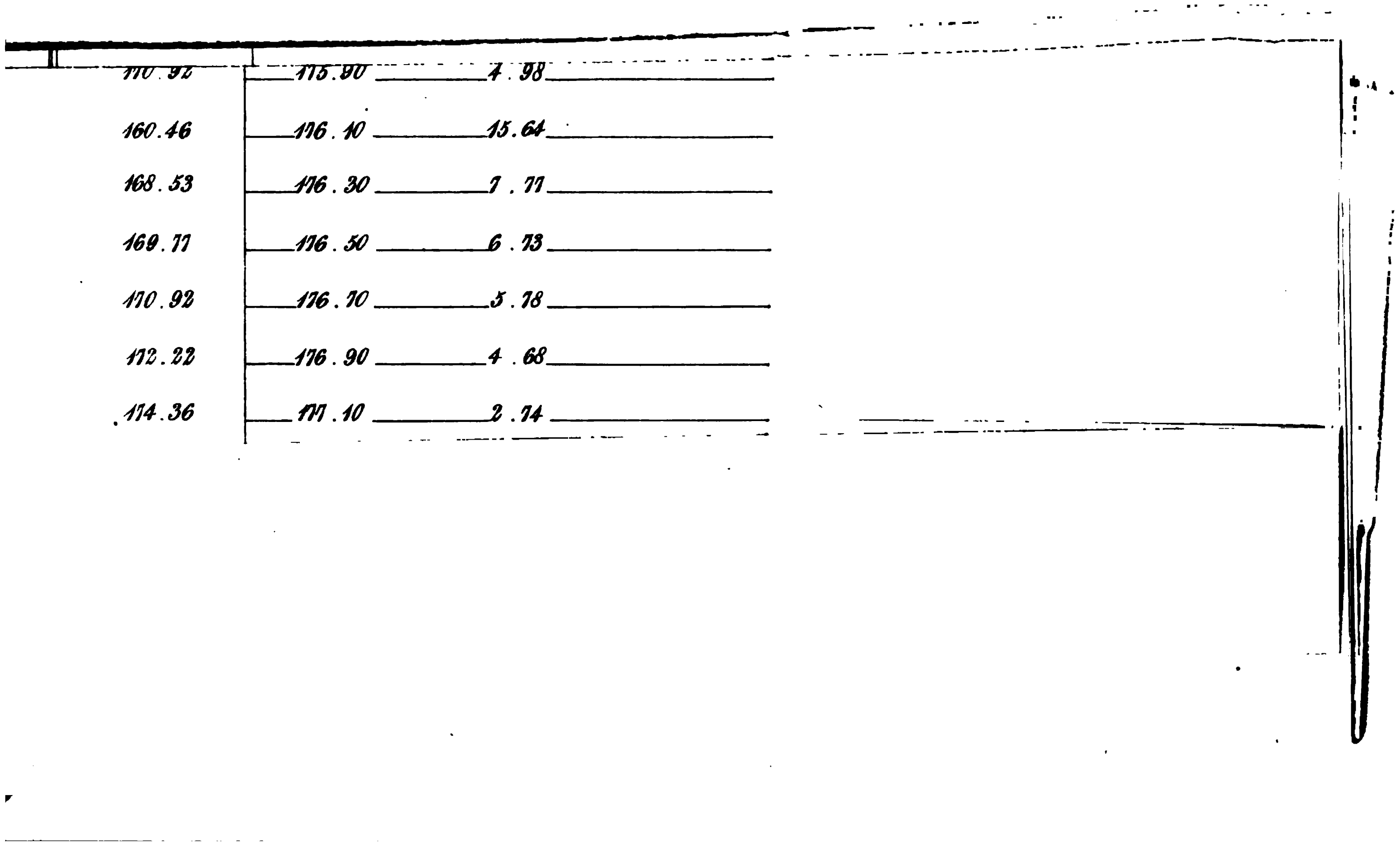


Fig. 145.
THE PLANIMETER.

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4.98
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176.70
5.78
172.22
176.90
4.68
174.36
177.10
2.74

ELLIOTT BROTHERS,

MANUFACTURERS OF

Mathematical, Optical, and Philosophical

INSTRUMENTS

TO THE ADMIRALTY, ORDNANCE, EAST INDIA COMPANY,
AND MILITARY COLLEGES,

30, STRAND,

FROM

56, STRAND, & 5, CHARING CROSS, LONDON.

THEODOLITES, LEVELS, TRANSIT INSTRUMENTS, &c.

	£	s.	d.
Four-inch plain Theodolite	13	13	0
Five-inch ditto ditto	16	16	0
Six-inch ditto ditto	18	18	0
Four-inch best Theodolite, divided on Silver	21	0	0
Five-inch ditto ditto	25	4	0
Six-inch ditto ditto	28	7	0
Seven-inch ditto ditto	34	13	0
Six-inch ditto ditto, with 2 Telescopes	34	13	0
Seven-inch ditto ditto ditto	39	18	0
Five-inch best Transit Theodolite	28	7	0
Six-inch ditto ditto	34	13	0
Seven-inch ditto ditto	39	18	0
Five-inch Transit Theodolite, without vertical circle	24	3	0
Six-inch ditto ditto	28	7	0
Seven-inch ditto ditto	34	13	0
Four-inch Everest Theodolite	21	0	0
Five-inch ditto ditto	25	4	0
Six-inch ditto ditto	29	8	0
Seven-inch ditto ditto	34	13	0
Eight-inch ditto ditto	39	18	0
Drainage Level, with Stand	4	4	0
Small Dumpy Level	6	10	0
Elliott's Improved Dumpy Level, 10 inch	14	14	0
Ditto ditto, with Compass	16	16	0
Ditto ditto, 14 inch	15	15	0
Ditto ditto, ditto with Compass	17	17	0
Ditto ditto, 18 inch	17	17	0
Ditto with Compass	21	0	0

Larger Theodolites and Levels made to Order.

		£	s.	d.
Ten-inch Y Level		9	9	0
Ditto ditto with Compass		10	10	0
Twelve-inch ditto		10	10	0
Ditto ditto with Compass		12	12	0
Fourteen ditto		12	12	0
Ditto ditto with Compass		14	14	0
Twenty-inch ditto ditto		18	18	0
Circumferenters, or Miner's dials	from £5 5s. to	18	18	0
Prismatic Compass in sling case		3	3	0
Ditto, with Azimuth Glass ditto		3	13	6
Ditto, large size ditto		4	14	6
Ditto, ditto, with silver Compass ditto		5	5	0
Stand for ditto		2	2	0
Four-inch Metal Sextant, divided on Silver		9	9	0
Five-inch ditto ditto		12	12	0
Six-inch ditto ditto		13	13	0
Eight-inch ditto ditto		14	14	0
Ditto ditto ditto with double frame		16	16	0
Quadrant		2	12	6
Ditto with Telescopes	£3 13s. 6d. to	6	6	0
Artificial Horizon	£2 12s. 6d. to	4	14	0
Pocket Sextant in sling case		4	10	0
Ditto, with Telescope ditto		5	0	0
Ditto, ditto, and Supplementary Arc ditto		6	0	0
Ditto, ditto, ditto, large size ditto		6	16	6
Optical Square		1	1	0
Pentagraph, 24 inch		5	15	6
Ditto, 30 inch		6	16	6
Ditto, 36 inch		7	17	6
Ditto, 42 inch		8	18	6
Ditto, 48 inch		10	10	0
Eidograph		10	10	0
Station Pointers	£6 6s. to	18	18	0
Transit Instrument		16	16	0
Ditto, with Iron Stand		21	0	0
Ditto, 24 inch, with Circular Brass Portable Stand		25	4	0
Ditto 30 inch, with Iron Stand		45	0	0
Large Transit Instrument, for permanent Stations		73	10	0
12-inch Azimuth, and Altitude Instrument		95	0	0
15-inch ditto, with Microscopes		120	0	0
Cross Staff	10s. 6d. to	0	16	0
Ditto, with Compass and Screw Staff	£2 2s. to	2	12	6

		£	s.	d.
Current Meter	£3 3s. to	5	5	0
Six-inch Circular Protractor, with Tangent Screw Adjustment		4	14	6
Six-inch ditto, divided on Silver ditto		5	15	6
Seven-inch ditto, ditto ditto		6	16	6
Eight-inch ditto, ditto ditto		7	17	6
Metal and Brass Circular and Semi-circular Protractors	10s. 6d. to	5	5	0
Portable Sliding Station Staff, 14 feet		2	12	6
Ditto ditto, Painted		3	0	0
Ditto, 17 feet		4	4	0
Gravatt's Level Staff	£3 3s. to	4	4	0
Papers for Level Staves	per foot	0	0	2
Ditto, in sets for 14 feet Portable Staff		0	3	0
Chain, 66 feet	10s. to	0	14	6
Ditto, extra strong wire, with 3 oval rings		0	18	0
Ditto, 25 feet		0	8	6
Ditto, 50 feet		0	14	0
Ditto, 100 feet		1	2	0
Patent Galvanized Chain, 66 feet		1	0	0
Standard Chains	£3 3s. to	5	5	0
Arrows, set		0	2	0
Tapes, 33 feet		0	8	0
Ditto ditto, divided decimally		0	9	0
Ditto, 50 feet		0	9	0
Ditto ditto, divided decimally		0	10	0
Ditto, 66 feet		0	11	0
Ditto ditto, divided decimally		0	12	0
Ditto, 100 feet		0	16	0
Pocket Tapes	3s. to	0	10	6
Steel Standard Tape Measures	25s. to	3	3	0
Heliograph or Spiral Instrument		8	8	0
Farey's Elliptograph		7	17	6
Opisometer or Map Meter, for measuring curved lines		0	8	6
Pencil Cutter		0	4	0
Card Protractors		0	2	6
Sang's Platometer, for calculating areas on plans		10	10	0
Amster's Planimeter ditto	£3 3s. to	3	13	6

ELLIOTT BROTHERS, in addition to the large Circular Dividing Engine (which they purchased from Mr. JONES, of Charing Cross), have received by order of Government, the Straight Line Dividing Engine from the Admiralty.

DRAWING INSTRUMENTS, &c.

		Brass.			German Silver.		
		£	s.	d.	£	s.	d.
Proportional Callipers, 12 inches		2	0	0	2	12	6
Ditto ditto, 9 inches		1	14	0	2	2	0
Proportional Compass, 12 inches		2	12	6	3	3	0
Ditto ditto, 9 inches		2	2	0	2	12	6
Ditto ditto, 6 inches		1	10	0	2	0	0
Ditto ditto, with Adjustment		2	0	0	2	10	0
Whole and half ditto		0	18	0	1	4	0
Triangle ditto		0	18	0	1	4	0
Tube Compass		1	15	0	2	5	0
Ditto Needle Points		1	18	0	2	8	0
Tube Beam Compass		2	0	0	2	11	6
Pocket Turn-in Compass		1	6	0	1	16	0
Ditto ditto, with handles		1	10	0	2	2	0
Ditto ditto, ditto, and Bars		2	2	0	2	12	6
Pillar Compass, with divided Sheath		1	11	6	2	2	0
Napier Compass		1	11	6	2	2	0
Pocket Dividers, with Sheath		0	8	0	0	10	0
Best double-jointed Compass		0	15	0	1	1	0
Ink Pencil, and Wheel Legs for ditto	each	0	5	6	0	7	0
Lengthening Bar		0	5	0	0	7	0
Hair Divider		0	10	0	0	13	0
Plain Divider		0	6	0	0	8	0
Tate's Bow		1	4	0	1	11	6
Double-jointed Bowpen and Bowpencil	each	0	10	6	0	13	0
Ditto ditto, Needle Pointed	each	0	12	0	0	15	0
Hair Bow Divider		0	9	0	0	12	0
Large Needle Bows	each	0	15	0	1	1	0
Spring Bowpen and Pencil	each	0	8	6	0	8	6
Spring Divider		0	7	6	0	7	6
Ivory Handle Drawing Pen		0	5	0	0	5	0
Ditto ditto, jointed		0	5	6	0	7	0
Ebony ditto		0	3	0	0	3	0
Double Pen		0	10	6	0	10	6
Six Wheels and Box		0	6	6	0	6	6
Six Pens and 1 Handle, in Case		1	10	0	1	10	0
Tracer		0	2	6	0	2	6
Crow Quill Holder		0	3	0	0	3	0
Needle Holder		0	2	6	0	3	0
Knife		0	1	6	0	1	6

	Brass.			German Silver.		
	£	s.	d.	£	s.	d.
Sector joined Compass, 4½, 5, or 6 inch	0	10	0	0	15	0
Divider	0	6	0	0	8	0
Ink, and Pencil Legs	0	11	0	0	14	0
Bar	0	5	0	0	7	0
Bowpen and Bowpencil	0	12	0	0	16	0
Marlow Compass	0	9	0	0	13	0
Divider	0	4	6	0	6	6
Ink, and Pencil Legs	0	9	0	0	13	0
Bar	0	4	6	0	6	6
Bowpen and Bowpencil	0	12	0	0	16	0
College Compass	0	7	0	0	10	0
Ink, and Pencil Legs	0	7	0	0	10	0
Divider	0	3	6	0	5	0
Bowpen and Bowpencil	0	7	0	0	10	0
Steel-joint Compass, with Pen and Pencil Legs	0	7	0			
Divider	0	2	6			
Bowpen and Pencil	0	6	0			
Drawing Pen	0	2	0			
Case of Instruments as supplied to the Royal Military College, Woolwich, and Ordnance School	2	12	6	3	13	6
Ditto as supplied to the Cadets at Addiscombe, £3 3s., £3 13s. 6d., and	4	4	0			
Ditto as supplied to Harrow School	1	17	6			
Ditto as supplied to King's College	3	0	0			
Ditto for Schools	1	1	0			
Architect's Case	3	13	6	4	14	6
Engineer's ditto £3 13s. 6d. to	6	16	6	10	10	0
Magazine Cases of Instruments, made of Brass, German Silver, or Silver, from	10	10	0	50	0	0

		£	s.	d.
Marquois Scales, Boxwood		0	12	0
Ditto, Ivory		2	12	6
Ditto, Metal		3	13	6
Ivory Plotting Scales, 12 inches		0	10	0
Ditto ditto 6 inches		0	5	0
Ditto Offset Scales, 2 inches		0	2	6
Ditto Architect's Scales, 12 inches		0	13	0
Ditto ditto 6 inches		0	7	0
Ditto Protractor, 12 inches		1	10	0
Ditto ditto, 6 inches		0	6	0
Ditto ditto, Rolling		0	18	0
Ditto Sector	6s. to	1	1	0
Ditto Parallel Rule, Brass Bars		0	4	6
Ditto ditto, German Silver Bars		0	5	6
Ditto Pocket Rules	4s. 6d. to	1	11	6
Slide Rules		0	5	0
Routledge's Slide Rule, with Book		0	9	0
Carret's Improved ditto		0	10	0
Hawthorn's ditto, with Book		0	12	0
Box Plotting Scales, 12 inches		0	4	0
Ditto Offsets		0	1	6
Ditto Architect's Scale		0	6	0
Ditto Pocket Rules	2s. 6d. to	0	14	0
Ebony Parallel Rules	per inch	0	0	4
Rolling Parallel Rules	do.	0	1	0
Ditto ditto, with Ivory Edges	do.	0	1	8
Steel Straight Edges	per foot	0	5	0
Lance-wood T-Squares	do.	0	3	0
Ebony T-Squares	do.	0	4	0
Computing Scales		0	16	0
Drawing Boards, Tachet's Patent	13s. to	1	2	0
Triangles, in Metal, Wood, and Ivory				
French Curves				
Shipwright's Curves		1	15	0
Centrolinead, small size		1	8	0
Ditto, large size		1	11	6
Sets of Radii Curves	£1 1 0 to	5	5	0
Trammels	£2 12 6 to	5	5	0
Beam Compasses	£1 0 0 to	5	5	0
Lalanne's Glass Slide Rule and Book		0	5	0

Scales, Rules, and Tapes, made to all Foreign Measures, also English and Foreign Standard Measures.

TELESCOPES, MICROSCOPES, &c.

		£	s.	d.
Twelve-inch Achromatic Telescope	15s. to	1	11	6
Eighteen-inch ditto	21s. to	2	2	0
Twenty-four-inch ditto	42s. to	3	3	0
Thirty-six-inch ditto	84s. to	5	5	0
Ditto in case, with Brass Stand		8	8	0
Twelve-inch Portable Acromatic Telescope		1	11	6
Eighteen-inch ditto ditto		2	2	0
Twenty-four-inch ditto ditto		2	12	6
Twelve-inch Portable Achromatic Telescope, German Silver.		2	2	0
Eighteen-inch ditto ditto ditto		2	12	6
Twenty-four-inch ditto ditto ditto		3	3	0
Thirty-inch ditto ditto ditto		4	4	0
Military Telescope, in Leather Case and Strap		3	13	6
Ditto Ordnance Pattern		2	10	0
Naval Telescope	from 24s. to	6	6	0
Thirty-six-inch Achromatic Telescope, on Stand, with Astronomical Eye-pieces		12	12	0
Forty-five-inch ditto, ditto ditto, 2¾ aperture		25	4	0
Ditto ditto ditto, 3¼ aperture		38	0	0
Ditto ditto ditto, 4-in. aperture		75	0	0
Powerful Double Race or Reconnoitring Glass much used as a Night Telescope	£3 13s. 6d. to	4	14	6
Gardner's Microscope		0	4	0
Compound ditto		1	1	0
Ditto ditto with Achromatic Lenses	£5 5s. to	30	0	0
Medical ditto	£4 4s. to	10	10	0
Solar ditto	£6 6s. to	20	0	0
Stereoscopes, with Diagrams	£1 1s. to	5	5	0

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Spectacles, mounted in Gold, Silver, Steel, and Shell, with Glasses, Pebbles, or Chamblant Glasses, which are highly approved of for Spectacles, Reading Glasses, &c., as they overcome all spherical aberration.
